Crack Growth-based Fatigue Life Prediction Using Spline Fictitious Boundary Element Method

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Abstract Fatigue life prediction is of great importance to the design and maintenance of structural components. A boundary element method (BEM)-based approach is proposed in this paper for fatigue life prediction using crack growth analysis. The proposed methodology is based on the well-known Paris equation for fatigue crack growth rate, which is related to the amplitude of the stress intensity factor (SIF) as a crack grows. The SIF is determined by the fracture spline fictitious boundary element method (SFBEM) based on the Erdogan fundamental solutions for plane cracked problems. The fusion of SFBEM and the Erdogan fundamental solutions is computationally efficient and provides a powerful tool for crack growth-based fatigue life prediction. A numerical example based on the mode-I crack problem is presented to validate the present method. The results show that the predicted fatigue life obtained by the present approach is accurate in comparison with the analytic solution.

Keywords Fatigue crack growth, life prediction, fracture mechanics, spline fictitious boundary element method

1. Introduction

According to a survey conducted by the ASCE Committee on Fatigue and Fracture Reliability [1], fatigue is the main reason that causes the failure in steel structures. Therefore, fatigue life prediction is an important task for the design and maintenance planning of structures. In general, there are two major types of approaches to predict the fatigue life [2]. The first is based on *S*-*N* curves combined with a damage accumulation rule. The second is based on the fracture mechanics and crack growth analysis. Generally, these two approaches are used sequentially. The one with *S*-*N* curves is used at the 'design' stage, and the one with fracture mechanics is used at the 'assessment' stage for existing structures [3]. From the point of view that initial flaws inevitably exist in engineering materials, the crack growth analysis based on fracture mechanics may be more suitable for refined fatigue life prediction of structural components.

Crack growth theories have formed the bridge that links fatigue and fracture mechanics concepts [4]. The most important contribution is the establishment of the relationships between the crack growth rate $\mathrm{d}a/\mathrm{d}N$ and the stress intensity factor (SIF). The most widely used fatigue crack growth model, commonly known as Paris law, was proposed by Paris and Erdogan [5]. The Paris law connects the crack growth rate with the amplitude of SIF through a simple power function, which makes the engineering application more easily. After that, various modifications and extensions to Paris law have emerged, and different forms of modified crack growth equations have been offered by Forman [6], Elber [7] and Walker [8], et al.

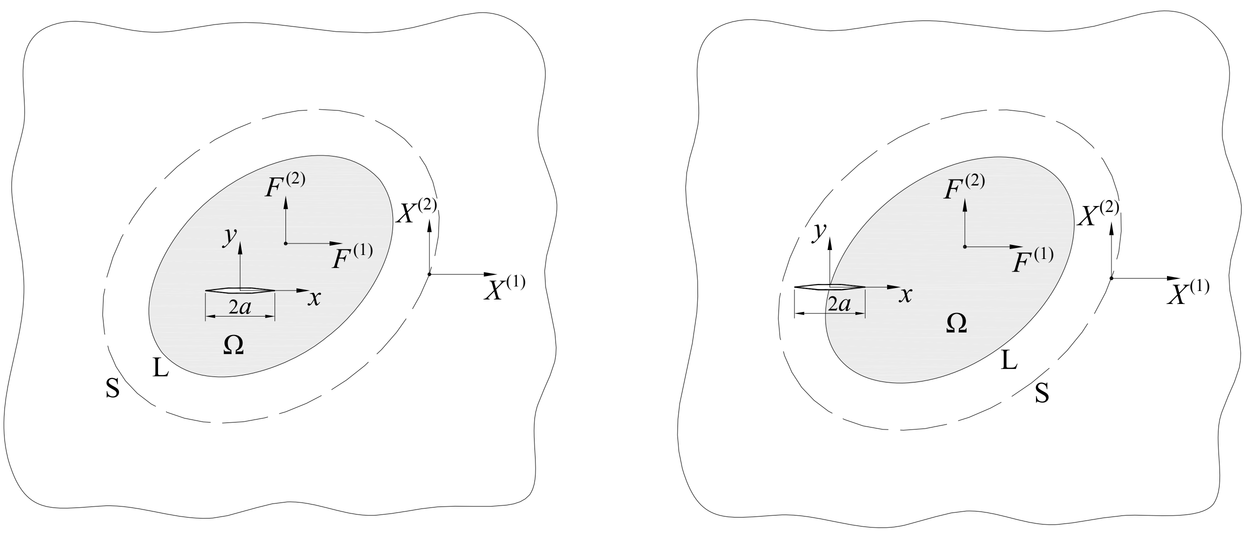
Another important task in the crack growth-based fatigue life prediction is the fracture analysis. Since few analytical solutions to SIFs are available, especially for engineering structures, numerical methods are widely employed. The boundary element method (BEM) is one of the most frequently used numerical methods. Its high precision and efficiency make it particularly suitable for fracture analysis.

In this paper, the Paris law is used to predict the crack growth-based fatigue life. To calculate the SIF, an efficient indirect boundary element method (IBEM), the spline fictitious boundary element method (SFBEM) [9-16], is adopted to perform the fracture analysis. Numerical examples are presented to illustrate the application of the proposed method.

2. Fracture analysis by SFBEM

As a modified IBEM, in SFBEM, nonsingular integral equations are derived rather than singular ones; spline functions with excellent performance are adopted as the trial functions to the unknown fictitious loads; and the boundary-segment-least-square technique is employed for eliminating the boundary residues. Because of these modifications, SFBEM is of high accuracy and efficiency in general. SFBEM was first applied to the solution of static plane elasticity problems [9], and so far it has been extended to multi-domain plane problems [10], orthotropic plane problems [11], plate bending problems [12], elastic fracture problems [13], stochastic elastostatic problems [14, 15] and probabilistic fracture mechanics [16].

In this study, a SFBEM based on the Erdogan fundamental solutions for infinite cracked plates [13, 16] is employed to conduct fracture analysis of linear-elastic cracked structures. As the Erdogan fundamental solutions [17, 18] are derived from an infinite plate containing a crack, when they are used in the formulation of BEM, the stress boundary conditions on the crack surface are automatically satisfied, and the singular behavior at the crack tip can be naturally captured. Therefore, no boundary elements are required to place along the crack surface. In addition, the SIF of the crack problem can be calculated directly from the corresponding fundamental solution of SIF, with no need of transformation from the displacement field around the crack tip, as is normally required in SIF analysis by the other numerical methods. The SFBEM in combination with the Erdogan fundamental solutions has been shown to be more computationally accurate and efficient and thus provides a powerful tool for fracture analysis.



(a) Inner crack (b) Edge crack

Figure 1. Plane domain embedded in an infinite plane with a crack

Consider an elastic plane domain with a crack, as shown in Fig.1. Let the domain studied be Ω, and assume the configuration boundary of the domain to be L, not including the crack surface. The body forces within the domain are assumed to be $F^{(l)}$ (l=1, 2), and the lengths of the inner crack and the edge crack are taken to be $2a$ and a, as shown in Fig.1(a) and Fig.1(b), respectively. Embed Ω into an infinite plane domain with a crack, the crack length being $2a$, and apply unknown fictitious loads $X^{(l)}$ (l=1, 2) along a fictitious boundary S outside Ω, whose shape is similar to that of the real boundary L, as also shown in Fig.1.

Due to the use of the Erdogan fundamental solutions [18], not only the governing differential equations within Ω but also the stress boundary conditions on the crack surface are satisfied automatically. Therefore, only the boundary conditions along the contour of Ω need to be considered. Under the combined action of the body force $F^{(l)}$ and the fictitious loads $X^{(l)}$ (l=1, 2), the boundary conditions along L can be expressed as

$$\sum_{l=1}^{2}\int_{\mathrm{S}} G_k^{(l)}\left(z_{\mathrm{L}};z_{\mathrm{S}}\right)X^{(l)}\left(z_{\mathrm{S}}\right)\mathrm{d}s+\sum_{l=1}^{2}\iint_{\Omega} G_k^{(l)}\left(z_{\mathrm{L}};z_{\Omega}\right)F^{(l)}\left(z_{\Omega}\right)\mathrm{d}V=H_k\left(z_{\mathrm{L}}\right)\qquad\left(k=1,2\right),\tag{1}$$

where $z_{\mathrm{L}}\in\mathrm{L}$, $z_{\mathrm{S}}\in\mathrm{S}$, $z_{\Omega}\in\Omega$; k=1,2 denotes that two boundary conditions exist along L for plane problems; H_k are the known boundary functions along L; and $G_k^{(l)}$ are the kernel functions consisting of the Erdogan fundamental solutions.

Eq. (1) are nonsingular fictitious boundary integral equations because the source points will never coincide with the field points in the kernel functions. However, analytic solutions to Eq. (1) are normally not available, and the integral equations should be solved on a numerical baisis. For this purpose, the unknown fictitious loads $X^{(l)}$ are expressed in terms of a set of B-spline functions, and the boundary-segment technique is used to eliminate the resulting boundary residues [10]. Then Eq. (1) turns into the following numerical equation as

$$\left[A\right]\left\{X\right\}+\left\{B\right\}=\left\{C\right\},\tag{2}$$

where $\left\{X\right\}$ is the column matrix consisting of the unknown spline node parameters of the fictitious loads along S; $\left[A\right]$ is influence matrix of $\left\{X\right\}$; and $\left\{B\right\}$ and $\left\{C\right\}$ are the known column matrices depending on the body forces $F^{(l)}$ within Ω and the boundary condition functions H_k along L, respectively. Usually Eq. (2) needs to be solved on a least-squares basis as generally overdeterminate collocation is conducted to achieve a better solution with more boundary segments while keeping the number of fictitious boundary elements to be at a lower level.

Once the spline node parameter $\left\{X\right\}$ is determined, the mode-I and II SIFs of the cracked problem can be obtained from the discrete forms of the following equations:

$$K_j=\sum_{l=1}^{2}\int_{\mathrm{S}} K_j^{(l)}\left(z_{\mathrm{S}}\right)X^{(l)}\left(z_{\mathrm{S}}\right)\mathrm{d}s+\sum_{l=1}^{2}\iint_{\Omega} K_j^{(l)}\left(z_{\Omega}\right)F^{(l)}\left(z_{\Omega}\right)\mathrm{d}V\qquad\left(j=\mathrm{I},\mathrm{II}\right),\tag{3}$$

where $K_j^{(l)}$ (j=I,II; l=1,2) are the Erdogan fundamental solutions of SIFs [18].

3. Crack growth-based fatigue life prediction

3.1. Paris law

Crack growth can occur under cyclic loading. Using Paris equation, the crack growth rate can be expressed as the function of the SIF range, and can be written as

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\left(\Delta K\right)^{m} \qquad \left(\Delta K > \Delta K_{\mathrm{th}}\right), \tag{4}$$

where a is the crack size; N is the number of cycles of the alternating stress; $\Delta K=K(\sigma_{\max})-K(\sigma_{\min})$ is the SIF range; ΔK_{th} is the fatigue threshold, which means if $\Delta K \leq \Delta K_{\mathrm{th}}$, the crack is assumed to be non-propagating; C and m are the material constants obtained from experiments.

By integrating Eq. (4), the crack growth-based fatigue life can be obtained as

$$N_{\mathrm{p}} = \int_{a_0}^{a_{\mathrm{c}}} \frac{\mathrm{d}a}{C\left(\Delta K\right)^{m}}, \tag{5}$$

where a_0 is the initial crack size; a_{c} is the critical crack size at fatigue failure and can be determined using the fracture toughness K_{Ic}.

Generally, the explicit solutions to SIFs for most engineering problems are not available. Therefore, numerical approaches are required for fatigue life prediction when using Eq. (5).

3.2. Fatigue life prediction using SFBEM

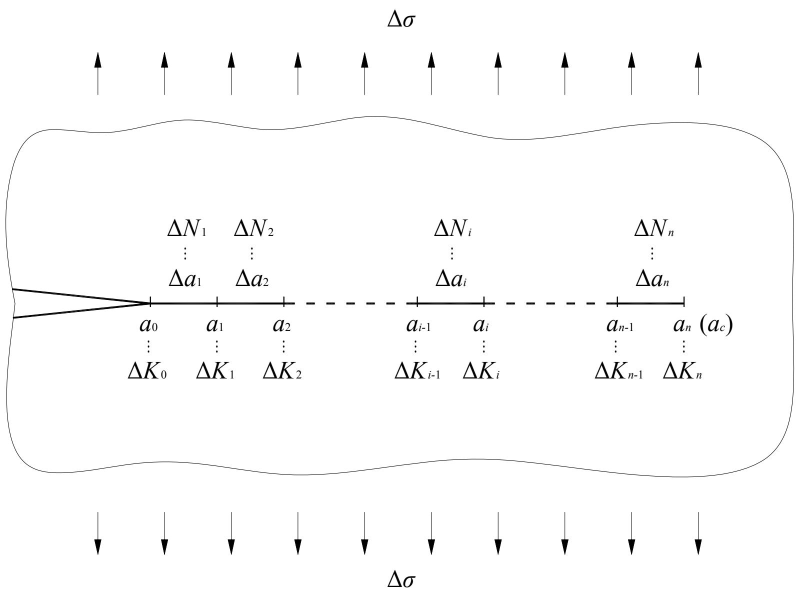


Figure 2. Propagation of a crack

The propagation of a crack from the initial crack size a_0 to the critical crack size a_{c} can be illustrated in Fig.2. The number of cycles of the alternating stress corresponding to the ith step of the crack growth can be approximately expressed as

$$\Delta N_i = \frac{\Delta a_i}{\overline{\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_i}}, \tag{6}$$

where Δa_i is the crack growth size of the ith step; $\overline{\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_i}$ is the average crack growth rate during the current step and can be determined using the Paris equation, that is

$$\overline{\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_i} = C\left(\overline{\Delta K_i}\right)^{m}. \tag{7}$$

In the above equation, $\overline{\Delta K_i}$ is the average SIF range of the ith step and can be defined as

$$\overline{\Delta K_i} = \left(\Delta K_{i-1} + \Delta K_i\right)/2\,, \tag{8}$$

where ΔK_{i-1} and ΔK_i are the amplitudes of SIF corresponding to a_{i-1} and a_i, respectively, and can be determined using the SFBEM presented in section 2.

The procedure for fatigue life prediction based on SFBEM is as follows:

1. Determine the initial SIF range ΔK_0 corresponding to the given initial crack size a_0 using the SFBEM in conjunction with the Erdogan fundamental solutions.

2. Check if $\Delta K_0 > \Delta K_{th}$. If yes, the propagation of the crack will occur.

3. Assume the crack growth size of the ith step to be $\Delta a_i = \eta a_{i-1}$ (i=1,2,…), in which η=0.1~0.01. Then the crack size of the ith step is $a_i = a_0 + \sum_{j=1}^{i} \Delta a_j$.

4. Determine the SIF range ΔK_i corresponding to a_i using SFBEM.

5. Calculate the average SIF range of the ith step $\overline{\Delta K_i}$ using Eq. (8).

6. Calculate the number of cycles of the alternating stress of the ith step ΔN_i using Eqs. (6) and (7).

7. Check if $K_i(\sigma_{max}) < K_{Ic}$. If yes, go to step 3. If no, then stop. Assume the final step number is n. Then the fatigue life can be obtained as

$$N_p = \sum_{i=1}^{n} \Delta N_i\,. \tag{9}$$

4. Numerical examples

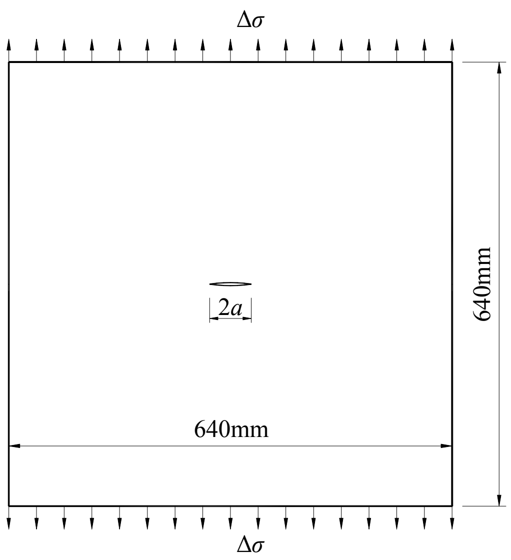


Figure 3. A square plate with a center crack

Fig.3 shows a square plate with a center crack subjected to a cyclic loading with $\Delta\sigma$=200MPa (σ_{max}=200MPa, σ_{min}=0). The fatigue threshold and the fracture toughness of the material are taken to be $\Delta K_{\rm th}$=5.5MPa·m$^{1/2}$ and $K_{\rm Ic}$=104MPa·m$^{1/2}$, respectively. The material constants C and m in the Paris equation are assumed to be C=6.9×10^{-12} m/cycle/(MPa·m$^{1/2}$)^3 and m=3, respectively. The initial crack size is taken to be a_0=0.5mm.

4.1. Analytic solution

Since the crack size is much smaller than the size of the plate, the SIF in this case can be approximately determined using the analytic solution for an infinite plate with a crack, that is

$$K = \sigma\sqrt{\pi a}\ . \tag{10}$$

It can be deduced from the above equation that

$$\Delta K_0 = \Delta\sigma\sqrt{\pi a_0} = 7.927\,\mathrm{MPa}\cdot\mathrm{m}^{\frac{1}{2}} > \Delta K_{\rm th} = 5.5\,\mathrm{MPa}\cdot\mathrm{m}^{\frac{1}{2}}, \tag{11}$$

and thus the crack propagation will occur.

From Eq. (10), we also have

$$a_{\rm c} = \frac{1}{\pi}\left(\frac{K_{\rm Ic}}{\sigma_{\max}}\right)^2 = 86.07\,\mathrm{mm}\ . \tag{12}$$

Therefore, the fatigue life can be obtained using Eq. (5) as follows:

$$N = \int_{a_0}^{a_{\rm c}} \frac{\mathrm{d}a}{C\left(\Delta K\right)^m} = \frac{1}{C\left(m/2-1\right)\left(\Delta\sigma\sqrt{\pi}\right)^m}\left(\frac{1}{a_0^{m/2-1}} - \frac{1}{a_{\rm c}^{m/2-1}}\right) = 2.688\times10^5\ \mathrm{cycle}\ . \tag{13}$$

4.2. Numerical solution

When using SFBEM to calculate the SIF of the plate, 16 fictitious boundary elements and 40 boundary segments are adopted, and the distance between the fictitious boundary and the real boundary is taken to be d=40mm. The value of the coefficient η in step 3 of section 3.2 is assumed to be 0.1. In the calculation, 53 steps of crack propagation have been involved in total. The results obtained by the present method are listed in Table 1. For the purpose of comparison, the results of the analytic solution are also presented in the table.

Table 1. Results of fatigue life prediction

Method	ΔK_0 (MPa·m$^{1/2}$)	$a_{\rm c}$ (mm)	$N_{\rm p}$ (cycle)
Present method	7.928	74.57	2.652×10^5
Analytic method	7.927	86.07	2.688×10^5

It can be seen from Table 1 that, for the fatigue life of the plate, the results obtained by both methods are in good agreement, while for the critical crack size, certain discrepancy occurs. Actually, as the crack size increases with propagation, the error of Eq. (10) corresponding to the case of infinite plate becomes larger, leading to the error of the critical crack size in the analytic solution, as shown in Fig.4. But as the crack size approaches the critical size, the number of cycles at this stage contributes much less to the fatigue life. Therefore, good agreement is still observed for the prediction of the fatigue life between the two methods, as can be seen from Fig.5.

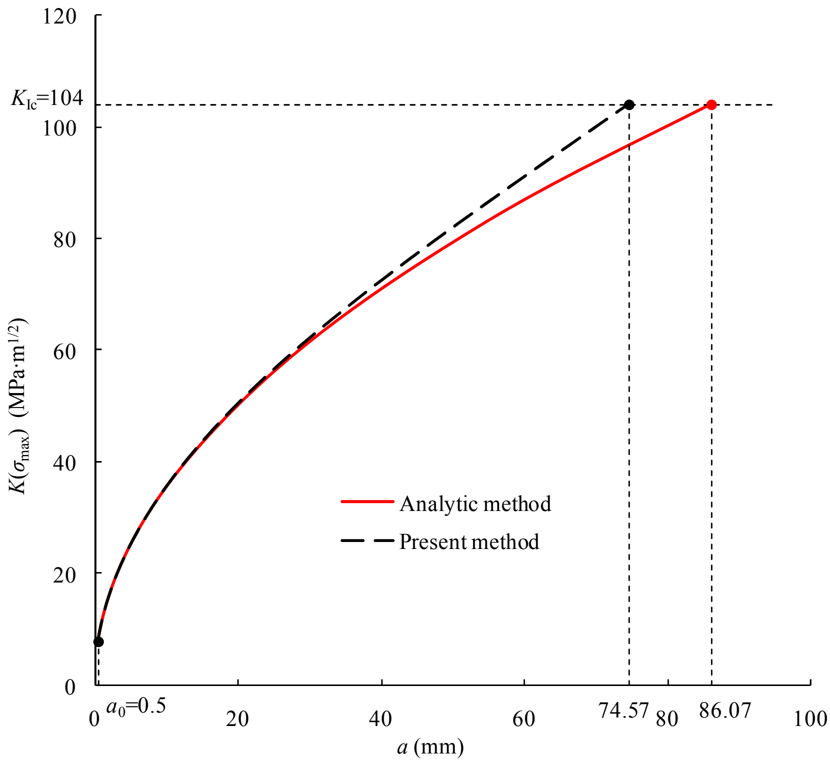


Figure 4. SIF versus the crack size

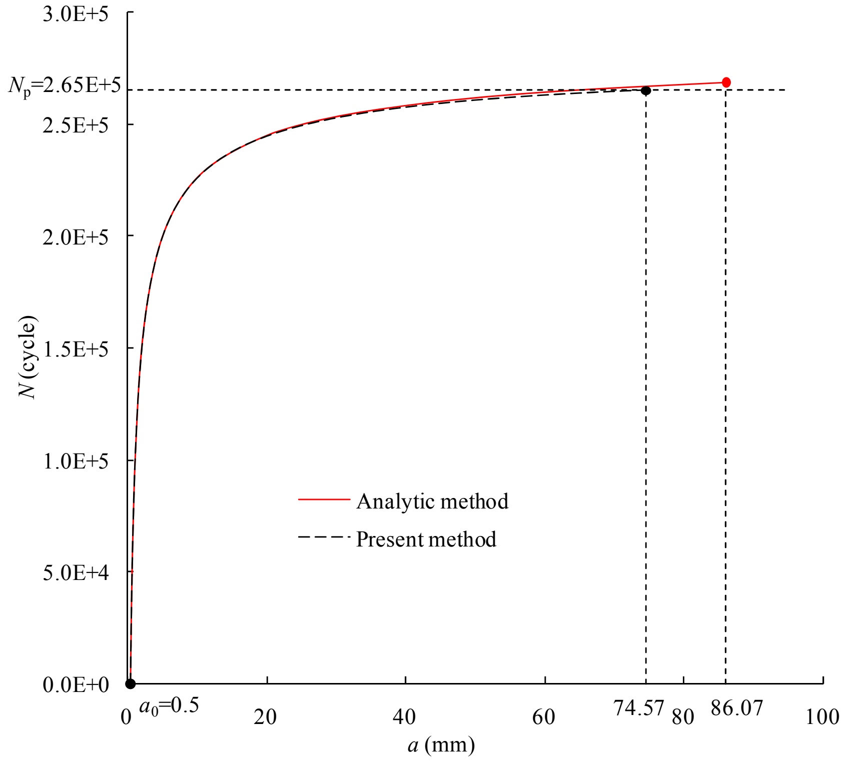


Figure 5. Number of cycles versus the crack size

5. Conclusions

The SFBEM based on the Erdogan fundamental solutions has high accuracy and efficiency for SIF analysis of linear-elastic cracked structures, and therefore can serve as an effective approach for crack growth-based fatigue life prediction using the Paris equation. A numerical example is presented to demonstrate the validity of the present method. The results show that the predicted fatigue life obtained by the present method agrees well with the analytic solution.

Acknowledgements

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