SOME DYNAMIC PROBLEMS OF STRUCTURAL FRACTURE MECHANICS

N.F. MOROZOV and Y.V. PETROV
Elasticity Department, St.-Petersburg State University, Petrodvorets,
St.-Petersburg, 198904, Russia

ABSTRACT

The problem of modeling of some of the specific effects of brittle fracture in the high loading rate conditions is discussed. An approach based on the system of fixed material constants describing macro-strength properties of the material is considered. New principles of material testing are analyzed. The corresponding incubation time criterion allows one to manage without the a priori given rate dependencies of dynamic strength and fracture toughness. New applications of the criterion to the problems of erosion and asymmetric impact loading are considered.

KEYWORDS

Dynamic fracture, incubation time, solid particle erosion, asymmetric impact loading, failure mode transition effect.

BASIC PARAMETERS OF STRUCTURAL FRACTURE MECHANICS

One of the principal parameters of linear fracture mechanics is the material structure size d describing the elementary cell of failure. The classical approaches by Griffith and Irwin consider this characteristic as a latent quality. It may be presented as dimensional combinations of surface energy, the critical stress intensity factor, static strength and elastic constants of the material:

$$d \sim \frac{\gamma \cdot E}{\sigma_c^2}, \qquad d \sim \frac{K_{Ic}^2}{\sigma_c^2} \tag{1}$$

The elementary cell of fracture has no unique physical interpretation. It may be interpreted in various ways, depending on the class of problems.

Neuber (1937) and Novozhilov (1969) suggested considering the material structure directly. The corresponding criterion requires that the mean normal stress in the range of material structure size d must be equal to the static strength of the material. In the plane deformation state case we have:

$$\frac{1}{d}\int_0^d \sigma(r)dr \le \sigma_c \qquad (2)$$

Assuming that in the simplest cases the criterion (2) gives the same results as the Irwin's critical stress intensity factor criterion, we obtain for the material structure size d the expression:

$$d = \frac{2}{\pi}\frac{K_{Ic}^2}{\sigma_c^2} \qquad (3)$$

Criterion (2) may be used in various cases in which the square root singularity and the appropriate energy balance do not work. Results obtained by means of the criterion (2) under the condition (3) are well confirmed by experiments in static cases (Morozov,1984).

Analysis of the dynamic experiments shows that the main contradictions of the traditional models appear when failure occurs during the short time intervals after the start of the loading process. Morozov and Petrov (1990) proposed an approach to the analysis of dynamic brittle failure based on the incubation time criterion:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_0^d \sigma(r,\theta,t')drdt' \le \sigma_c \qquad (4)$$

Here d and τ are material structure size and structure time of failure respectively, σ_c is static strength of the material, (r,θ) are polar coordinates, $\sigma(r,\theta,t)$ is tensile stress at the crack tip ($r=0$). The material structure size d is to be determined in accordance with the data of quasi-static tests of specimens containing cracks and in the case of plane strain it may be expressed by the simple formula (3). The material structure time τ is responsible for the dynamic peculiarities of the macro-fracture process and for each material it should be found from experiments. In accordance with this approach, σ_c, K_{Ic} and τ constitute the system of fixed material constants describing macro-strength properties of the material. Petrov (1991) has shown that the criterion (4) reflects the discrete nature of dynamic fracture of brittle solids.

In the case of virgin materials, the criterion (4) reduces to the form:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\sigma(t')dt' \le \sigma_c \qquad (5)$$

This form will be used later for the analysis one of the particular problems.

The analysis of the particular problems of dynamic fracture mechanics is associated with the appropriate choice of the parameter τ. We shall mention two basic cases:

a) The incubation time is defined by the material structure size of fracture:

$$\tau = \frac{d}{c} = \frac{d\sqrt{\rho}}{k} \qquad (6)$$

where c is the maximum wave velocity, ρ is the density of continuum, k is the constant depending on the deformation material properties. According to this definition, the incubation time has a physical meaning of the minimum time period required for the interaction between two neighboring material structure cells. Morozov *et al.* (1990) have shown that the definition (6) provides a good analogy between the incubation time criterion and the well-known experiments in the case of "defectless" materials.

b) The incubation time does not directly depend on the material structure size of failure. This takes place when a problem of initiation of the macro-crack growth is considered. Petrov and Morozov (1994) proved that in the case of macro-cracks, the material structure time τ can be interpreted as an incubation time in the well-known minimum time criterion proposed and explored by Kalthoff and Shockey (1977), Homma *et al.* (1983), and Shockey *et al.* (1986):

$$\tau = t_{inc} \qquad (7)$$

The aforementioned dependence of the fracture toughness on the loading history (Ravi-Chandar and Knauss, 1984) and the specific behavior of the short loading pulse threshold amplitudes can be explained and effectively analyzed by means of the incubation time criterion under the condition (7) (Petrov and Morozov, 1994).

PRINCIPLES OF THE MATERIAL STRENGTH PROPERTIES TESTING

In this section we outline some of the possible methods of description of the material strength properties. Table 1 represents the basic parameters and criteria to be used in testing of the materials. In Table 1 σ_c, K_{Ic} are the material constants, $\sigma_c^{dyn}(v)$, $K_{Id}(v)$ are the material functions that represent the dependencies of critical characteristics on the loading rate v.

The classical dynamics approach, resulting directly from the static strength theory and linear fracture mechanics, is based on two strength characteristics $\sigma_c^{dyn}(v)$, $K_{Id}(v)$, that are supposed to be material functions found from experiments.

Table 1

No.	Method	Material parameters	Criteria
1	Classic static	σ_c, K_{Ic}	$\sigma \le \sigma_c$, $K_I \le K_{Ic}$
2	Classic dynamics	$\sigma_c^{dyn}(v)$, $K_{Id}(v)$	$\sigma(t) \le \sigma_c^{dyn}$, $K_I(t) \le K_{Id}$
3	SRI International	$\sigma_c^{dyn}(v)$, $K_{Id}(v)$, t_{inc}	$\sigma(t) \le \sigma_c^{dyn}$, *minimum time criterion*
4	Incubation time approach	σ_c, K_{Ic}, τ	*Incubation time criterion*

The minimum time theory proposed by J.F. Kalthoff, D.A. Shockey and co-workers is based on the incubation time notion. It allows us to explain some of the principal dynamic fracture effects. On the other hand, the minimum time technique turns out to be too sophisticated for practical engineering.

It is seen from the Table 1 and the aforementioned results that the incubation time criterion combines the simplicity of the classical static method with the effectiveness of the SRI International approach. Basing on the system of fixed material constants, it enables us to predict the behavior of dynamic strength and dynamic fracture toughness from a unified viewpoint. Thus, the rate strength dependencies may be considered as calculated characteristics. The criterion may be applied for both the "defectless" and macro-cracked specimens.

APPLICATION TO THE PROBLEM OF EROSION

The solid particle impact velocity at the beginning of target material loss in the steady state erosion process can be considered as a critical or threshold velocity. In this section, the relation between the threshold velocity W and the incubation time τ is investigated. The possibility of using the incubation time criterion in determining the threshold erosion characteristics is established.

One of the principal features of the erosion process is that the target material surface is subjected to extremely short impact actions. The incubation time criterion (5) is an effective instrument for the analysis of this process. Here we shall consider the simplest way to obtain some of the basic threshold erosion characteristics.

Let a spherical particle of radius R fall with velocity v on the surface of an elastic half-space. Using the classical Hertz impact theory approximation (Kolesnikov and Morozov, 1989), we describe the motion of the particle by the following equation:

$$m\frac{d^2h}{dt^2} = -P, \tag{8}$$

where

$$P(t) = k(R)h^{3/2}(t), \qquad k(R) = \frac{4}{3}\sqrt{R}\frac{E}{(1-v^2)} \tag{9}$$

At the beginning of the impact event we have $dh/dt=v$. The maximum penetration h_0 occurs when $dh/dt=0$. Solving the equation (8), we obtain

$$h_0(v,R) = \left(\frac{5mv^2}{4k}\right)^{2/5}, \qquad t_0(v,R) = \frac{2h_0}{v}\int_0^1\frac{d\gamma}{\sqrt{1-\gamma^{5/2}}} = 2{,}94\frac{h_0}{v} \tag{10}$$

where t_0 is the duration of the impact event. The penetration function $h(t)$ can be approximated by the simple formula (Kolesnikov and Morozov, 1989):

$$h(t) = h_0 \sin(\pi\, t / t_0) \tag{11}$$

The maximum tensile stress occurring at the edge of the contact area is given by the expression (Lawn and Wilshaw, 1975):

$$\sigma(t,v,R) = \frac{1-2v}{2}\frac{P(t,v,R)}{\pi\, a^2(t,v,R)}, \qquad a(t,v,R) = \left[3P(t,v,R)(1-v^2)\frac{R}{4E}\right]^{1/3} \tag{12}$$

where the contact force $P(t,v,R)$ can be found by means of Eqs.(9)-(11).

Let $v=W$ denote the threshold velocity corresponding to the beginning of failure. We consider the function:

$$f(\tau,v,R) = \max_t \int_{t-\tau}^{t}\sigma(s,v,R)ds - \sigma_c\tau \tag{13}$$

According to (5), we determine the threshold velocity $v=W$ as the minimum positive root of the equation:

$$f(\tau,v,R)=0 \tag{14}$$

where τ is the incubation time for the target material.

The corresponding calculations were performed for the aluminum alloy B95 and the incubation time was determined according to formulae (3), (6): $\sigma_c = 460MPa$, $K_{Ic} = 37MPa\sqrt{m}$, $c = 6500m/s$, $\tau = 2K_{Ic}^2/(\pi\sigma_c^2 c) \approx 0.6\mu s$. The calculated dependence of the threshold velocity W on the value of radius R is presented in the Fig.1 by the solid curve. The static branch shows a weak dependence of the threshold velocity on the value of radius. On the contrary, the dynamic branch, corresponding to the small particles and very short loading pulses, represents a strong dependence of the critical velocity on the radius of particles. This behavior of the threshold velocity is observed in numerous experiments (Polezhaev, 1986), but it can not be explained on the basis of the traditional fracture mechanics. The dependence following from the conventional critical stress theory is also presented in Fig.1 by a dashed line.

FRACTURE AT THE CRACK TIP DUE TO ASYMMETRIC IMPACT LOADING

The incubation time criterion (4) provides an effective way of determination of the direction of failure in crack containing materials. The corresponding example can be given with respect to the shear band paradox (Kalthoff and Winkler, 1987; Lee and Freund, 1990). The particular boundary value problem to be analyzed is shown schematically in Fig.2. It can be described analytically by the following equations:

$$u_x = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}, \quad u_y = \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x}$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} - a^2\frac{\partial^2\varphi}{\partial t^2} = 0, \quad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - b^2\frac{\partial^2\psi}{\partial t^2} = 0$$

$$a = 1/c_d = \sqrt{\rho/(\lambda + 2\mu)}, \quad b = 1/c_s = \sqrt{\rho/\mu} \qquad (15)$$

$$\sigma_x(-l,y,t) = 0, \quad \sigma_{xy}(-l,y,t) = 0, \quad y < 0$$

$$u_x(-l,y,t) = \int_0^t v(t')dt', \quad \sigma_{xy}(-l,y,t) = 0, \quad y > 0$$

$$\sigma_y(x,\pm 0,t) = 0, \quad \sigma_{xy}(x,\pm 0,t) = 0, \quad x < 0$$

Here ϕ, ψ and a, b are longitudinal and shear displacement potentials and inverse waves speeds respectively; λ, μ and ρ are the Lame constants and the mass density; $v(t)$ is the loading velocity prescribed for non-negative values of its argument t. In our particular case we shall consider $v(t)=VH(t)$, where $H(t)$ is a Heaviside step function. The boundary value conditions are expressed in terms of the stress components.

The corresponding asymptotic solution for the stress components at the crack tip is given by the expression:

$$\sigma_{ij} = \frac{K_I(t)}{\sqrt{2\pi r}} \cdot f^{(I)}{}_{ij}(\theta) + \frac{K_{II}(t)}{\sqrt{2\pi r}} \cdot f^{(II)}{}_{ij}(\theta) + \sigma_{ij}^{(R)}(r,\vartheta,t) + O(r^{1/2}), r \to 0 \quad (16)$$

Here (r, θ) are polar coordinates at the crack tip. The asymptotic solution we are going to use for the analysis consists of both singular and regular parts. To determine the direction of crack propagation, we have to apply a criterion of fracture that is irrespective to the presence of singularity. The incubation time criterion provides us such opportunity.

To determine the direction of crack propagation we shall calculate time to fracture t_* on each line extended at the angle $\theta(-\pi/2 \le \vartheta \le \pi/2)$ from the crack tip. We assume that the crack grows in a direction θ for which time to fracture is minimum.

The singular part of the asymptotic solution was analyzed by Lee and Freund (1990). The corresponding expressions for the stress intensity factors in this particular problem can be taken from their paper. Combining it with the regular terms of the asymptotic (16) and using the criterion (4) we receive the following.

(a) At low loading velocity the crack does not propagate.
(b) When loading velocity exceeds the certain value, a failure is observed that in general corresponds to the fracture in quasi static pure mode-II conditions. In this instance, the crack catastrophically propagates at the angle around -78 deg with respect to the ligament, and its instability can be effectively predicted by the classical fracture mechanics (Lee and Freund, 1990).
(c) The further increase of loading velocity leads to the specific dynamic effect. The regular part of the asymptotic solution is very important for the early time range of the impact event, as the corresponding transient wave, produced by the impact, bears a tension that arises faster than the stress intensity factors. Thus, when loading velocity is big enough the crack propagates in a direction that is controlled by both parts of the

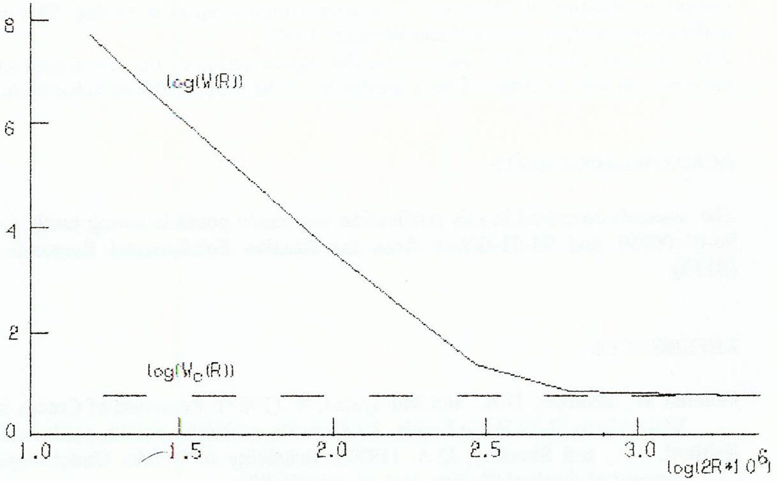

Fig.1. Dependence of the threshold velocity W (m/s) on the value of radius R (m) of erodent particles calculated for aluminium alloy B95. The dependence corresponding to the classical fracture criterion is plotted by dashed line.

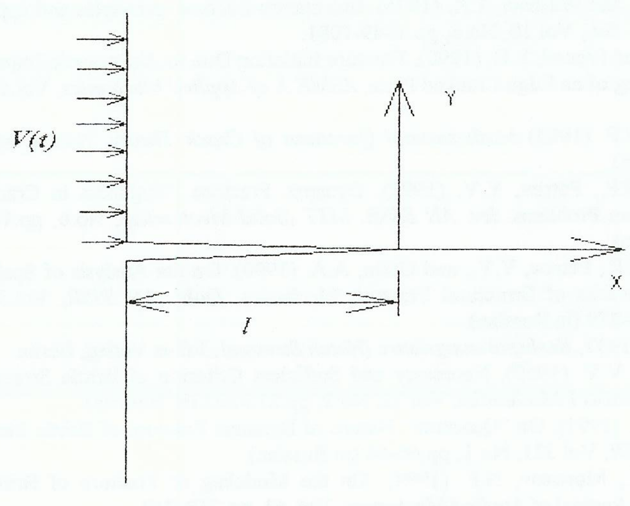

Fig.2. Asymmetric impact loading problem

asymptotic solution. It turned out to be approximately equal to +4 deg. This was observed in the experiments by Kalthoff and Winkler (1987).

Therefore, the analysis has shown that the regular terms of the stress field asymptotic at the crack tip may be essential for a prediction of the specific failure behavior in dynamic.

ACKNOWLEDGMENTS

The research described in this publication was made possible owing partly to the Grants 96-01-00391 and 95-01-00644 from the Russian Fundamental Research Foundation (RFFI).

REFERENCES

Homma, H., Shockey, D.A., and Murayama, Y. (1983). Response of Cracks in Structural Materials to Short Pulse Loads. *J. Mech. Phys. Solids*, Vol.31, pp.261-279.

Kalthoff, J.F., and Shockey, D.A. (1977). Instability of Cracks Under Impulse Loads. *Journal of Applied Physics*, Vol.48, pp.986-993.

Kalthoff, J.F., and Winkler, S. (1987). Failure Mode Transition at High Rates of Shear Loading, *Impact '87 (International Conference on Impact Loading and Dynamic Behavior of Materials)*, May, Bremen, West Germany.

Kolesnikov, Y.V., and Morozov, E.M. (1989). *Contact Fracture Mechanics*, Moscow, Nauka (in Russian).

Lawn, B.R., and Wilshaw, T.R. (1975). Indentation fracture: principles and application. *J. Mater. Sci.*, Vol.10, No.6, pp.1049-1081.

Lee, Y.J., and Freund, L.B. (1990). Fracture Initiation Due to Asymmetric Impact Loading of an Edge Cracked Plate, *ASME J. of Applied Mechanics*, Vol.57, pp.104-111.

Morozov, N.F. (1984) *Mathematical Questions of Crack Theory*. Nauka, Moscow (in Russian).

Morozov, N.F., Petrov, Y.V. (1990). Dynamic Fracture Toughness in Crack Growth Initiation Problems. *Izv. AN SSSR. MTT (Solid Mechanics)*, No.6, pp.108-111 (in Russian).

Morozov, N.F., Petrov, Y.V., and Utkin, A.A. (1990). On the Analysis of Spalling in the Frameworks of Structural Fracture Mechanics. *Dokl. AN SSSR*, Vol.313, No.2, pp.276-279 (in Russian).

Neuber, H., 1937, *Kerbspannungslehre (Notch Stresses)*, Julius Verlag, Berlin.

Novozhilov, V.V. (1969), Necessary and Sufficient Criterion of Brittle Strength, *Prikl. Matematika I Mechanika*, Vol.33, No.2, pp.212-222 (in Russian).

Petrov, Y.V. (1991). On "Quantum" Nature of Dynamic Fracture of Brittle Media. *Dokl. AN SSSR*, Vol.321, No.1, pp.66-68 (in Russian).

Petrov, Y.V., Morozov, N.F. (1994). On the Modeling of Fracture of Brittle Solids. *ASME Journal of Applied Mechanics*, Vol. 61, pp.710-712.

Polezhaev, Y.V. (1986). *Thermo-Gas-Dynamic Testing of Aircraft's*. Moscow, Izd. MAI (in Russian).

Ravi-Chandar, K., and Knauss, W.G. (1984). An Experimental Investigation into Dynamic Fracture: 1. Crack Initiation and Arrest. *Int. J. Fract.*, Vol.25, pp.247-262

Shockey, D.A., Erlich, D.C., Kalthoff, J.F., and Homma, H. (1986). Short Pulse Fracture Mechanics. *Eng. Fract. Mechanics*, Vol.23, pp.311-319.