Based on cycle strain damage prediction of the crack growth behavior of elliptic surface crack

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Abstract According to the low cycle fatigue strain damage, a unit average damage parameter over the cyclic plastic zone along the crack extending direction is defined, and a new fatigue crack growth rate prediction model (LCF-FCGM) is established. Then combined with Newman-Raju formula, simulation of the fatigue crack growth problem of a half elliptic surface crack in tension plate under cyclic loading is fulfilled. The fatigue crack growth rate curve of Al 7075-T6 alloy predicted by LCF-FCGM agrees well with that obtained from the corresponding test. Assuming that the shape of the surface crack front keeps elliptic and is controlled by two critical points: the deepest point and the surface point, a numerical approach based on material's low cycle fatigue properties is developed to analyze the configuration evolution of surface cracks during fatigue crack growth. The new approach is well applied to Al 7075-T6 alloy surface cracked plate, and six different initiate crack shapes are discussed and all reflect the same phenomenon of round first to oval again, which are consistent with the test results.
Keywords fatigue crack propagation; low-cycle fatigue damage; Newman-Raju formula; elliptic surface crack; Al 7075-T6 alloy

1. Introduction

In engineering the crack is always in three-dimensional stress status, especially for the surface crack in pressure vessel pipe. So study on the surface crack growth problem has great importance. Because of the uncertainty of the crack front shape, it brings much difficulty on solving these nonpenetrating crack problems. Nowadays, there are three methods have been proposed: conservative estimate method, shape assuming method and crack shape tracking method. ASME X1(1997) assumed the crack shape parameter α=a/c is constant. And BSI PD6493（1980）assumed that the crack width c keeps still until the shape changes to round and the round shape kept till the end. Known as the representative of crack shape assuming method, Newman-Raju formula[1] (Newman and Raju, 1981) has been wildly used in solving the surface crack problem, in which the crack shape parameter α is assumed to be changeable. By using the crack shape tracking method, the simulation of nonpenetrating crack is fulfilled with 3D finite element method[2] (Smith and Copper, 1989), the 3D crack propagation problem is studied by boundary element method[3] (Deng Jiangang et al., 2003). Based on the continuum damage mechanics method the fatigue crack growth behavior of surface crack is discussed[4] (Feng Xiqiao and He Shuyan, 1997), and by using S-version FEM the process of surface crack growth under mixed mode cyclic loading condition is simulated[5] (Masanori et al., 2011). What's more, many researchers have proposed different approaches in mathematic way to predict the growth behavior of surface crack. Take energy approach for example, Song, Sheu and Shieh[6] (2002) have well applied it to predict the surface crack growth in 2024-T4 aluminium alloy. And, an approach with data obtained from the testing of through-thickness cracks was established to predict surface crack growth, in which the effects of crack closure[7] was considered (Liu Yanping et al., 2010). Therefore, the study on surface crack growth behavior is still hot to solve the fatigue fracture problem in engineering like the pressure vessels and pipes. However, the past researches on the crack growth behavior of surface crack are all based on the Paris formula[8] (Paris and Erdogan, 1963). As we know, the fatigue crack growth rate behavior and the low cycle fatigue behavior are just different way to describe material's fatigue properties, so there should be a connection to each other[9] (Cui Weicheng, 2002). And many theoretical models have been proposed based on material's fatigue properties[10-12] (Castro J.T.P., 2005 & Li D.M, et al., 1998 & Ramsamooj D.V., 2003), but they are still involved with some human debugs. Therefore, combined Newman-Raju formula, a fatigue crack growth prediction model based on the material's low cyclic fatigue properties (LCF-FCGM) is proposed to predict the extending process of the surface crack. The proposed LCF-FCGM has been well discussed in author's previous studies[13-14] (Chen Long et al., 2012). Notice that the influence of the extending direction on the fatigue crack growth properties is ignored.

2. Analysis Theory of Elliptic Surface Crack Growth

2.1. The Proposed LCF-FCGM Theory

Based on the HRR field, the stress-strain field near the crack tip was modified to describe the cyclic crack tip stress-strain field[15] (Schwalbe, 1974),

$$\begin{cases} \Delta\varepsilon_{\mathrm{P}}(r) = \dfrac{2\sigma_{yc}}{E}\left(\dfrac{\mathrm{PZ}c}{r}\right)^{1/(1+n)} \\ \Delta\sigma(r) = 2k\left(\dfrac{\Delta\varepsilon_{\mathrm{P}}(r)}{2}\right)^{n} \end{cases} \tag{1}$$

in which E is Young's modulus, σ_{yc} is cyclic yield stress, k is cyclic hardening coefficient, n is cyclic hardening exponent, r is the distance to the crack tip, and PZc is the cyclic plastic zone size that can be calculated as follows.

$$\mathrm{PZ}c = \frac{1}{4\pi\kappa^2(1+n)}\left(\frac{\Delta K}{\sigma_{yc}}\right)^2 \tag{2}$$

Where, $\kappa = \begin{cases} 1 & plane-stress \\ \dfrac{1}{1-2v} & plane-strain \end{cases}$, v is poison ratio and ΔK is stress intensity factor amplitude.

In fact, according to amount of FEA analyses, the curvature of the crack tip is non-zero, and the plastic strain of the crack tip is finite. So, a fatigue blunting factor x_1 is introduced into (1), and the cyclic plastic strain amplitude can be further described as follows.

$$\Delta\varepsilon_{\mathrm{P}}(r+x_1) = \frac{2\sigma_{yc}}{E}\left(\frac{\mathrm{PZ}c}{r+x_1}\right)^{1/(1+n)} \tag{3}$$

Informed researches show that material near the crack tip can be considered as a serial of fatigue elements under cyclic loading[15] (Schwalbe, 1974). According to the fatigue theory, the relationship between fatigue life Nf and the cyclic plastic strain amplitude $\Delta\varepsilon_{\mathrm{p}}$ can be described as follows.

$$\frac{\Delta\varepsilon_{\mathrm{P}}}{2} = \varepsilon'_f\left(2N_f\right)^c \tag{4}$$

Where, ε'_f and c are plastic hardening coefficient and plastic hardening exponent, respectively.

According to the Miner accumulative damage theory, combined (3) and (4), the damage D of the material per one cycle is defined as $1/N_{\mathrm{f}}$, where N_{f} is associated with the plastic strain amplitude. Therefore the distribution of the plastic strain damage along the crack growth direction in the cyclic plastic zone can be described as follows.

$$D(r+x_1) = 2\left(\frac{\sigma_{yc}}{E\varepsilon'_f}\right)^{-1/c}\left(\frac{\mathrm{PZ}c}{r+x_1}\right)^{-1/(1+n)c}, 0 < r \le \mathrm{PZ}c - x_1 \tag{5}$$

According to the fatigue striations phenomenon of the fatigue fracture image, assuming that each step of the crack advancement size equals to the cyclic plastic zone size (PZc-x_1) along the growth direction. The plastic strain amplitude is much bigger than the elastic strain amplitude in the cyclic plastic zone, so the damage of elastic strain can be ignored. Then, a unit average damage parameter over the cyclic plastic zone along the crack extending direction is defined as $\overline{D}=\frac{\int_0^{\mathrm{PZ}c-x_1} D(r+x_1)\mathrm{d}r}{\mathrm{PZ}c-x_1}$.

According to the Miner accumulative damage theory, when $\overline{D}=1$ the crack will grow forward one step. Therefore, the life and the rate of each step can be calculated as follows.

$$\begin{cases} N_{fi}=\dfrac{1}{\overline{Di}}=\dfrac{\mathrm{PZ}c-x_1}{\int_0^{\mathrm{PZ}c-x_1} D(r+x_1)\mathrm{d}r} \\ \left(\dfrac{\mathrm{d}a}{\mathrm{d}N}\right)_i=\dfrac{\mathrm{PZ}ci-x_1}{N_{fi}}=\int_0^{\mathrm{PZ}c-x_1} D_i(r+x_1)\mathrm{d}r \end{cases} \tag{6}$$

From (6), it is found that the fatigue crack growth equals to the sum of the nodes' damages in the cyclic plastic zone. Combined (5) and (6), a new FCG prediction model based on the mean plastic strain in the cyclic plastic zone can be given as (7).

$$\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_i=2\left(\frac{E\varepsilon_f'}{\sigma_{yc}}\right)^{1/c}\cdot\mathrm{PZ}ci\cdot\frac{c+cn}{c+cn+1}\cdot\left[1-\left(\frac{x_1}{\mathrm{PZ}ci}\right)^{1+1/(c+cn)}\right] \tag{7}$$

As known to all, in the initiation stage of the FCG curve, where the stress intensity factor amplitude $\Delta K=\Delta K_{\mathrm{th}}$, no crack growth occurs approximately. So, the fatigue blunting factor x_1 can be calculated through (8).

$$x_1=\mathrm{PZ}c\mathrm{th}=\frac{1}{4\pi\kappa^2(1+n)}\left(\frac{\Delta K_{\mathrm{th}}}{\sigma_{yc}}\right)^2 \tag{8}$$

From (8), the fatigue blunting factor equals the cyclic plastic zone size corresponding to ΔK_{th}. Then a zero range of cyclic plastic strain damage is obtained from (8) and (5) nearby the crack initiation, which means there is no damage accumulated under this condition. Then the LCF-FCGM can be further developed as (9),

$$\begin{aligned}\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)&=\frac{\mathrm{PZ}ci-x_i}{N_{fi}} \qquad i=1,2,3\ldots\ldots \\ &=\frac{c}{2\pi\kappa^2(1+c+cn)}\bullet\left(\frac{E\varepsilon'_f}{\sigma_{yc}}\right)^{1/c}\bullet\left(\frac{\Delta K_i}{\sigma_{yc}}\right)^2\bullet\left[1-\left(\frac{\Delta K_{\mathrm{th}}}{\Delta K_i}\right)^{2+2/(c+cn)}\right]\end{aligned} \tag{9}$$

where i is the serial number.

2.2. Simulation of Surface Crack Growth Theory

As for the surface crack front, the crack growth behavior can not be described as one point, which must be considered as a whole curve, two controller points are defined: the crack deepest point B and the crack surface point A, shown in Fig. 1. Follow the assumption on surface crack by Newman and Raju, during the extending process, the crack front shape always can be described with ellipse function. Therefore, the prediction of the surface crack front shape can be achieved by applying the LCF-FCGM at the two controller points A and B. The maximum average plastic strain damage $\{\overline{D}_{\mathrm{A}},\overline{D}_{\mathrm{B}}\}$max decides the main controller point, and the corresponding minimum growth life {$N_{f\mathrm{A}}$,

N_{fB} }min is chose as the crack growth life. What's over, the corresponding extending steps of the two controller points A and B have the following relationship.

$$\frac{(\mathrm{d}c)_{\mathrm{A}}}{(\mathrm{d}a)_{\mathrm{B}}}=\frac{\mathrm{PZ}c_{\mathrm{A}}-x_1}{\mathrm{PZ}c_{\mathrm{B}}-x_1}\bullet\frac{N_{f\mathrm{B}}}{N_{f\mathrm{A}}}$$
$$=\left(\frac{\Delta K_{\mathrm{A}}}{\Delta K_{\mathrm{B}}}\right)^2\bullet\left[1-\left(\frac{\Delta K_{\mathrm{th}}}{\Delta K_{\mathrm{A}}}\right)^{2+2/(c+cn)}\right]\Bigg/\left[1-\left(\frac{\Delta K_{\mathrm{th}}}{\Delta K_{\mathrm{B}}}\right)^{2+2/(c+cn)}\right] \tag{10}$$

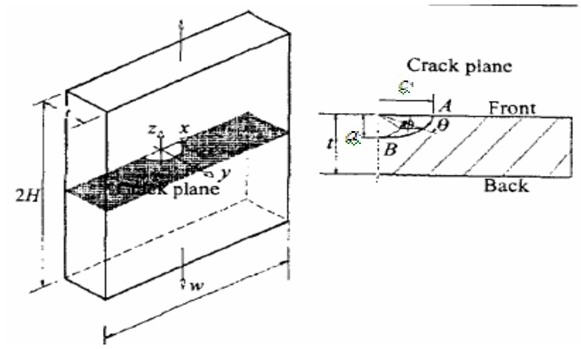


Fig. 1. scheme of flat surface crack under tension loading

The famous Newman-Raju formula[1] (Newman and Raju, 1981) is applied to obtain the stress intensity amplitude along the surface crack front, as Eq..11,

$$K=\left[(\sigma_t+H\sigma_b)\sqrt{\pi a}/E(\kappa)\right]F_s(a/c,a/t,c/W,\varphi) \tag{11}$$

where $E(\kappa)$ can be approximated by

$$E(\kappa)=\begin{cases}\left[1+1.464(a/c)^{1.65}\right]^2, a/c\le 1\\ \left[1+1.464(c/a)^{1.65}\right]^2, a/c>1\end{cases} \tag{12}$$

and the load combination factor H can be calculated through

$$\begin{aligned}&H=H_1+(H_2+H_3)\sin^p\varphi, p=0.2+a/c+0.6a/t\\&H_1=1-0.34(a/t)-0.11(a/c)*(a/t)\\&H_2=1+G_1(a/t)+G_2(a/t)^2\\&G_1=-1.22-0.12(a/c)\\&G_2=0.55-1.05(a/c)^{0.75}+0.47(a/c)^{1.5}\end{aligned} \tag{13}$$

The geometry modifying factor F_s is obtained by FEA as

$$F_s=\left[M_1+M_2(a/t)^2+M_3(a/t)^4\right]gf_\varphi f_W \tag{14}$$

in which

$$M_1=\begin{cases}1.13-0.09(a/c) & ,a/c\le 1\\ \sqrt{c/a}\left[1+0.04(c/a)\right], & a/c>1\end{cases} \tag{15}$$

$$M_2=\begin{cases}-0.54+0.89/\left[0.2+(a/c)\right] & ,a/c\le 1\\ 0.2(c/a)^4 & ,a/c>1\end{cases} \tag{16}$$

$$M_3 = \begin{cases} 0.5 - [1/(0.65 + (a/c))] + 14(1 - a/c)^{24} & , a/c \le 1 \\ -0.11(c/a)^4 & , a/c > 1 \end{cases} \tag{17}$$

$$g = \begin{cases} 1 + \left[0.1 + 0.35(a/t)^2\right](1 + \sin\varphi)^2 & , a/c \le 1 \\ 1 + \left[0.1 + 0.35(c/a)(a/t)^2\right](1 + \sin\varphi)^2 & , a/c > 1 \end{cases} \tag{18}$$

The function f_φ, an angular function from the embedded elliptical-crack solution[16] (Green and John, 1965), is

$$f_\varphi = \left[(a/c)^2 \sin^2\varphi + \cos^2\varphi\right]^{0.25} \tag{19}$$

The function f_W, a finite-width correction from reference[17] (Newman, 1976), is

$$f_W = \left[\sec\left(\pi c/(2W)\sqrt{a/t}\right)\right]^{0.5} \tag{20}$$

so, the corresponding stress intensity amplitude can described as $\triangle K=(1-R)K$. The specific prediction approach is given in Fig. 2.

3 Results and Discussion

3.1 Comparable Analysis of The Proposed Model

The crack growth behavior of the Al 7075-T6 alloy elliptic surface crack in tension plate was tested[18] (Putra and Schijve, 1992), and the shape change of the crack front had been obtained, shown in Fig. 4. A literature [19] (Noroozi, 2005) had studied the fatigue properties of Al 7075-T6 alloy, and the specific fatigue data are shown in Table. 1, and the feasibility of LCF-FCGM is tested as shown in Fig. 3. It is found that the prediction result by LCG-FACGM is consistent with the reference result [19](Noroozi et al., 2005).

Based on the proposed prediction theory, the shape change of the elliptic crack front with different initiate crack shape is predicted by following the prediction process, which has been compared with the reference paper in Fig. 4. The geometry scale and the material properties are the same as the reference paper, in which the initiate crack depth a_0 is fixed as 0.2 mm. Seen from Fig. 4, the prediction results for different initiate crack shape are consistent with the test results, especially for the initiate shape parameter α_0 equals 0.8,0.4 and 0.2.

Table. 1. Geometry size and low cycle fatigue properties

Material	E /GPa	σ_{yc} /MPa	n	ε_f'	c	R	$\triangle K_{th}$ /MPa.m$^{1/2}$
Al 7075-T6	71	469	0.0865	0.19	-0.52	0.1	1.45
	H /mm		α_0		W /mm		t /mm
	65		2.5,1,0.8 ,0.6,0.4,0.2		20		10

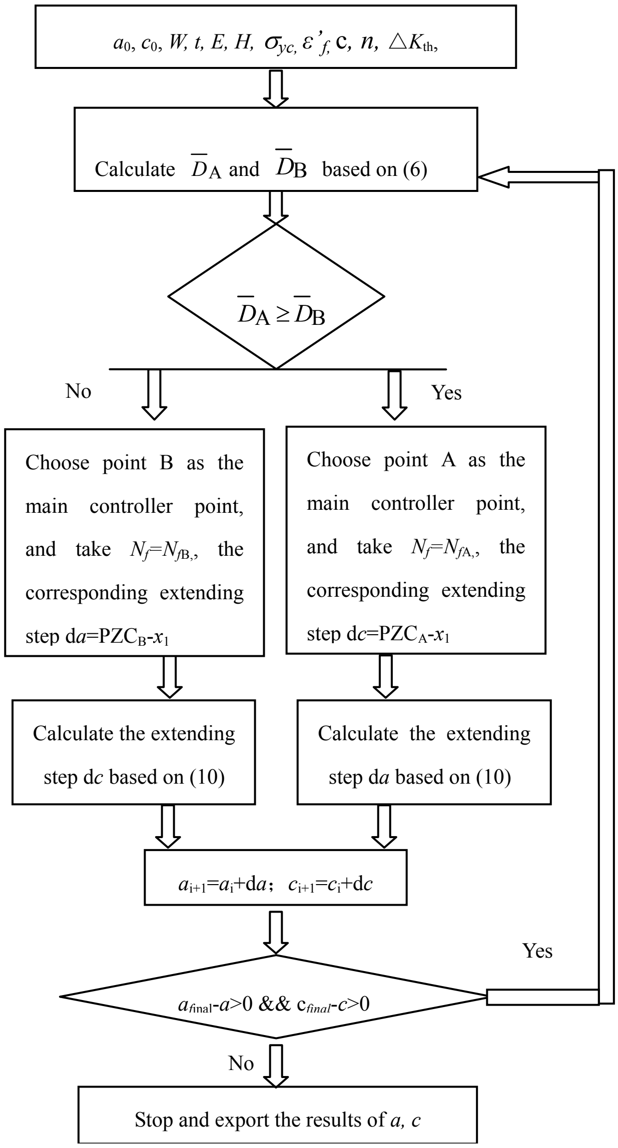


Fig. 2. the specific simulation approach of surface crack

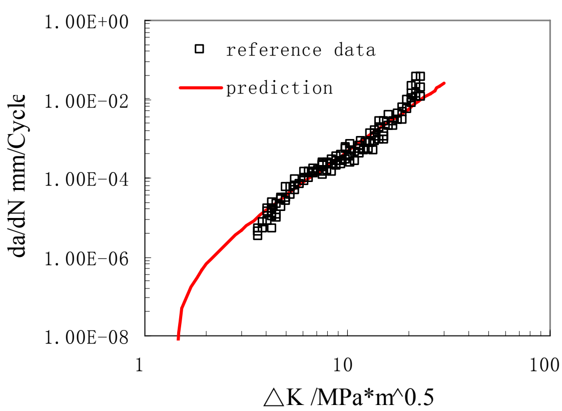


Fig. 3. Comparison of FCG result of Al 7075-T6 alloy

3.2 Prediction Results of Different Initiate Crack Shape

Taking the Al 7075-T6 alloy for example, the crack front change of the elliptic surface crack with different shape parameters is simulated. The initiate geometry parameters are: thickness d equals 10 mm, width W equals 20 mm, and the shape parameters α0 are 2.5, 1, 0.8, 0.6, 0.4 and 0.2 respectively. The prediction results of shape change of the crack front during the extending process are shown in Fig. 5. Meanwhile, the relationship curves between the shape parameter α and the depth ratio $\beta=a/t$ are also shown in Fig. 6. Seen from Fig. 6, when initiate shape parameter $\alpha_0<1$, the maximum damage is found in the controller point A, which defines the corresponding crack growth life of the whole surface crack. Otherwise, the maximum damage turns to the deepest point B. Combined with Fig. 4, it shows that the same crack growing trend is found with different initiate crack shape parameter, the crack front shape all become oval after getting round, which is consistent with results in some recent reports[20-21] (Wu zhixue, 2007; Brennan *et al*., 2008). What's more, every two parameters of the three initiate factors: shape parameter α_0, crack depth a_0 and crack width c_0 control the change of the surface crack front.

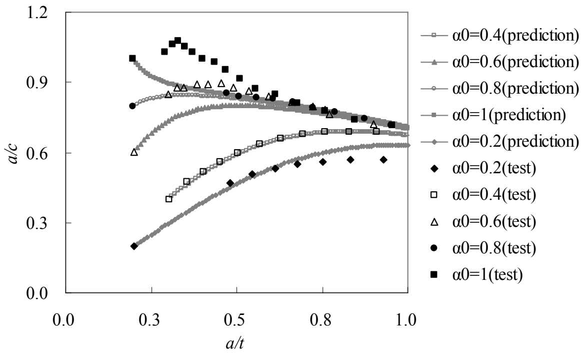
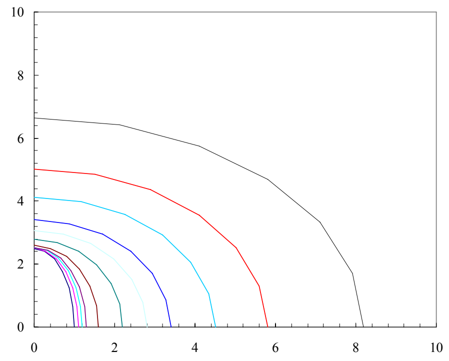


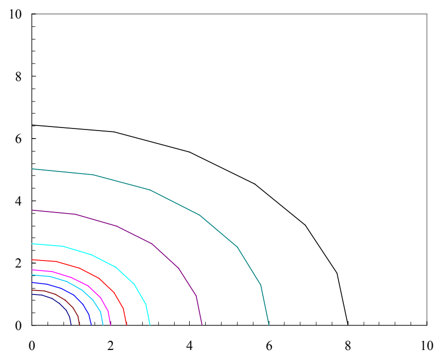
Fig. 4. Comparison of the shape change of the elliptic crack front

4 Conclusions

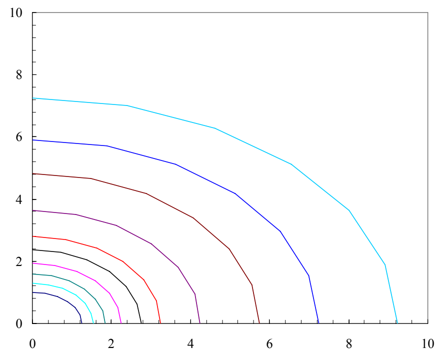
From the point of low cycle fatigue strain damage, this paper proposed a new fatigue crack growth rate prediction model. It has been well testified with Al 7075-T6 alloy. Meanwhile, combined with the famous Newman-Raju formula used to solve the stress intensity along the surface crack front, a numerical approach is obtained to predict the fatigue crack growth behavior of elliptic surface crack. The prediction results of different initiate crack shape are all consistent with the reference test, and they all present a conversion process of rounding first to oval again. And every two parameters of the three initiate factors: shape parameter α_0, crack depth a_0 and crack width c_0 control the change of the surface crack front.



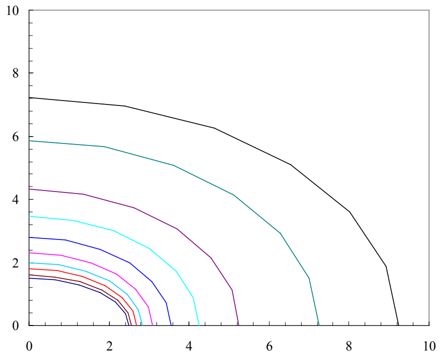
a) $\alpha_0=a/c=2.5$



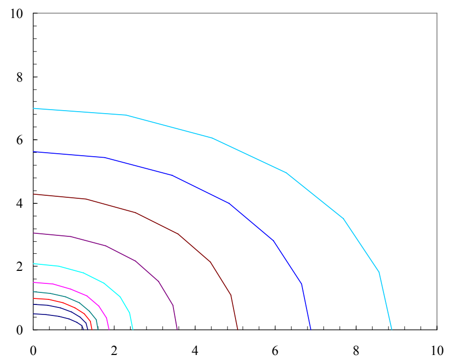
b) $\alpha_0=a/c=1$



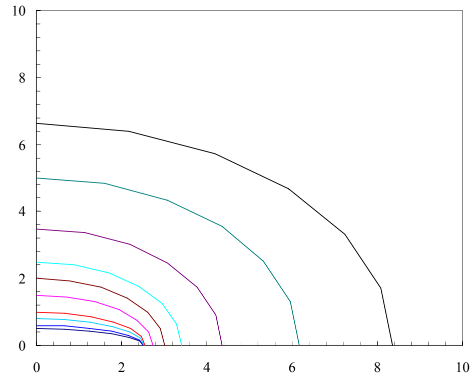
c) $\alpha_0=a/c=0.8$



d) $\alpha_0=a/c=0.6$



e) $\alpha_0=a/c=0.4$



f) $\alpha_0=a/c=0.2$

Fig. 5. the prediction shape change of crack front

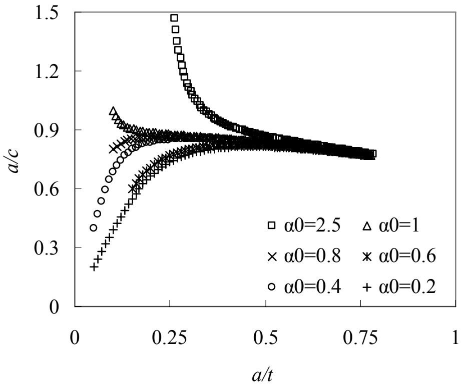


Fig. 6. $\alpha\sim\beta$ curve during the crack growth process

Acknowledgements

The financial support of the National Nature Science Foundation of China, Grant NO.11072205, is gratefully acknowledged. We are also grateful for the valuable assistance of Key Laboratory of Strength and Vibration, the SWJTU Laboratory.

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