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ON MIXED-MODE DYNAMIC CRACK INITIATION IN BRITTLE SOLIDS

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ABSTRACT

Mixed mode dynamic crack initiation in brittle solids has been investigated. The selected materials were glass and PMMA. The experiments involved systematic combinations of mode I and II loading at various impact velocities. Specific experiments were carried out which favored dominance of one of the two modes using sharp-notched specimens. Results show that regardless of the dominant mode, specimen geometry and material, the initial kink angle between the crack and the original notch line increases with the impact velocity. These results are presented and discussed in terms of a simple maximum (potential) energy release rate criterion which dictates the operating fracture mechanism (opening or shear). Good agreement is obtained between the observed and predicted initial kink angle values. Conclusions regarding dynamic crack initiation are drawn.

KEYWORDS

Mixed mode, PMMA, glass, dynamic crack initiation, kink angle, fracture time.

INTRODUCTION

Experimental techniques for the investigation of crack-tip dynamic fields are of two basic kinds. The first is purely experimental and aims at direct crack-tip monitoring using e.g. optical methods (Kalthoff, 1988 ; Rosakis, 1993). The second relies on the combination of numerical calculations and experimental measurements to access the sought data (Kobayashi, 1987).
In this spirit, Bui and Maigre (1988) developed the path independent H-integral which can be used for the direct determination of the dynamic stress intensity factors (SIF) from a recording of the experimental forces and/or displacements on the boundaries of a cracked structure. This concept is valid for two-dimensional linear elastic materials and stationary cracks subjected to transient loading. The H-integral was successfully applied to the testing of small scale yielding (or brittle) materials using the compact compression specimen (CCS) and a Kolsky bar (Bui *et al.*, 1992; Rittel *et al.*, 1992). In these experiments, the symmetric specimen is impacted on one side only so that the loading is predominantly of mode I type with a minor mode II component. The contribution of each mode was assessed using linear superposition techniques (Maigre and Rittel, 1993).

The dependence of the initial crack path on the mode mixity has long been investigated for quasi-static loading using a maximum normal stress criterion, maximum mode I stress intensity factor or minimum strain energy density. In fact it was shown that these various criteria predict rather similar results (Tirosh, 1977).
Recent work on mode II dynamic crack initiation in high strength steels (Kalthoff, 1988) and polycarbonate (Ravi-Chandar, 1995) disclosed interesting phenomena about the dependence of the crack orientation on the impact velocity and the fracture mechanism. The analysis of Lee and Freund (1990) showed that this kind of experiments includes a negative mode I component. However this study did not address the issue of the initial kink angle.

Therefore, the purpose of this work is to report and analyze systematic comparative experiments about dynamic crack initiation in notched specimens subjected either to dominant mode I or to mode II conditions. The experiments are analyzed using the same framework and a simple criterion is adopted to describe the observed initial crack trajectories.

THEORETICAL

For a structure with boundary S, made of a linear-elastic material containing a stationary crack of length a, it has been shown that the experimental forces $\underline{T}^{exp}$ and displacements $\underline{u}^{exp}$ can be convoluted (*) with reference fields $\underline{T}^{ref}$ and $\underline{u}^{ref}$ to yield the following path-independent H-integral:

$$H(\tau) = \frac{1}{2}\int_S \left\{\underline{T}^{exp} * \frac{\partial \underline{u}^{ref}}{\partial a} - \underline{u}^{exp} * \frac{\partial \underline{T}^{ref}}{\partial a}\right\} dS = \frac{1-\nu^2}{E} K_{\alpha d}^{exp} * K_{\alpha d}^{ref} \quad \alpha=\mathrm{I,II} \qquad (1)$$

From eq. 1 it appears that the (global) forces and displacements are related to the (local) SIFs so that the experimental SIFs can be extracted by solving a linear convolution equation for any experimental crack length of length a to a +da.
Furthermore, writing now H as the superposition of pure mode I and II components, the contribution of each mode can be exactly identified (Maigre and Rittel, 1993).
Another approach for the identification of the dynamic stress intensity factors relies on superposition (Freund, 1990). Denote by $\hat{k}_T$ and $\hat{k}_U$ the dynamic SIF corresponding to a unit impulse force or displacement, respectively, with appropriate boundary conditions for mode I or II. The evolution of the dynamic stress intensity factors $K_T(t)$ and $K_U(t)$ are given by:

$$K_{T\alpha}(t) = \int_S T * \hat{k}_T \, dS \quad \text{or} \quad K_{U\alpha}(t) = \int_S u * \hat{k}_U \, dS \qquad \alpha=\mathrm{I \ or \ II} \qquad (2)$$

Maigre and Rittel (1995) showed that using both the experimental recordings of the forces and the displacements on the specimen's boundaries in eqs. (1) or (2), one can determine the "bulk" fracture time as the time at which the specimen halves start to separate. In other words, "bulk" fracture time is that time at which the evolution of K_T and K_U start to diverge.
Whereas either of the two approaches can be applied to solve the same problem, a common key parameter is the accurate determination of the fracture time.

EXPERIMENTAL

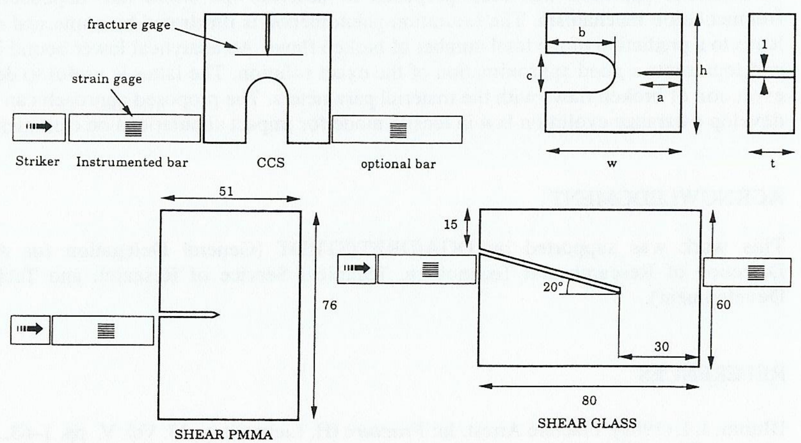
Two experimental materials were chosen for this study: glass and PMMA (polymethylmetacrylate). Both are assumed to behave as linear elastic materials at high strain rates.
Dominant mode I experiments were carried out on Compact Compression Specimens (CCS) (Rittel and Maigre (1993)). Dominant mode II experiments were carried out on notched plates. Notches with approximately 0.2mm root radius were machined in the PMMA samples. 1mm wide square-ended slits were machined in the glass samples.
Transient loads were applied and measured using a conventional Kolsky bar for the glass specimens and a single instrumented bar for the PMMA specimens (one point impact). The same basic equations are used to determine the interfacial forces and displacements using once all three incident reflected and transmitted signals (Kolsky bar) and once the incident and reflected signals only. The experimental setups and specimens dimensions are shown in figure 1 and table 1 respectively. A total number of 12 glass and 4 PMMA specimens of each kind (CCS and notched plates) were tested.

The onset of crack propagation was detected using single wire fracture gages and the "bulk" fracture time assessment technique. In some experiments with the glass specimens we used a synchronized array of 4 high speed video cameras to obtain a series of 4 pictures of the specimen under impact. These pictures were used for qualitative assessment of the crack-tip state, i.e. presence or absence of a propagating crack at selected times.

Table 1: Dimensions of the compact compression specimens (in mm)

CC-Specimen	PMMA	GLASS
a	12.7	20
b	25	35
c	17	20
h	46	80
w	51	70
t	12.7	15

Figure 1: Experimental setup and specimens



RESULTS

The results obtained for selected PMMA and glass specimens are summarized in tables 2 and 3 respectively. Mode I experiments were analyzed using eq. (1) whereas mode II experiments were analyzed using eq. (2).

Table 2: Experimental results for PMMA specimens. Mode I: compare observed and predicted angles. Mode II: note the transition from shear to opening initiated kink with increased impact velocity.

spec.	V_imp	t_{gage}	t_{bulk}	obs. α	pred. α	failure mode
	[m/s]	[μs]	[μs]	[°]	[°]	
CCS5	10	87-87	≤ 80	20 - 22	0-30	opening
CCS4	20	83-97	≤ 70	61 - 62	≥ 30	opening
SHEAR2	14	not used	not used	≈10	≈ 10	shear
SHEAR3	22	not used	not used	69 - 74	> 50	opening

Table 3: Experimental results for glass specimens. In this series of experiments, advantage is taken from photographic recordings showing the presence or absence of cracks.

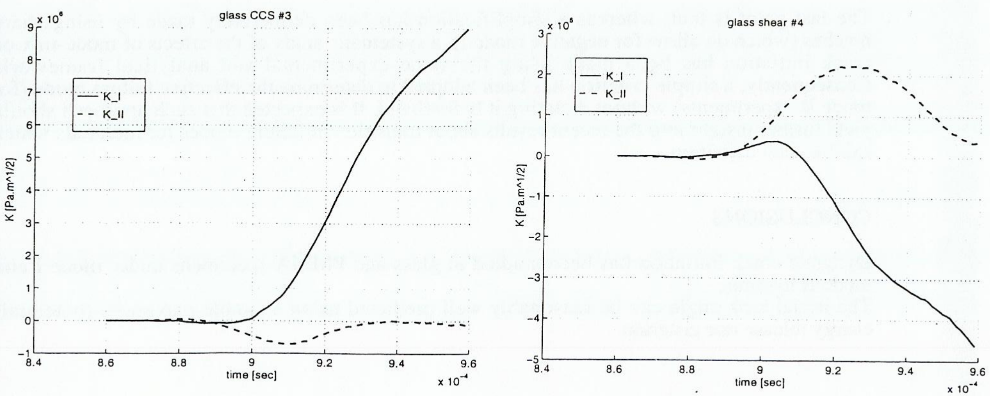
spec.	V_imp	tgage	tbulk	obs. α	pred. α	failure mode	observed crack
	[m/s]	[μs]	[μs]	[°]	[°]		
CCS5	2.1	161	≈ 110	40 - 54	>30	opening	at t=63μs
CCS9	1.4	206	≈ 175	21 - 42	≈ 10	opening	at t=116 - 124μs
SHEAR1	1.8	324	not used	65	60 – 75	opening	at t=31 - 61μs
SHEAR5	1.0	∞	not used	15	–	unknown	no, up to t=120μs

The results show that as the impact velocity is increased, the kink angle between the propagating crack and the original notch line increases both in mode I and mode II experiments regardless of the tested material and experimental setup.

For mode I (CCS) experiments, a very good agreement is noted between the two determinations of the fracture time ("bulk" and fracture gages) for glass specimens. For the more brittle glass specimens, the agreement is less good and readings from the fracture gages were noted to lag behind the "bulk" fracture time.
For the mode II experiments, the fracture gages provide little reliable information as they seldom fracture. In fact, it appears that fracture time determination based on specimen separation (either from fracture gages or from numerically equivalent procedures such as bulk fracture time) suffers from a lack of accuracy when net separation (crack opening) of the specimen does not occur. This becomes particularly evident in some low impact velocity mode II experiments in which the crack propagated for a few millimeters before arresting, without total separation of the specimen in two (several) pieces. This observation is further substantiated when the apparent fracture times of glass specimens (from the gage readings) were confronted with photographic recordings showing the nucleation and early propagation of cracks long before the fracture gages reacted!

Typical evolution of the dynamic stress intensity factors for glass specimens are shown in figure 2 for mode I and mode II experiments.

Figure 2: Evolution of K_I and K_{II} for glass CCS #3 and SHEAR #4 specimens



For CCS experiments it is noted that while mixed-mode is present, mode I is by far the dominant mode. Also, as expected, mode I is positive meaning that the crack tends to open. Examination of typical curves for mode II experiments show that regardless of the material and specimen's geometry, mode II is dominant while mode I is negative (after a short positive period with the glass plates). Consequently a compressive closure mode is superposed on the dominant shearing mode.

DISCUSSION

Dominant mode I: The experiments carried out on glass and PMMA show the same trend for an increase in the kink angle with impact velocity. There appears to be a well defined correspondence between the bulk and the gage fracture times for PMMA and to a lesser extent for glass.
Dominant mode II: Here too, the kink angle increases with impact velocity. However, the determination of fracture time is much more delicate in this series of experiments. The fracture gages yield little or no reliable data, due to the nature of the failure process.

As previously stated, for dynamic fracture testing a key parameter is the determination of the onset of crack propagation which in turn determines the value of the dynamic fracture toughness (from the evolution of the stress intensity factor(s)).

Despite the scarcity of experimental methods to assess this parameter one can think of additional tests to validate the result obtained from e.g. gage readings. For this purpose, one can check for consistency between the observed value of the initial kink angle and the value predicted from a mixed-mode fracture criterion, as discussed next.
Whereas, there is no unique criterion to interpret mixed mode crack initiation, a useful criterion is that of the maximum energy release rate (G) criterion. Keeping in mind that G is strictly defined for an increment of (usually self similar crack) propagation, one can define g, the angular distribution of the energy release rate *prior to crack propagation* (Lawn (1975)), as:

$$g(\theta, K_I, K_{II}) = \frac{1-\nu^2}{E}\begin{cases} K_I'^2(\theta) + K_{II}'^2(\theta) & \text{if} \quad K_I'(\theta) \geq 0 \\ K_{II}'^2(\theta) & \text{if} \quad K_I'(\theta) < 0 \end{cases} \qquad (3)$$

$$\text{where} \quad K_I'(\theta) = K_I f_{\theta\theta}^{I} + K_{II} f_{\theta\theta}^{II} \quad \text{and} \quad K_{II}'(\theta) = K_I f_{r\theta}^{I} + K_{II} f_{r\theta}^{II}$$

Nuismer (1975) showed that for positive K_I the maximum g criterion is equivalent to the maximum hoop stress ($\sigma_{\theta\theta}$) criterion of Erdogan and Sih (1963).
However, when the mode I component becomes negative this criterion can no longer be used as such to account for attempted crack closure and possible friction between the crack faces (Broberg, 1983; Mellin, 1986).
For actual notches the negative mode I will cause a finite amount of closure prior to interpenetration of the notch flanks. Assuming that the crack will initiate during the closure phase, it appears that no energy can be dissipated through the (negative) mode I component. Consequently it is proposed that the energy will be dissipated by a shear mechanism only (see also Ravi-Chandar, 1995). Here, we neglect frictional problems as we consider only the very initial phase of the crack initiation from the notch tip (noting that this might not be possible for an actual preexisting sharp crack). Therefore it is proposed that the crack will initially propagate in a direction which maximizes g(θ) through the mode II component only.
More generally stated, the crack will select the initial propagation direction (angle, θ_{kink}) which maximizes the (potential) energy release rate g, thus selecting an opening or a shearing mechanism. It is also noted that in such criterion, both the opening and shearing mechanism are equally weighted as potential fracture mechanisms. Therefore, the criterion can be written as

$$g(\theta_{kink}, K_I, K_{II}) = \max_{\theta}\{g(\theta, K_I, K_{II})\} \qquad (4)$$

This criterion is represented graphically in figure 3. In figures 4 and 5 we have plotted the above mentioned criterion for the experimental results on PMMA specimens (table 2). It should be emphasized that in this analysis, the failure mechanism is not supposed *a priori*. Rather, it is naturally selected through (potential) energy considerations.

Figure 3: Kink angle vs mixity ratio (arctan $[K_{II}/K_I]$).

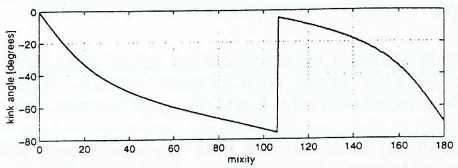


Figure 4: Kink angle and evolution of K_I and K_{II} for PMMA CCS #5 and #4. Fracture mode is opening.

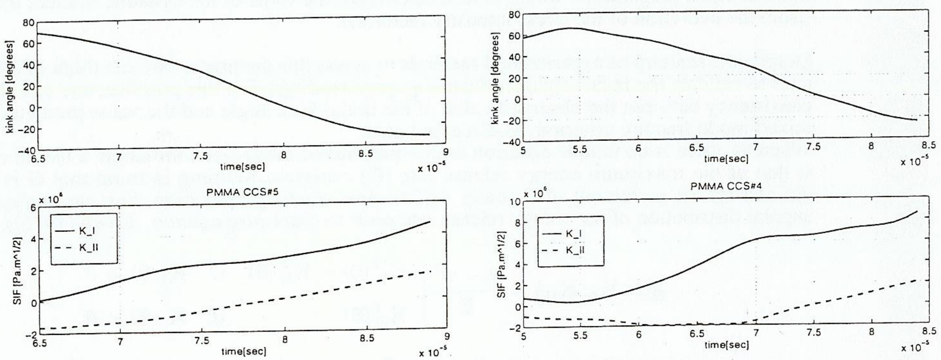
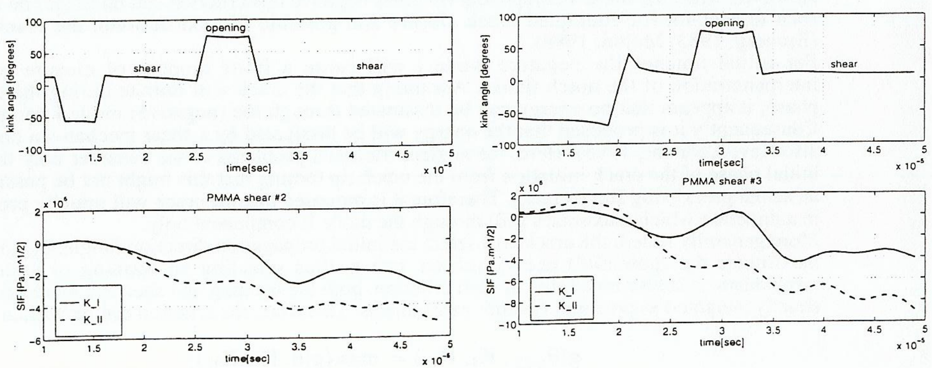


Figure 5: Kink angle and evolution of K_I and K_{II} for PMMA SHEAR #2 and #3 specimens. Fracture mode alternates between opening and shear.



Dominant mode I: In this type of experiments, both the glass and the PMMA specimens fail by opening mode. When comparing the observed kink angle with that corresponding to bulk fracture time, a good agreement is noted (table 2 and figure 4a and 4b).
For the glass specimens (table 3 and evolution not shown here), the fracture gages indicate rather long fracture times. For the compact specimen #5 a crack was observed to run at the early time of t=63µs. The observed angle value of 40°-54° is quite consistent with the predicted range of fracture times and angles. Yet, it contradicts fracture gage readings. For the other specimen (#9) neither the bulk nor the gage fracture time correspond to the observed angle of 21°–42°. Such values are observed for much earlier fracture times in the range of 65–75 µs.

Dominant mode II: For this type of experiments, the fracture mode may either be opening or shearing, contrary to the previous case.
For the PMMA specimens, its is noted that the lower velocity impact induces a shear failure for mode II specimen #2 for which the predicted and observed angle values are in excellent agreement at bulk fracture time. This also applies to the higher velocity impact which this time induced an opening type of failure (table 2 and figures 5a and 5b).
Concerning mode II glass specimens (table 3 and evolution not shown here), an undetected crack at t=31µs was clearly observed to propagate at t=61µs in specimen #1. By looking at the predictive graph, it appears that the large value of the observed angle can only correspond to an opening mode of failure, as observed in the case of PMMA specimens.
By contrast, for specimen #5 (and others alike) which was impacted at a smaller velocity, no crack could be detected for times up to 120µs. Since the analysis could not be carried out for such long times (due to multiple wave reflections in the specimen) the failure mechanism could not be accurately determined.

The present study suggests that the actual fracture process under dynamic conditions comprises a nucleation and a growth step. Whereas it may be difficult to distinguish these steps in quasi-static loading, dynamic fracture tests of brittle materials show that specimen separation is due to the later (perhaps final) stages of the flaw growth. This is particularly evident in the more brittle of the two materials tested, i.e. glass. For this material there is no correspondence between the bulk or gage fracture times and the fact that cracks are observed in the specimen at much earlier times. Moreover, these initial cracks propagate at an angle which matches the values predicted by the maximum (potential) energy release rate criterion. For the less brittle PMMA, there is a much better correspondence between the measured fracture times and the kink angles, regardless of the loading mode. These observations call for an accurate definition of fracture (nucleation, growth or both stages).

Secondly, the consistency between the kink angle and the fracture time should be of assistance in discriminating the true effects of the loading rate on the fracture toughness from the influence of the method adopted for the determination of fracture time.

The last point is that, whereas a simplification has been deliberately made by using sharp notches (which do allow for negative mode I), a systematic study of the effects of mode-mix on crack initiation has been made using the same experimental and analytical framework. Consequently, a simple criterion has been adopted to determine the effective failure mode (for mode II experiments) without dictating it beforehand. It is expected that such approach should yield further insight into the recent results about transition in failure modes for materials which exhibit such transition.

CONCLUSIONS

Dynamic crack initiation has been studied in glass and PMMA specimens under mode I and mode II loading.
The initial kink angle can be reasonably well predicted using a simple maximum (potential) energy release rate criterion.

The observation of the initial kink angle is an important experimental data which should corroborate other experimental results such as fracture gage readings, thus assisting in fracture time determination.
For actual fatigue cracks in which no closure is allowed, this analysis should be modified to account for friction between the crack faces.

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