Investigations of Subcritical Crack Propagation under High Cycle Fatigue

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Abstract

Most of the fatigue life in high cycle fatigue (HCF) is expended in growing subcritical defects to a detectable damage size, followed by rapid growth to failure. This is a significant challenge in design as this leads to concerns with respect to the efficacy of planned inspections and routine repair operations. The variability of the damage phenomena in HCF can impact the characterization of the long fatigue life regimes, as the extent and severity of damage depends on the intensity of the operating loads, and on the intrinsic variations in material characteristics at different scales.

In this paper we propose a methodology to account for stochastic variability in HCF by using the framework of the Kitagawa-Takahashi diagram, and the El-Haddad formulation as the design curve for subcritical crack propagation in fatigue and in environmentally assisted cracking (EAC). Moment-based reliability, based on first order and second order reliability methods, as well as Monte Carlo simulation is used to handle stochastic variability in applied stress and defect size. The calculation of a probability of defect propagation at each instant of the subcritical growth process is the goal of the developed framework. Preliminarily validation with available data and an application of the framework to stochastic defect growth for block loading is presented.

1. Introduction

Designing against HCF can be challenging, particularly for dynamic components subjected to a large number of small amplitude or vibratory solicitations. While the growth of cracks from the subcritical level to a detectable size level has been extensively investigated in literature, current models show difficulties in estimating the fatigue life in HCF regimes and innovative solutions are sought [1-4].

The characterization of the fatigue behavior of materials in the long fatigue life regime is critical for fatigue life prediction models, but the investigation of damage phenomena in the proximity of the endurance stress can be experimentally intensive. Furthermore, crack growth under high R-ratios and near threshold is not well captured by currently employed frameworks [5, 6].

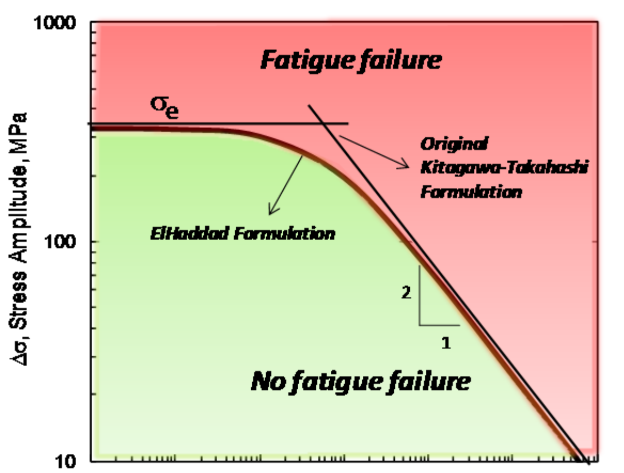

Figure 1: Kitagawa-Takahashi diagram for fatigue.

Therefore, certification becomes difficult for damage tolerance of components predominantly subjected to HCF during the component lifetime and with limited or no possibilities of secondary load paths. The difficulty in characterizing the long life fatigue region interferes with the calculation of component retirement time, and the difficulties in crack growth predictions highly impacts the inspection intervals spacing [5].

The Kitagawa diagram, as modified by El Haddad to address crack growth threshold for short cracks (Figure 1), has been shown to be a powerful framework for the rationalization of initiation vs propagation of subcritical defects in metals and structural materials [7-10]. El Haddad introduced the correction factor a_0 into the Kitagawa diagram formulation, which can be interpreted as a subcritical defect size above which the long crack growth threshold holds. By dividing the stress-crack length space into two main areas, one where subcritical defects or crack are not expected to grow, and one where both fatigue and LEFM methodologies predict defects or cracks to grow in fatigue, the Kitagawa diagram can be used as a framework for the investigation of defect growth in HCF from the subcritical level to a detectable size level [3].

A statistical framework for the Kitagawa-Takahashi diagram in fatigue is here developed, with the goal of determining the likelihood that a nucleated defect propagates when subjected to a stress at a specific structure or component location, given a material behavior defined in terms of fatigue endurance, σ_e, and crack growth stress intensity threshold, ΔK_{th}. Additionally, a failure diagram in the form of a modified Kitagawa diagram for EAC is also developed by considering the corrosion fatigue (CF) and stress corrosion cracking (SCC) behavior of the material. The goal of this framework is to address whether a nucleated defect is likely to grow by CF by considering the concomitant SCC damage phenomena. The two frameworks are described in Section 2.

Probabilistic methods such as first order reliability method (FORM), second order reliability method (SORM) and the Monte Carlo simulation (MCS) can be employed in conjunction with fatigue and fracture mechanics to estimate the probability of the growth of a critical/propagating defect [11]. The reliability methods as used in the developed frameworks are described in Section 3.

The developed framework for fatigue is validated with available experimental data and finally an application example to HCF subcritical crack propagation is described.

2. Developed Framework in Fatigue and Environmentally Assisted Cracking

The Kitagawa diagram was introduced by Kitagawa and Takahashi [7] showing the transition as crack-size decreases from LEFM controlled growth (ΔK), to stress controlled behavior as the fatigue limit ($\Delta\sigma$) is approached. The Kitagawa diagram conveys two different thresholds: the minimum threshold stress intensity range for crack growth (ΔK_{th}) for a fracture mechanics specimen, and the endurance limit of a smooth specimen, that characterizes the minimum stress amplitude

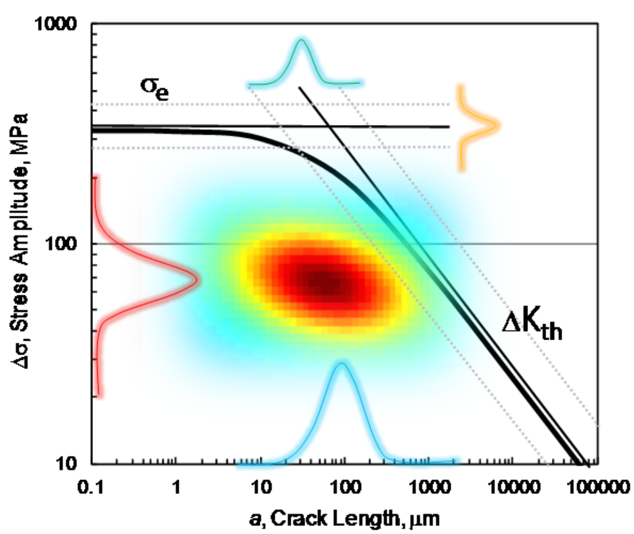

Figure 2: developed stochastic framework for the Kitagawa-Takahashi diagram.

(σ_e) to failure [8].

As mentioned above, in developing a statistical framework for the Kitagawa diagram, the limit state function used for reliability calculation is based on the mathematical formulation developed by El Haddad [8]. The limit state function defines the region in the stress-crack length space where the onset of crack propagation occurs for a nucleated defect, after which a flaw in HCF is expected to rapidly grow to a detectable length.

The El-Haddad formulation intrinsically considers the change in ΔK_{th} for cracks shorter than a_0, and elegantly describes this change in limiting condition by a simple mathematical formulation:

$$\Delta\sigma = \Delta K_{th}/(Y(a + a_0)\sqrt{\pi(a + a_0)}\,) \tag{1}$$

Although ΔK_{th} is nominally independent of crack length, at small crack sizes this function tends to a limit given by the fatigue endurance stress.

A statistical framework was initially developed for the Kitagawa diagram to consider statistical variability of stress and crack lengths; the extended framework allows including variability in any of the parameters (ΔK_{th}, a, $\Delta\sigma$,…), upon consideration of their relative interdependencies [11], as shown in Figure 2. The limit state is defined as the threshold of Kitagawa diagram in combination with El Haddad's correction which differentiates the non-propagating from propagating crack.

The limit state function for the fatigue framework developed can be written as follows:

$$g(X)_{Fatigue} = \Delta\sigma - \frac{\Delta K_{th}}{\sqrt{\pi(a+a_0{}^{Fatigue})}\,Y(a+a_0{}^{Fatigue})} \tag{2}$$

The criteria for crack propagation is defined as g(X)<0 with a probability of crack growth:

$$P_{c.p.} = \int_{g(x)\le 0} f_X\,(x)dx \tag{3}$$

and reliability is expressed as $R=1-P_{c.p.}$

The failure diagram for corrosion is developed along the lines of the modified Kitagawa-Takahashi diagram for fatigue, as described elsewhere [3, 10, 12]. For SCC, by considering that in aggressive environment the material will be exposed to different concentrations of chemicals/ions involved in the corrosion damaging process, a limiting stress condition dependent upon the concentration level can be found. At concentration C, for example, the limiting threshold stress for crack propagation and failure of a smooth specimen, σ_{th}, forms a limiting condition in the SCC

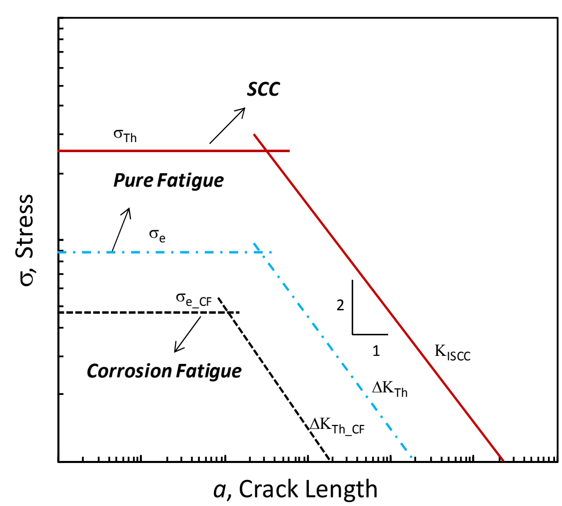

Figure 3: developed framework for EAC failure diagram.

diagram, as shown in Figure 3. For a fracture mechanics specimen the limiting threshold for crack propagation in SCC is K_{ISCC}, as calculated by EAC testing. Similarly to the original Kitagawa diagram, these two limiting conditions, σ_{th} and K_{ISCC}, can be used to develop an envelope diagram describing where crack propagation by SCC is expected to occur. By considering both stress related propagation and long crack condition, the diagram describes an SCC "safe" area and an SCC "unsafe" area.

The failure diagram for SCC can be described by the following equation:

$$\sigma_{Th} = K_{ISCC}/(Y(a + a_0{}^{SCC})\sqrt{\pi(a + a_0{}^{SCC})}\,) \tag{4}$$

Similarly, for corrosion fatigue, a Kitagawa like diagram can be envisioned, where the endurance stress in environmentally assisted fatigue, $\Delta\sigma_{e\_CF}$, is combined with the stress intensity threshold in aggressive environment, ΔK_{th_CF}. The failure diagram will have the form:

$$\Delta\sigma_{CF} = \Delta K_{th\_CF}/(Y(a + a_0{}^{CF})\sqrt{\pi(a + a_0{}^{CF})}\,) \tag{5}$$

Conceptually, a combined Kitagawa-like diagram with more than one mode of failure can be constructed for any material; it is the interest of this paper to ilustrate a failure diagram for CF, SCC and pure fatigue. The diagram can be pictured to look like Figure 3: the outermost Kitagawa-like diagram is expected to be the one representing SCC, with σ_{th} and K_{ISCC} as limiting values, the innermost Kitagawa-like diagram is expected to be the one representing CF, with $\Delta\sigma_{e\_CF}$ and ΔK_{th_CF} as limiting values. The Kitagawa diagram for pure fatigue is pictured to lie between CF and SCC, with $\Delta\sigma_e$ and ΔK_{th} as limiting values. A failure diagram in aggressive environment can be constructed by considering only the outermost and innermost limiting conditions: CF and SCC.

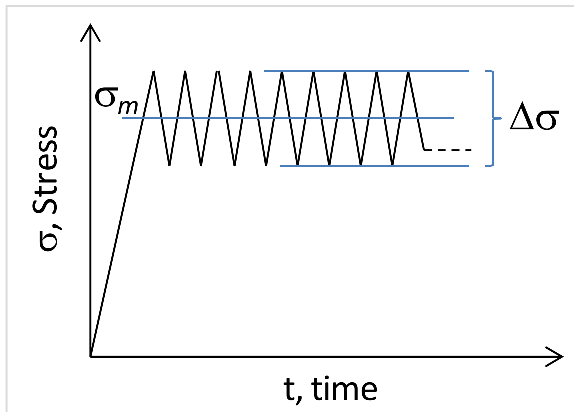

Figure 4: example loading in EAC.

Let's suppose that a component is subjected to fatigue loading in aggressive environment by cyclic stressing at a high R-ratio, as shown in Figure 4. In this situation we can infer that the component will be subjected to corrosion fatigue at a stress amplitude $\Delta\sigma$ and mean stress σ_m. Since the cyclic loading occurs at a high R-ratio, we can also infer that the component is subjected to stress corrosion cracking at a stress level σ_m. In this situation the Kitagawa-like diagram that can be applied are two: one for CF and one for SCC.

Two limit state functions need to be written for the failure diagram in EAC, one considering the CF and one the SCC damaging process; the limit states can be written as follows:

$$\begin{cases} g(X)_{CF} = \Delta\sigma - \dfrac{\Delta K_{th\_CF}}{\sqrt{\pi(a+a_0{}^{CF})}Y(a+a_0{}^{CF})} \\ g(X)_{SCC} = \sigma_m - \dfrac{K_{ISCC}}{\sqrt{\pi(a+a_0{}^{SCC})}Y(a+a_0{}^{SCC})} \end{cases} \tag{6}$$

When considering both phenomena occurring simultaneously, it needs to be considered that the probability of a propagation event in CF or SCC is mutually influenced by both damaging phenomena. A possible approach to obtain probability bounds for a system subjected to both CF and SCC and the relative probability of each mode of propagation is the use of failure tree diagrams and common cause failure, as described by the authors elsewhere [1].

Note that all of the Kitagawa like diagrams shown correspond to a specific R-ratio level; multiple diagrams can be constructed for a single material at different R-ratios.

3. Reliability Methods

The modeling of the intrinsic variability of the controlling variables in a system is the foundation for the determination of a measure of reliability of a structure. The development of a statistical framework for a developed model is advantageous since it introduces mathematical and statistical concepts into the model and attempts to derive the statistical variability of the desired output directly computing the uncertainties associated with the parameters used in the model [13, 14].

First order and second order reliability methods are two analytical approaches that are used in this work to handle the variability of the stochastic inputs in the developed framework. The goal is the estimation of the probability of onset of HCF subcritical defect growth, in fatigue or CF and SCC, depending on the framework.

The name of First Order Reliability Method (FORM) comes from the fact that the performance function *g(X)* is approximated by the first order Taylor expansion (linearization), around the Most Probable Point (MPP). Two steps are involved in these approximation methods to make the probability integration easy to compute and to obtain the probability of exceeding a specified threshold**.**

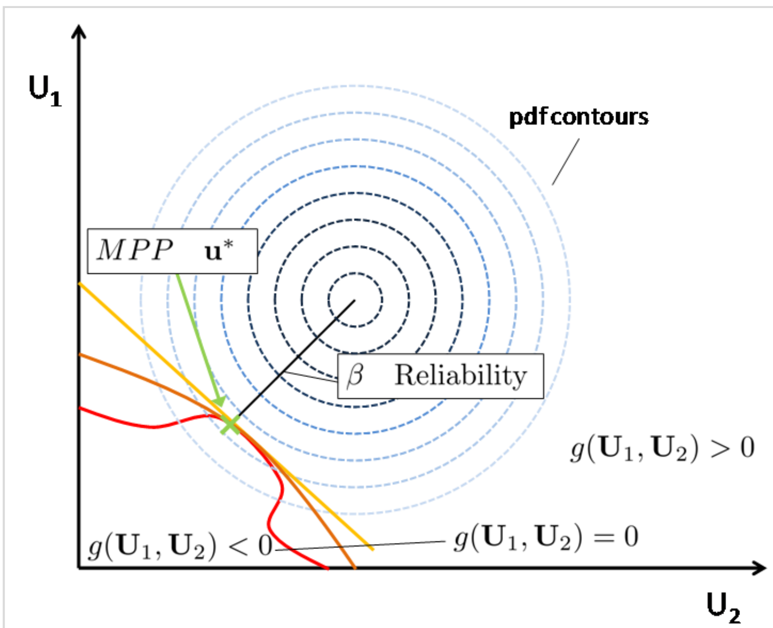

Figure 5: schematic of FORM and SORM calculation methods.

The first step is to simplify the joint probability distribution, i.e. the function to be integrated, so that its contours become more regular and symmetric. The second step involves the approximation of the integration boundary on the limit state function *g(x)* at limit state *g(x) = 0*. After these two steps, an analytical solution to the probability integration can be obtained.

For ease of analytical development, all the variables are transformed into their standard forms. The transformation of the joint probability distribution is performed by mapping the physical space to a standard space. Let $\boldsymbol{X}_i$ be the vector of stochastic variable under considerations, $\boldsymbol{U}_i$ is the vector of its transformed, as shown below.

$$\mathrm{F}_{\mathbf{X}_\mathrm{i}}(\mathbf{x}_\mathrm{i}) = \Phi(\mathrm{u}_\mathrm{i}) \tag{7}$$

The transformation is performed on the premises that the cumulative density functions, c.d.f.s, of the random variables remain the same before and after the transformation.

$$\mathrm{U}_\mathrm{i} = \Phi^{-1}\left[\mathrm{F}_{\mathbf{X}_\mathrm{i}}(\mathbf{X}_\mathrm{i})\right] \tag{8}$$

The form of the limit state function consequently changes, and the calculation of the probability of failure remains the integration of the transformed limit state in the standard space domain where its value is less than 0:

$$p_f = P\{g(\boldsymbol{U}) < 0\} = \int_{g(\boldsymbol{U})<0} \phi_{\boldsymbol{U}}(\boldsymbol{u})d\boldsymbol{u} \tag{9}$$

FORM further approximate the integrand above by linearizing the limit state function $g(\mathbf{U})$ through Taylor series expansion around a point u*, called most probable point (MPP), as shown in Figure 5. SORM methods use a second order expansion around the MPP point to better approximate the limit state curvature. Reliability is then computed as the distance β from the center of the joint distribution to the MPP.

Another method that can be used and is here employed to handle the variability in the limit state function inputs is through numerical simulations, i.e. Monte Carlo methods. Although the application of direct simulations is more computationally intensive than the use of moment based methods FORM and SORM, Monte Carlo methods can be useful in many instances. As an example, the transformation of the distributions into the standard normal space can be challenging and simulations can be easily performed to calculate a measure of reliability, without the need of inverting $\mathrm{F}_{\mathrm{X}_\mathrm{i}}(\mathrm{X}_\mathrm{i})$ to map the c.d.f.s to standard normal distributions. Additionally, by using Importance Sampling methods [15], results from FORM can be used to better target the Monte Carlo simulations and improve on the accuracy of the result.

In Monte Carlo simulations, typically uniformly distributed pseudo-random numbers are generated and then mapped into the proper distribution for each of the variables in the model. Finally, the pseudo-random values are used in the limit state function to obtain the limit state function output. Let n_f be the number of simulation cycles when $g(x)$ is less than zero and let N be the total number of simulation cycles, an estimate of the probability of crack propagation can be expressed as [16]:

$$P_{c.p.} = \int \ldots \int I[g(x) < 0] f_x(x)dx \approx \frac{1}{N}\sum_{i=1}^{N} I[g(x_i) < 0] \approx \frac{n_f}{N} \tag{10}$$

The number of random variables in the problem does not affect the accuracy of Monte Carlo simulation, therefore the current framework allows to consider multiple input variability.

4. Verification and Validation of the Kitagawa Diagram for Fatigue

In this section, available experimental data are compiled and used for preliminary verification and validation of the developed stochastic framework for the Kitagawa diagram in fatigue.

A study on Type 403 (12% Cr) stainless steel focused on the development of pits in turbine blades developing into defects, eventually growing as fatigue cracks in service. An experimental data set is here compiled, and the stochastic Kitagawa diagram is used as the framework to determine whether the experimentally produced pits would cause a fatigue crack to grow during cyclic loading and possibly lead to failure.

The experimental study focused on fatigue testing pre-damaged smooth specimens by introducing a surface pit in their test section. The specimens are cycled in fatigue at different stress levels up to a very large number of cycles (10^7) or until failure occurs. The framework described in this paper was used to analyze the experimental data by calculating the probability of failure of each specimen cycled in fatigue, given the initial crack size defect (half pit width = *a*), and their statistical variation, including the statistical variation of threshold stress intensity factor ΔK_{th}.

Half pit width (*a*) and applied alternating stress ($\Delta\sigma$) are considered to be normally distributed, while stress intensity threshold distribution is lognormal, with mean and standard deviation as reported from experimental data. For each test a probability of crack propagation, calculated as a probability of failure using FORM is shown in the plot along with the experimental data point in the graph shown in Figure 6. Probability of failure calculated with Monte Carlo and SORM methods are within a 5% tolerance.

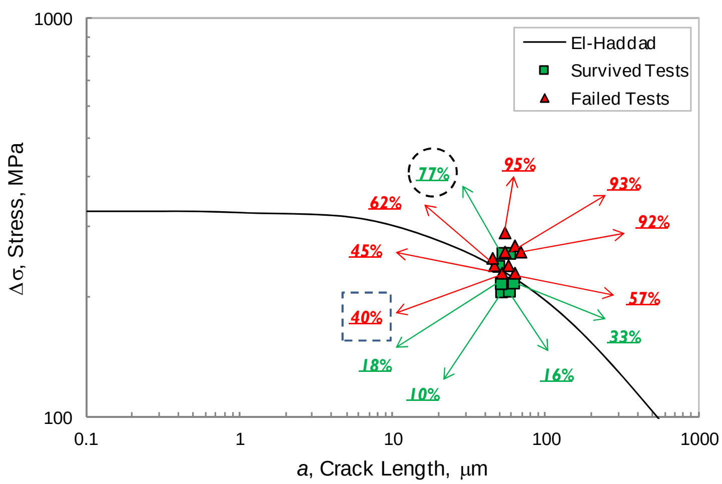

Figure 6: compiled data for framework validation.

The average probability of crack propagation for the tests shown, calculated from the experiments as the ratio between the number of tests where cracks grew to failure and the number of tests where cracks did not propagate is 0.57, and the average probability of crack propagation using the developed framework has been calculated as 0.569.

As observed from this experimental data set, on average the performance of the model reflects the results from experimental data. In some cases however, for tests with high probability of failure, experimental data showed no failure (circled results), or for others with low probability of failure (squared result), failures are observed. More data are needed in regions of lower probability of failure or higher probability of failure to further validate this framework, and to test whether the populations described by the model are in agreement with experimentally observed populations. Additionally, sensitivity analysis studies are planned to understand the influence of the statistical distributions of the inputs with respect to the performance of the framework described.

5. Application to Fatigue Crack Growth in HCF

An example application of the developed framework is illustrated for fatigue; similar concepts and applications can be extended for the combined SCC, pure fatigue and CF framework. For simplicity it is supposed that all solicitations have R=0, therefore only one R-ratio is used for the Kitagawa diagram. Note that with appropriate material information and available data, this can be extended to any R-ratio level.

It is here assumed that a component subjected to fatigue loading has an initial crack length a_1. This component is subjected to HCF blocks of loading at constant R-ratio = 0. The blocks of loading are such that there is alternation of low amplitude stresses and higher amplitude ones. Within the framework of the Kitagawa diagram for fatigue the average point (σ, a), stress amplitude and crack length, will be moving from the safe zone into the fatigue crack propagation zone and vice versa frequently. This type of situation is illustrated in Figure 7, where the Kitagawa diagram for fatigue is shown with the failure diagram for static loading, whose bounds are the ultimate strength σ_U and the fracture toughness for the material K_{Ic}. The blocks of loading are shown in the top right corner of Figure 7.

As shown, the first block of cycles is well within the no-growth region of the Kitagawa diagram, therefore no crack propagation is expected. For the following block it is expected that fatigue crack propagation will occur, as the increase in stress level will bring the (σ, a) point outside the non-propagating area. The initial defect a_1 grows in fatigue to a dimension a_2 at the end of the block. In a similar way the third block will cause fatigue crack growth from a_2 to a_3. Note that at the time the defect has grown to a_3, the stress is reduced to the same level of the first loading block. Given the different crack length a_3, block 4 is near the limit condition described by the Kitagawa diagram. Nominally, no growth is expected for block 4 as well, but eventually the alternation of loading blocks of different stress amplitude will determine the (σ, a) point to lie outside the safe zone even for stress levels 1 and 4. Eventually the crack reaches the critical length for failure, i.e. when the (σ, a) point lies on the failure diagram for static loading.

It is here proposed to use the developed stochastic framework for fatigue to calculate for each loading block a probability of crack propagation, P_{CP}. Stress and crack length are taken as random variables within the current context, whose distribution needs to be estimated from available data. A probability of crack propagation can be calculated for each (σ, a) using the reliability methods described above and by considering for simplicity the correction factor Y constant and equal to unity. In the example described the P_{CP} will be very small for the first loading block, and it is expected to be much higher for the second and third block. When the crack has grown to a_3 however, even if the stress is decreased to a value equal to the one corresponding to the first block, the P_{CP} is expected to increase.

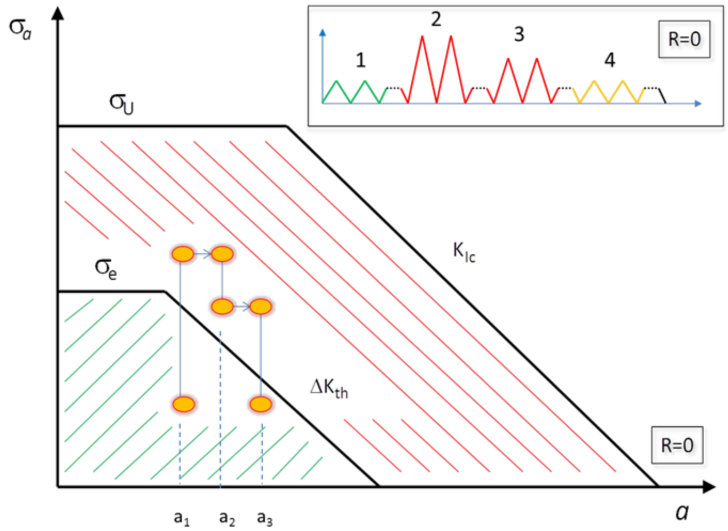

Figure 7: example of block loading and application of the developed framework for fatigue.

By considering that for every loading block the probability of crack propagation can be estimated using the framework developed in this paper, a probability tree can be constructed where each branch event (or block) has an associated probability of crack propagation. Cracks are assumed to grow following Paris' law at the ΔK corresponding to the stress $\Delta\sigma$. It is assumed that when (σ, a) lies within the "safe zone" of the Kitagawa diagram the crack will grow at the threshold level ΔK_{th}.

Each node of the tree will divide in two branches, one for a propagation event and one for a non-propagation event. By selecting consecutively at each node the branch with highest probability, a most probable crack growth path can be constructed. By selecting the branch that implies crack propagation at each node, no matter how probable the event is, a worst case scenario crack path can be constructed. Similarly a best case scenario can be constructed, by selecting the branch that implies no-propagation, creating an envelope for the most probable crack growth path.

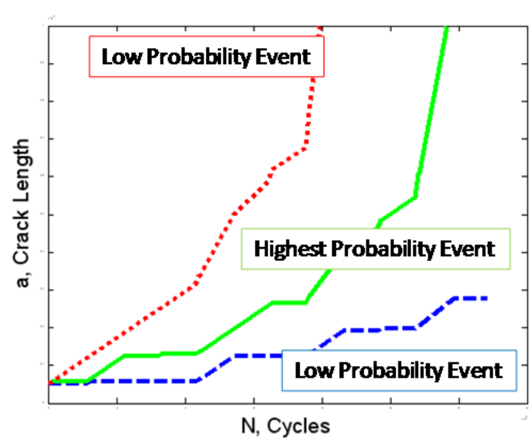

Figure 8: application example of the developed framework for subcritical crack growht in HCF block loading.

An example is shown in Figure 8 where different blocks at different stress levels are applied and crack propagation probabilities consecutively calculated for each loading block. Cracks are grown using Paris' law and three curves are calculated, a most probable path by choosing the highest probabilities, and a worst and best case scenarios.

6. Conclusions

In this paper subcritical crack propagation diagrams have been illustrated for fatigue and environmentally assisted cracking. The limit condition for subcritical crack propagation in the proposed framework is represented by Kitagawa-like diagrams combined with the El Haddad correction factor. Reliability methods for the estimation of probability of crack propagation for the diagrams shown have been proposed. Probability of crack propagation has been evaluated by using first and second order reliability methods (FORM/SORM) as well as Monte Carlo simulation.

Preliminary validation of the developed framework for fatigue has been shown, and more experimental data are currently being gathered for further validation of this diagram and for EAC diagrams. An example application of the developed framework to probabilistic subcritical crack growth has been illustrated. Further work is needed to apply the framework to multiple R-ratios data, and to extend to variable amplitude loading. Sensitivity analysis is planned to study the effects of selected stress and defect distributions on the reliability of the system.

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