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Theoretical Analysis of Effects of Confining Pressure on the Stress Intensity Factors for Cracked Brazilian Disk

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Abstract: In order to investigate effects of confining pressure on the stress intensity factors for a cracked Brazilian disk, the closed-form expressions of the stress intensity factors were obtained by using the weight function method in fracture mechanics for the cracked Brazilian disk loaded by the confining pressure, and then the explicit expressions of the stress intensity factors were achieved under both the confining pressure and diametrical forces loading condition. Based on the formulas of the stress intensity factors, effects of confining pressure on stress intensity factors of the cracked Brazilian disk was analyzed. The analyzed results show that the confining pressure has no effects on the mode II stress intensity factor, however, mode I stress intensity factors decrease with the increasing confining pressure. In addition, effects of confining pressure on the loading condition of pure mode II crack were also investigated. The analyzed results show that the critical loading angle for pure mode II crack decreases gradually to 0 degree with the increasing confining pressure.

Key Words: cracked Brazilian disk, stress intensity factor, confining pressure, critical loading angle

1. Introduction

Fracture toughness is an important parameter to describe the mechanical behavior of brittle materials and fracture toughness test is one of the main tasks of the fracture mechanics. How to accurately test fracture toughness of brittle materials has been studied by many researchers. It is very important to select a Suitable specimen for performing fracture toughness tests. The cracked Brazilian disk has been widely used for fracture toughness test of brittle materials as its closed-form solutions of the stress intensity factors can be achieved [1,2], and it can be easily achieved pure mode I, pure mode II and mixed-mode (I-II). Liu et al.[3] used the cracked Brazilian disk to measure the fracture toughness of fiber-reinforced composite material. Dong et al.[4,5] conducted the Brazilian disk test to investigate the static and dynamic fracture mechanical properties of PMMA. Nasseri et al.[6] conducted the Brazilian disk test to study the anisotropy of granite fracture toughness. However, most of the fracture toughness tests were conducted in a situation without confining pressure by now. This is not consistent with the actual working condition of some materials. As we know, many practical engineering structure materials serve in situation with confining pressure, such as underground rocks, structures in the deep water and so on. However, the material mechanical properties under the action of confining pressure are different from the properties under without confining pressure [7]. In order to investigate effects of confining pressure on the fracture toughness of brittle materials, many researchers have performed the fracture toughness experiments under the confining pressure loading conditions. But there were some unsolved problems in their experimental principle. For example, Al-Shayea et al.[8] have studied the effects of confining pressure on fracture toughness of a limestone rock using straight notched Brazilian disk specimens and hydraulic fracturing technique. They calculated rock mode I and mode II stress intensity factors by using the approximation formulas proposed by Atkinson et al.[1] and the variation of fracture toughness was found to be linearly proportional to confining pressure from 0 to 28MPa. In addition, they compared their experimental results with some similar results proposed by other researchers and found that the tendency of fracture toughness increased with the increasing confining pressure was basically consistent but the corresponding relation of fracture toughness for different rocks was not the same, and some relationship were nonlinear. It is necessary to point out that this method belongs to Brazilian disk experiment under both confining pressure and diametrical forces loading condition. The validity and reliability of the results obtained by using the stress intensity factor formulas which only consider concentrated load regardless of confining pressure to calculate the stress intensity factors(or fracture toughness) under both the confining pressure and diametrical forces loading condition are debatable. And, whether or not the critical loading condition (or critical loading angle) for pure mode II crack under both the confining pressure and diametrical forces is the same with the critical loading condition (or critical loading angle) for pure mode II crack under diametrical forces has to be discussed. Lou et al.[9] designed a test system of the mode I and mode II fracture toughness for rocks using Brazilian disk specimens and hydraulic fracturing technique on MTS multifunctional material testing machine, and calculated the mode I and mode II stress intensity factors by using the approximation formulas proposed by Atkinson et al.[1]. They analyzed the effects of specimen size, confining pressure and other factors on the mode I and mode II fracture toughness of rocks according to the experimental data. The results showed that: both of the mode I and mode II fracture toughness increased with the increasing confining pressure in a certain pressure range, but the trend was not obvious under high confining pressure. Their experiment principle and calculation formula of stress intensity factors are similar to Al-Shayea’s method and formula [8], and have similar problems.

Therefore, how to evaluate effects of confining pressure on the stress intensity factors and critical loading angle for pure mode II crack is an important problem, and is worth of further investigating. It will provide the theoretical foundations for conducting fracture toughness tests under confining pressure. In the present paper, based on the formula of the stress intensity factors for central cracked circular disk subjected to diametrical forces, the closed-form expressions of the stress intensity factors for central cracked circular disk subjected to confining pressure was obtained firstly and then the explicit expressions of the stress intensity factors were achieved under both the confining pressure and diametrical forces loading condition. Finally, the effects of confining pressure on the stress intensity factors and critical loading angle was analyzed.

2. Stress intensity factors for Cracked Brazilian Disk under confining pressure

2.1 Stress components for uncracked Brazilian Disk under confining pressure

The schematic diagram of an uncracked Brazilian disk under confining pressure is shown in Fig.1. The thickness of the disk is B and the radius is R .

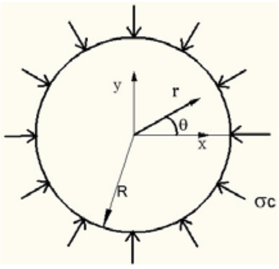


Fig.1 Uncracked Disk under confining pressure

The disk is assumed to be made of isotropic elastic material and obey the small deformation hypothesis. The stress distributions in the disk relative to a polar coordinate system can be obtained based on the elasticity theory [10] as

$$\sigma_r = -\sigma_c \ , \sigma_\theta = -\sigma_c \ , \ \tau_{r\theta} = 0 \qquad (1)$$

2.2 Stress intensity factors for cracked disk under confining pressure

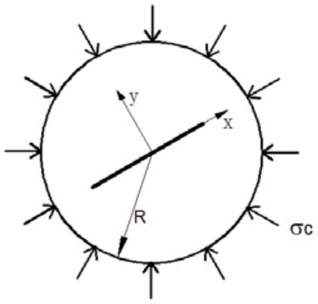


Fig.2 Cracked Brazilian Disk under confining pressure loading

The weight function method is used to calculate the stress intensity factors for the cracked disk. The cracked Brazilian disk subjected to confining pressure σ_c is shown in Fig.2. The x-axis lies in the crack plane. The coordinate origin is located at the center of the disk. a is the half length of the crack, R is the disk radius and B is the thickness. Then the $K_I^{'}$ and $K_{II}^{'}$ can be given as

$$K_{\mathrm{I}}^{'} = \int_0^a h_1(x,a)\sigma_\theta(x)dx \qquad (2)$$

$$K_{\mathrm{II}}^{'} = \int_0^a h_2(x,a)\tau_{r\theta}(x)dx \qquad (3)$$

where h_1 and h_2 are weight functions, $\sigma_\theta, \tau_{r\theta}$ are the stress components of the uncracked Brazilian disk under confining pressure. Specifically, h_1 and h_2 can be expressed as [11]:

$$h_1(x,a) = \frac{2}{\sqrt{\pi a}}[\frac{1}{\sqrt{1-\rho_1^2}} + c_{11}\sqrt{1-\rho_1^2} + c_{12}(1-\rho_1^2)^3] \qquad (4)$$

$$h_2(x,a) = \frac{2}{\sqrt{\pi a}}[\frac{1}{\sqrt{1-\rho_1^2}} + c_{21}\sqrt{1-\rho_1^2} + c_{22}(\sqrt{1-\rho_1^2})^3] \qquad (5)$$

where

$$c_{11} = \frac{8-4\alpha+3.8612\alpha^2-15.9344\alpha^3+24.6076\alpha^4-13.234\alpha^5}{\sqrt{1-\alpha}} - 8 \qquad (6)$$

$$c_{12} = \frac{-8+4\alpha-0.6488\alpha^2+14.1232\alpha^3-24.2696\alpha^4+12.596\alpha^5}{\sqrt{1-\alpha}} + 8 \qquad (7)$$

$$c_{21} = \frac{5-2.5\alpha+1.4882\alpha^2-2.376\alpha^3+1.1028\alpha^4}{\sqrt{1-\alpha}} - 5 \qquad (8)$$

$$c_{22} = \frac{-4+2\alpha+0.4888\alpha^2+0.81112\alpha^3-0.7177\alpha^4}{\sqrt{1-\alpha}} + 4 \qquad (9)$$

$(\alpha = a/R, \rho_1 = x/a)$

By substituting Eqs.(1) and (4) into Eq.(2), respectively, the following explicit formula of the stress intensity factors can be yielded as

$$K_{\mathrm{I}}^{'} = -\sigma_c \sqrt{\pi a}\, f_{11} \tag{10}$$

where $f_{11} = 1 + \frac{1}{2}c_{11} + \frac{3}{8}c_{12}$. It is clear, for $\tau_{r\theta} = 0$, $K_{\mathrm{II}}^{'} = 0$.

2.3 Stress intensity factors under both confining pressure and diametrical forces loading condition

The cracked disk subjected to a pair of diametral-compressive force P and ambient confining pressure σ_c is shown in Fig.3. a is the half length of the crack, the disk radius is R and thickness is B, θ is the loading angle (the angle of inclination of the crack relative to the line of loading). The stress intensity factors for cracked Brazilian disk subjected to a pair of diametral-compressive P have been given as [2]

$$K_{\mathrm{I}}^{''} = \sigma\sqrt{\pi a}[f_{11} + 2\sum_{i=1}^{n} A_{1i} f_{1i} \alpha^{2(i-1)}] \tag{11}$$

$$K_{\mathrm{II}}^{''} = 2\sigma\sqrt{\pi a}\sum_{i=1}^{n} A_{2i} f_{2i} \alpha^{2(i-1)} \tag{12}$$

where $\sigma = \frac{P}{\pi BR}$, coefficients f_{ji} and angle coefficients A_{ji} (j=1,2; i=1,2,…...n) are shown in the literature[2].

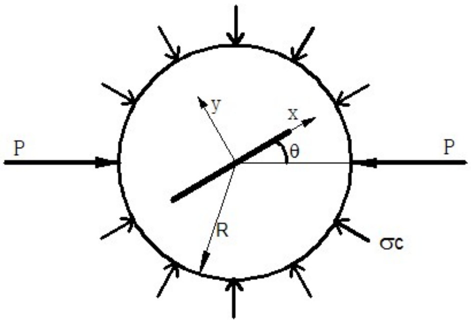


Fig.3 Cracked Brazilian Disk under both confining pressure and diametrical forces loading

Now, based on the diametral-compressive P load, the confining pressure (σ_c) was applied. By employing the superposition principle of stress intensity factors [12], the stress intensity factors for cracked Brazilian disk under both confining pressure σ_c and diametrical forces P can be obtained

$$\begin{aligned} K_{\mathrm{I}} &= K_{\mathrm{I}}^{'} + K_{\mathrm{I}}^{''} \\ &= -\sigma_c\sqrt{\pi a}\, f_{11} + \sigma\sqrt{\pi a}[f_{11} + 2\sum_{i=1}^{n} A_{1i} f_{1i}\alpha^{2(i-1)}] \\ &= (\sigma - \sigma_c)\sqrt{\pi a}\, f_{11} + 2\sigma\sqrt{\pi a}\sum_{i=1}^{n} A_{1i} f_{1i}\alpha^{2(i-1)} \end{aligned} \tag{13}$$

By letting $\sigma_c / \sigma = t$, then $\sigma_c = t\sigma$. Where t is a dimensionless scaling factor (simply called confining pressure coefficient) and different confining pressure coefficients are correspond to different confining pressures. Then K_I can be written as

$$K_{\mathrm{I}} = \sigma(1-t)\sqrt{\pi a}\, f_{11} + 2\sigma\sqrt{\pi a}\sum_{i=1}^{n} A_{1i} f_{1i}\alpha^{2(i-1)} \tag{14}$$

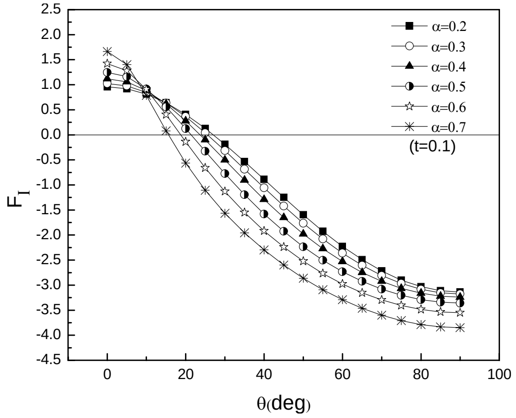
The normalized stress intensity factor, F_I may be written as

$$F_{\mathrm{I}} = \frac{K_{\mathrm{I}}}{\sigma\sqrt{\pi a}} = (1-t)\, f_{11} + 2\sum_{i=1}^{n} A_{1i} f_{1i}\alpha^{2(i-1)} \tag{15}$$

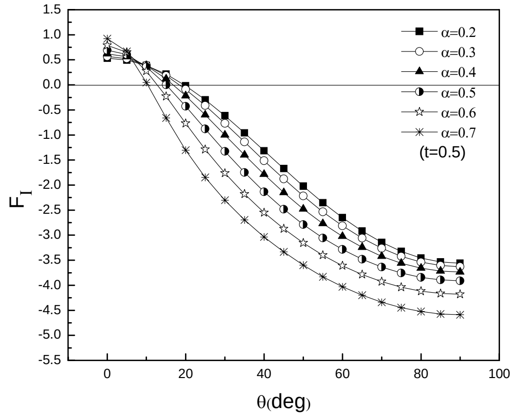
For $K_{II}^{'}=0$, $K_{II}=K_{II}^{'}+K_{II}^{''}=K_{II}^{''}$, which means that confining pressure σ_c has no effects on mode II stress intensity factor. Eq. (12) can still be used to calculate mode II stress intensity factors under both confining pressure and diametrical forces loading condition.

3. Effects of confining pressure on stress intensity factors

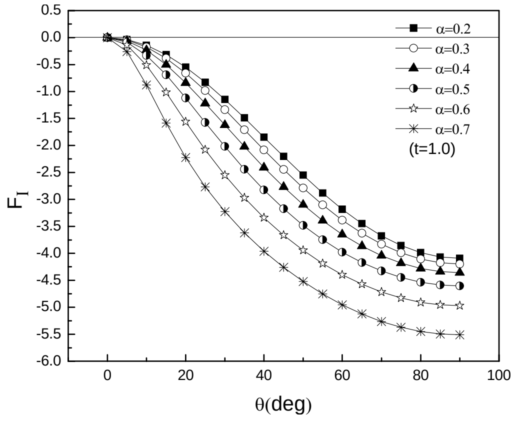
To investigate the effects of the confining pressure on the stress intensity factors, according to Eq. (15), we take t=0. 1、0. 5、1. 0, respectively, and calculate the mode I stress intensity factors F_I for different relative crack length α and loading angle θ. The calculated results are shown in Fig.4.



(a) t=0.1



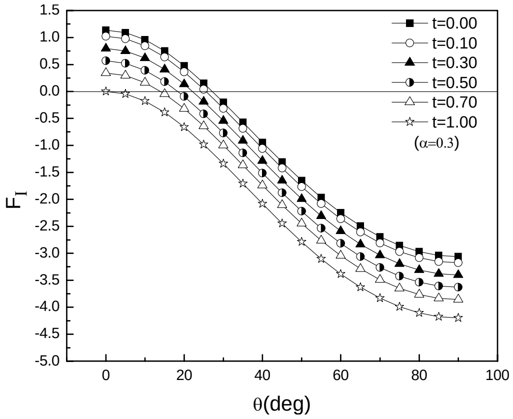
(b) t=0.5



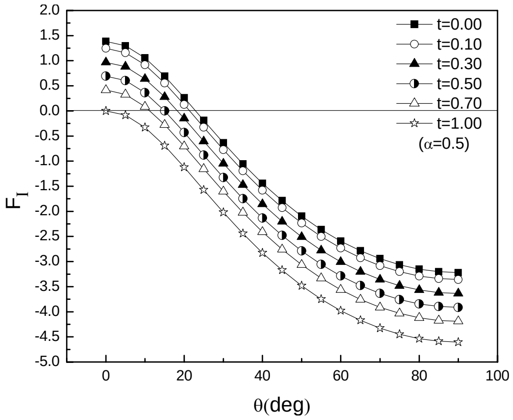
(c) t=1.0

Fig.4 Normalized stress intensity factors F_{I} for different confining pressures

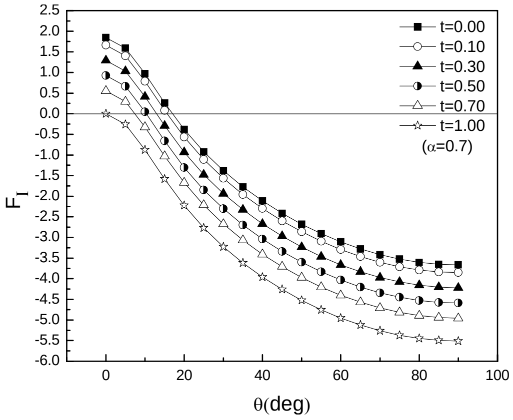
In addition, in order to further analyze the effects of confining pressure on the stress intensity factors, taking α =0.3、0.5、0.7 as example, we calculate the mode I stress intensity factors F_I for fixed relative crack length but different confining pressures. Results are shown in Fig.5.



(a) $\alpha = 0.3$



(b) $\alpha = 0.5$



(c) $\alpha = 0.7$

Fig.5 Effects of confining pressure on stress intensity factors for a fixed relative crack length

The Fig.4 and Fig.5 show that confining pressure significantly influences the mode I stress intensity factors for cracked Brazilian disk. The mode I stress intensity factors decrease with the increasing confining pressure at any relative crack length. When the confining pressure coefficient (t) is less than 1, the mode I stress intensity factors for cracked Brazilian disk always change from positive to negative with the increasing the loading angle. However, the mode I stress intensity factors will always be negative values for any relative crack length while the confining pressure coefficient t>1. Due to the mode I stress intensity factors are negative, crack will be closed, and hence, we cannot test the pure mode I fracture toughness when the confining pressure coefficient t>1.

4. Effects of confining pressure on pure mode II crack loading condition

According to the definition of the pure mode I loading and pure mode II loading, the pure mode II crack can be determined from the conditions that $K_{\mathrm{I}} = 0$, $K_{\mathrm{II}} \neq 0$. By letting $K_{\mathrm{I}} = 0$ in Eq.(14), we get

$$(1-t) f_{11} + 2\sum_{i=1}^{n} A_{1i} f_{1i} \alpha^{2(i-1)} = 0 \qquad (16)$$

It's not easy to directly achieve the closed-form solutions from Eq. (16), but the solution is $\theta = \theta(t, \alpha)$ which means that the pure mode II loading angle is the function of α and t. According to Eq. (16), we develop a small program to compute the critical loading angle θ_c for various α and different coefficients t, the results are listed in Table 1.

Tab.1 the critical loading angles θ_c (°) for pure mode II crack with different confining pressures

α	θ_c (°)						
	t=0[2]	**t=0.1**	**t=0.3**	**t=0.5**	**t=0.7**	**t=1.0**	**t>1**
0	**30.00**	**28.32**	**24.73**	**20.70**	**15.89**	**0**	No real solution
0.1	**29.67**	**28.00**	**24.43**	**20.44**	**15.67**	**0**	No real solution
0.2	**28.72**	**27.06**	**23.56**	**19.66**	**15.05**	**0**	No real solution
0.3	**27.23**	**25.60**	**22.20**	**18.45**	**14.07**	**0**	No real solution
0.4	**25.27**	**23.70**	**20.43**	**16.89**	**12.81**	**0**	No real solution
0.5	**22.93**	**21.40**	**18.30**	**15.02**	**11.31**	**0**	No real solution
0.6	**20.18**	**18.72**	**15.82**	**12.84**	**9.58**	**0**	No real solution
0.7	**16.96**	**15.57**	**12.92**	**10.33**	**7.60**	**0**	No real solution
0.8	**13.07**	**11.79**	**9.49**	**7.41**	**5.35**	**0**	No real solution
0.9	**8.17**	**7.05**	**5.32**	**3.99**	**2.81**	**0**	No real solution

From Table 1 we can know that the confining pressure significantly influences the loading condition of pure mode II crack. As the confining pressure increases, the critical loading angle for pure mode II crack decreases gradually to 0 degree.

5. Discussion

5.1 Mode I fracture toughness under confining pressure

According to the experiment results in the literature [8], the fracture toughness increases with the increasing confining pressure (Fig.6). Al-Shayea et al.[8] used the five-term approximation formula proposed by Atkinson et al.[1] to calculate the mode I stress intensity factors, which is equivalently letting n=5 in Eq.(11). They ignored the effects of confining pressure on stress intensity factors; therefore, the failure load $P_{\max}$ increased with the increasing confining pressure and thus the test values of the fracture toughness increased with the confining pressure. However, according to Eq.(14) presented in this paper, mode I stress intensity factors decrease with the increasing confining pressure. Under confining pressure loading condition, though the test values of the failure load increase, but part of the load has to offset the ‘negative effect’ caused by confining pressure on K_I. So according to Eq.(14), though the failure load show an increase with the increasing confining pressure, the fracture toughness may be unchanged actually.

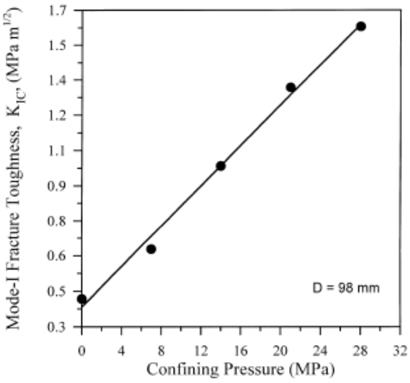


Fig. 6 Variation of mode-I fracture toughness under confining pressure **[8]**

5.2 Mode II fracture toughness under confining pressure loading

For loading angle $\theta = 0°$, we usually think that materials will produce the pure mode I fracture. Now, this viewpoint may be reasonable only under without or low confining pressure loading conditions, the fracture mode of materials may be pure mode II fracture around $0°$ loading angle under high confining pressure. On the other hand, according to the literature [2], for $\theta = 0°$, $K_{II} \equiv 0$, so the failure of the specimen is no longer pure fracture mechanics problem when the confining pressure coefficient $t \geq 1$.

In the literature [8], the critical loading angle of the pure mode II crack was calculated by using the five-term approximation formula proposed by Atkinson et al.[1] which ignored the effects of confining pressure. For example, when α=0.3, they used $\theta_c = 29°$. However, according to the present paper, the critical loading angle for pure mode II crack decreases with the increasing confining pressure.

6. Conclusion

(1) The stress intensity factors are obtained by using the weight function method in fracture mechanics for the cracked Brazilian disk loaded by the confining pressure, and then the explicit expressions of the stress intensity factors are achieved under both the confining pressure and diametrical forces loading condition, which can be used to calculate the stress intensity factors for any relative crack length and any confining pressure.

(2) Analyzed results show that confining pressure significantly influences the mode I stress intensity factors for the cracked Brazilian disk. The mode I stress intensity factors decrease with the increasing confining pressure, however, the confining pressure has no effects on the mode II stress intensity factors.

(3) Calculation results show that confining pressure significantly influences the loading condition for the pure mode II crack: as the confining pressure increases, the critical loading angle for pure mode II crack decreases gradually to 0 degree.

(4) The fracture mode of materials may be pure mode II around $0°$ loading angle under high confining pressure loading.

Acknowledgements

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