SOME TOPOLOGICAL ASPECTS OF TRANSITION FROM DAMAGE TO FRACTURE AS NON-LINEAR PROBLEM OF MICROCRACK ACCUMULATION

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ABSTRACT

The non-linear character of defect interaction in high-strength nonhomogeneous materials, gives rise to instability and localisation leading to stochasticity, which is typical of the behaviour of essentially nonequilibrium systems. The instability involving localization is known to be fractal in nature. This suggests that fractal analysis of the system behaviour should be correlated with the non-linear kinetics of the growing defect ensembles. In effect, an attempt has been made unify approaches from fracture mechanics and damage mechanics.

KEYWORDS

Non-linearity of microcrack accumulation, fractal character of fracture.

1. STATISTICAL MODEL

The existence of different types of microcracks and diverse mechanisms of their generation and development requires adequate choice of parameters characterizing the microcracks. The parameters determining the volume concentration and the preferential orientation of the microcracks may be represented by the symmetric tensor $p_{ik} = n\langle s_{ik}\rangle$, where n is the number of microcracks per unit volume (Naimark et al., 1991). The tensor s_{ik} characterizes the volume and orientation of the disk-shaped microcracks with the base $\vec{S}_0 = S_0\vec{\nu}$ and vector of the displacement jump (normal to the base) in transition from one face to another $\vec{b} = b\vec{\nu}$. The volume of microcrack is $s = Sps_{ik}$. Barenblatt and Botvina (1983) described the process of damage accumulation assuming self-similarity of microcrack distributions at various damage stages. The self-similarity allows us to take into consideration average dimensions of defects and their energy characteristics. The potential energy of microcracks may be calculated as (Naimark, 1992)

$$E = E_0 - H_{ik}s_{ik} + \alpha s_{ik}^2, \tag{1}$$

where E_0 is a term depending on p_{ik}, $H_{ik} = \gamma\sigma_{ik} + \lambda p_{ik}$ is the effective (mean) force field acting on the microcrack, σ_{ik} is the macroscopic stress tensor, and α, λ and γ are material parameters. The form of H_{ik} reflects the fact that reconstructions of the material structure are determined by the level of the local stresses, which may differ considerably from the macroscopic ones. The term λp_{ik} describes the force action, which causes defect growth in the overstressed fields from the adjacent defects. It is to be noted that a solid with defects is a non-linear system which is far from equilibrium. By analogy with such systems one may conclude that p_{ik} plays the part of the order parameter. For this case, the distribution function of defects with respect to their sizes and orientations was taken as $W = Z^{-1}\exp(-E/Q)$, where Z is a normalizing parameter and Q is the fluctuating force intensity determined by the potential relief of the initial or induced (by the deformation processes) structure. Averaging s_{ik} with the distribution function W, we obtain the self-consistency equation for p_{ik}

$$p_{ik} = n\int s_{ik}Z^{-1}\exp(-E/Q)dsd^3\vec{v}, \tag{2}$$

where $Z = \int \exp(-E/Q)dsd^3\vec{v}$. Equation (2) is the constitutive equation of the quasi-brittle behaviour of the medium with microcracks when there is preferential growth of

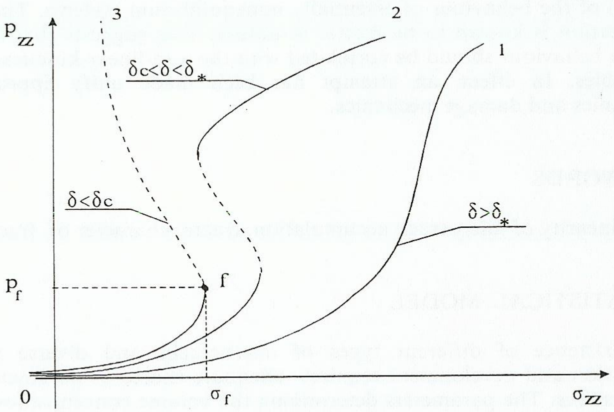


Fig.1. Characteristic responses of solid on the microcrack growth.

of the mean size of defects at a constant number of the latter n (Naimark et al., 1984). An analysis of (2) for the case of uni-axial loading has shown that depending on the value of the dimensionless parameter $\delta = 2\alpha/\lambda n$ there are three responses of material to the defect growth (Fig.1): monotonous $(\delta > \delta_*)$, metastable $(\delta_c < \delta < \delta_*)$ and

unstable $(\delta < \delta_c)$. The value of δ is determined by the natural scale of structural heterogeneity and the correlation radius of fields of overstress from microcracks. In the case of monotonous response $\delta > \delta_*$, which is characteristic of fine-grained materials, the applied stress corresponds to the constant value of microcrack concentration. In the case of metastable response $(\delta_c < \delta < \delta_*)$ p_{ik} experieces a step change in the metastable region which is accompanied by ordering of the defect system. The unstable response $(\delta < \delta_c)$ is characteristic of coarse-grained materials, which initially contain large nuclei of microcracks. The stress scale in this case is divided into two regions: the metastable region $(\sigma < \sigma_f)$ when p_{ik} tends to infinity in the presence of structural disturbances, and the region of absolute instability $(\sigma > \sigma_f)$ which allows the existence of large defects.

2. NON-LINEAR KINETICS OF THE MICROCRACK ACCUMULATION.

From the analysis of a microcrack system as a statistical ensemble, it is readily seen that the transition from disperse to macroscopic fracture takes place under conditions of non-equilibrium (kinetic) transition, when microcrack growth is governed by the non-linear character of microcrack interaction. In the analysis of the nonequilibrium situation, the part *of* of curve 3 in Fig.1 describes the thermodynamic branch of the system evolution that controls the behaviour of the defect ensemble until a certain stress level is reached. Outside of this region $(\sigma > \sigma_f)$ the kinetics of growing defects becomes unstable and may lead to infinite microcrack growth and spatial organization effects. When analyzing the regularities of transition from disperse microcrack accumulation to the formation of macrocrack centers in the damaged medium, it is essential to take into consideration the spatially nonhomogeneous microcrack distribution. The continuum problem of quasi-brittle fracture includes a kinetic equation for tensor p_{ik} (Naimark and Davydova, 1993; Naimark et al., 1984)

$$\frac{\partial p_{ik}}{\partial t} = -\frac{1}{\tau_p}\frac{\partial \Psi}{\partial p_{ik}} + \frac{\partial}{\partial x_l}\left(\rho\frac{\partial p_{ik}}{\partial x_l}\right) \tag{3}$$

and a constitutive equation of elastic medium with microcracks

$$u_{ik} = C_{iklm}\sigma_{ik} + \gamma p_{ik} \tag{4}$$

where τ_p is characteristic time, ρ is a parameter of nonlocality, Ψ is the free energy of material with defects, and u_{ik} and C_{iklm} are the strain and elastic compliance tensors.

It was shown by Naimark and Davydova (1994) that there are several types of self-similar solutions, corresponding to a localized infinite growth of p_{ik} over a range of constant or "growing" space scales. This situation is typical for solids in case of initiation of stabilizing or extending fracture centers. The evolution of dissipative structures for equation (3) is described by the self-similar solution (Kurdumov, 1988; Samarskii, 1984)

$$p(x,t) = g(t)f(\varsigma), \quad \varsigma = x/\phi(t), \tag{5}$$

where $g(t)$ governs the growth law of parameter p and $\phi(t)$ defines variations over the half-width of localization region. From equation (5) it follows that the time dependence of p remains self-similar, i.e. it just extends along the x and p-axes. Substitution of (5) into equation (3) allows us to clarify the form of the function $g(t)$

$$g(t)=G\left(1-\frac{1}{\tau_c}\right)^{-m}, \tag{6}$$

where τ_c is the so-called "peak time" ($p\to\infty$ at $t\to\tau_c$ (Kurdumov, 1988)); $G>0, m>0$ are the parameters of nonlinearity that characterize the rate of free energy release $\partial\Psi/\partial p$ with an increase in the volume fraction of microcracks in the region $p>p_f$, $(\delta<\delta_c)$. Concurrently, an eigenvalue problem is formulated for the eigenfunction $f(\varsigma)$. Its solution gives the spectrum of eigenforms $f_i(\varsigma)$"living" during time τ_c^i in the discrete ranges of eigenvalues ς_i, specifying the damage localization scales. The solution (5) refers to the class of nonlinear singular solutions that describe infinite growth of $p(t)$ over localization scale ς_i (Kurdumov, 1988) at $t\to\tau_c$.

The high-strength nonhomogeneous materials with multiple interacting microcracks are dissipative systems, the behaviour of which changes from a regular to a random one at small variations of certain parameters. This phenomenon is caused by local instabilities of p_{ik} beyond the thermodynamic branch of $p(\sigma)$ relation for $\delta<\delta_c$. Local instabilities in ensemble of defects are accompanied by alteration of the topological properties of the system. It is interesting that the same form of the relation (curve 3, Fig.1.) between the scalar measure of damage P and stress was proposed by Bolotin (1984)

$$P=F_r(g(\sigma,P)), \tag{7}$$

where

$$F_r[g(\sigma,P)]=1-\exp\left[-(g(\sigma,P)/r_c)^{\alpha}\right] \tag{8}$$

is the Weibull distribution function of the short-time strength, $g(\sigma,P)$ is the rèal stress in the structural elements, $r_c=r_s(V_s/V_c)^{1/\alpha}$ is a characteristic strength of the structural elements, r_s is the strength of the specimen, and α is the Weibull modulus. Assuming $g(\sigma,P)=\sigma\exp(\beta,P)$, we obtain a concrete form of equation (7)

$$P=(\sigma/r_c\exp(\beta P))^{\alpha}, \tag{9}$$

where β is the parameter controlling the effective stress growth under damage accumulation. The relation such as equation (9) follows from (2) for the equilibrium condition of elastic medium with microcracks $\frac{d\Psi}{dp_{ik}}=0$. Phenomenological analog of expression (3) appears from the above

$$\dot{P}=-\frac{1}{\tau_p}\left(P-\left(\frac{\sigma}{r_c}\exp(\beta P)\right)^{\alpha}\right) \tag{10}$$

3. FRACTALITY AND DAMAGE LOCALIZATION

The regularities of transitions from damage to fracture were examined numerically for carbon-carbon composites. Tensile loading tests of carbon-carbon specimens demonstrate characteristic features of deformation and fracture: the presence of microcracks in the bulk of the specimen; the influence of microcracking on the deformation behaviour of materials; fragmentation of the specimen across the regions subject to damage and highly statistical scattering of the specimen strength (Fig. 2). For the case of uni-axial tension only the component p_{yy} of the tensor p_{ik} is sufficient to characterize the microcrack accumulation process.

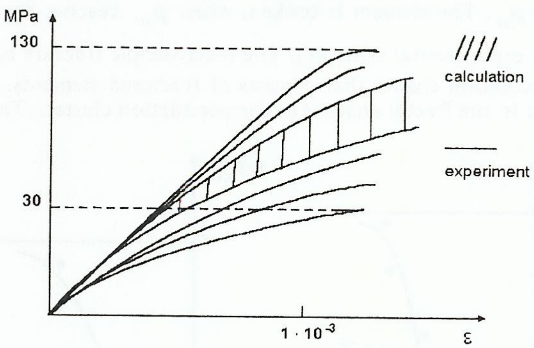


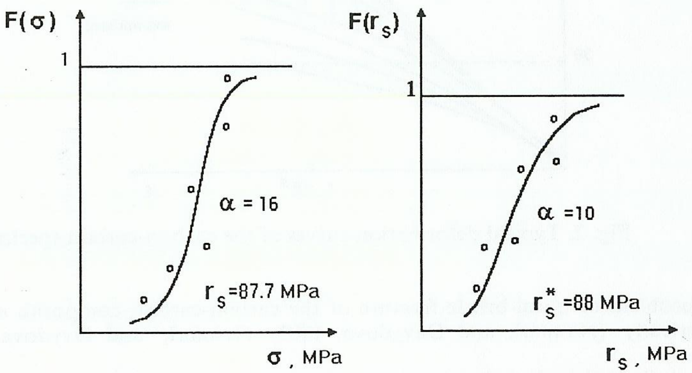
Fig. 2. Typical deformation curves of the carbon-carbon specimens.

The problem of quasi-brittle fracture of the carbon-carbon composite has been solved numerically (Naimark and Davydova, 1993; Naimark and Dvydova, 1994) by the

Table 1. Statistics of tensile test measurements

N series	dispersion	characteristic strength r_s, MPa	Weibull modulus α
1	360	79	4
2	130	83	8
3	45	88	16
4	25	77	17
5	43	117	20
6	19	81	23
7	20	91	25

finite element method based on the equilibrium equation $\partial\sigma_{ik}/\partial x_{ik}=0\ (k=1,2)$, the consti-tutive equation of elastic medium with microcracks (4), the kinetic equation of damage accumulation (10), and boundary condition and initial conditions. Phenomenological parameters (α,β,r_c) were determined from the results of statistical analysis (Table 1, Fig. 2, Fig. 3) of tensile test measurements on carbon-carbon composite specimens (Fig. 2). Simulation of the deformation and fracture processes starts with random assignment of (according to Table 1 and Fig.3) the strength r_s and Weibull modulus α to each element of the finite element approximation. At every step of the time we calculate a new value of the elastic modulus taking into account the influence of microcrack accumulation, and solve the elasticity problem and define the value of parameter p_{yy}. The element is broken, when p_{yy} reaches the critical value p_c ($p_c=3\cdot10^{-4}$ is an experimental estimate). The macroscopic fracture corresponds to the formation of a percolation cluster that consists of fractured elements. The final step of fracture simulation is the fractal analysis of the percolation cluster. The cluster appears



a) b)

Fig. 3. Distribution function of the specimen strength for one series (a); distribution function of characteristic strength for all series (b).

to be fractal in nature and with an increase of linear dimension L of the damaged array (Feder, 1988) its mass M (the number of failured elements) increases on the average as:

$$M(L)=AL^{D}, \tag{11}$$

where D is the fractal dimension, A is the effective amplitude. The mean value of A is obtained by averaging over the manifold realization of the percolation cluster. This approach was used to simulate failure development in carbon-carbon specimens with an initial macroscopic defect located in the center (the macrocrack is normal to the tension direction) with characteristic size N_a (Fig. 4). The dependence $M(L)$ consists of two linear parts with the slopes determined by the fractal dimension D. Simulation of damage has demonstrated that under loading the initial stage is accompanied by preferential failure of elements located in the vicinity of the macroscopic defect ($D=1$). The percolation cluster across the specimen results from coalescence of the

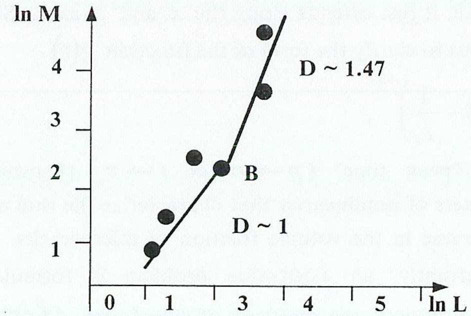


Fig. 4. Topological characteristics (fractal dimensions) of the damage cluster growth.

cluster originating from the initial macrodefect with clusters in its immediate neighbourhood ($D=1.4-1.7$). This is indicative of the qualitative change in the topology of damage accumulation process and the fracture mechanism replacement. The fractal dimension $D=1$ supports the validity of approaches of fracture mechanics only at the initial stage of crack evolution.

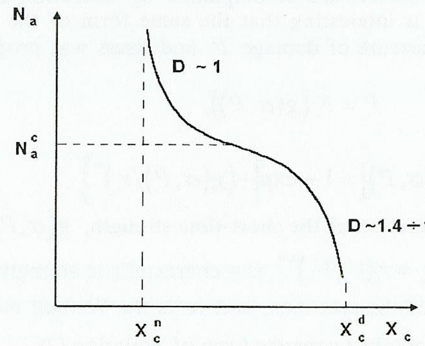


Fig.5. Dependence of critical concentration of failed elements x_c on initial macrodefect size N_a.

The results of statistical simulation for the time of complete formation of the main cluster are plotted in Fig. 5. This dependence involves two parts with two asymptotics x_c^n and x_c^d. The right part corresponds to the formation of branched cluster with the fractal dimension $D=1.4-1.7$. The transient region between these parts defines the critical size N_a^c of the initial defect, which specifies two qualitatively different mechanisms of fracture according to the size of initial defects. It means that the highest reliability of materials is reached when the size of initial defect is not larger than N_a^c. In this case, the material exhibits the maximum of "dissipative capacity" and the damage accumulation in the specimen is more homogeneous.

A fracture zone formation is connected with the nucleation of localized damage zones in the form of dissipative structures which are developed in a peak regime. This is accompanied by the generation of simple and complex structures of the localized failure and it reflects the self-similarity of damage development at various scale levels.

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