### DYNAMICS OF BRITTLE FRACTURE

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#### **ABSTRACT**

The basic problem of dynamic fracture is examined. It is shown through experimental observations of the fracture surface morphology and its interpretation that the primary reason for the inability of the elastodynamic fracture theory to predict the behavior of brittle cracks is the key role that the crack tip process zone evolution plays in controlling the energy dissipation rate. Fracture surface observations at different scales reinforce these conclusions. A nucleation and growth modeling approach to the dynamic fracture problem is illustrated.

#### **KEYWORDS**

Dynamic fracture, brittle materials, fractography, microcracks

#### INTRODUCTION

Brittle fracture in solids has attracted much attention over most of this century from physicists as well as engineers. Two principal avenues have been followed in investigations of this problem; the first avenue concerns the determination of the dynamic strength of a bulk material and began with the investigations of the father-son team of Hopkinsons, (Hopkinsons, 1901, 1912). Their interest was in determining the strength of materials under dynamic loading conditions. Subsequent work by Rhinehart and Pearson, (1956), Kolsky (1963) and others illuminated the mechanisms responsible for spalling due to tensile waves. A detailed characterization of the damage generated by tensile stress pulses was obtained by Curran *et al.*, (1977), who performed a very innovative experiment wherein short tensile pulses of different amplitudes were sent through a specimen and the resulting microcrack statistics characterized. From these experiments Curran *et al.*, (1977) concluded that the mechanism for brittle fracture in a bulk material was the nucleation, growth and coalescence of microcracks. A complete survey of the current state-of-the-art of this problem can be found in Zukas (1990).

The second avenue of investigations considers the growth of a dominant crack under dynamic loading; the main impetus to this line of inquiry appeared with the development of a special high speed camera by Schardin and Struth (1938) and with the resulting spectacular discovery that

cracks in glass grew at a constant speed of about 50% of the Rayleigh wave speed. The applied mechanics community has pursued, slowly at first, but with intensity in the 70's and 80's the problem of a dynamically growing crack. The development of high speed photography aided in the experimental measurement of crack speeds and the formulation of continuum models of cracks through elastodynamic analyses facilitated a description of the crack tip state. The current state of the analytical description of dynamic fracture is expounded in a monograph by Freund (1990). The continuum description of dynamic fracture incorporates an idealized point dissipation process at the crack tip and hence ignores the dynamics of the evolution of the dissipation process from the dynamic crack growth modeling. A number of dynamic fracture observations in brittle materials still remain unresolved by this continuum model; qualitative mechanistic models, however, are available for the interpretation and understanding of the observed fracture behavior and these will be the focus of this paper. While most of the early experiments in dynamic fracture were concerned with determining the crack speed and relating it to a "crack driving force" and thereby generating a dynamic crack growth criterion, (Dally, 1979, Kobayashi and Mall, 1983, Kalthoff, 1983, Rosakis and Zehnder, 1985 among others), Ravi-Chandar and Knauss (1984a-d), and Knauss and Ravi-Chandar, (1985) in a series of papers examined the underlying mechanisms in a dynamic fracture process and suggested that dynamic crack growth problem is intimately linked to the problem of multiple flaw generation, growth and coalescence. In this view, the macroscopic crack provides primarily a highly localized stress and strain concentration associated with locally high strain rates to which the material at the crack tip responds by microfracture and/or hole growth. The dynamics of crack growth will then be dictated not only by the dynamics of the transport of energy to the process region by waves from the loading boundaries, but also by the dynamics of the evolution of the state of the process zone; this latter part has been neglected in all dynamic fracture modeling of brittle materials until recently. In an effort to understand brittle fracture from an atomistic point of view, lattice dynamics (Marder and Gross, 1995) and molecular dynamics (Abraham et al., 1994, Nakano et al., 1995) descriptions of the bond breaking process have been employed recently. However, the scaling from the atomic level to the scale of the process zone (tens of microns) to the macroscopic level of millimeters is not easily accomplished and thus these models only provide a qualitative or phenomenological interpretation of dynamic fracture.

In this paper the classical continuum theory of dynamic fracture is first briefly described to provide the point of departure for the subsequent exposition. The range of validity of the continuum theory and the main outstanding issues in the continuum theory are then described. This is followed by a detailed mechanistic investigation of the dynamic fracture process. It is demonstrated that the microcracking in the process zone plays a crucial role in the dynamics of brittle fracture; the microcracks themselves are shown to have a substructure at an even smaller scale. Links to the results of atomistic modeling and possible continuum modeling are described.

# CLASSICAL ELASTODYNAMIC FRACTURE THEORY

In order to describe the classical elastodynamic fracture theory, it is helpful to consider the specific example of a two-dimensional, isotropic, homogeneous elastic medium with a semi-

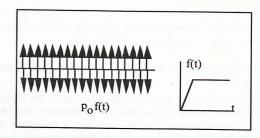


Figure 1. Geometry and loading configuration of the pressurized semi-infinite crack problem. In the experiments, the specimen was large enough to eliminate the influence of the reflected waves until 150 µs after initial loading; the rise time of the pulse was about 25 µs.

infinite crack as illustrated in Figure 1. The crack is initially at rest at the origin of the coordinate system and the applied loading corresponds to a time dependent pressure on the surfaces of the semi-infinite crack. In addition, a criterion for the onset and subsequent growth of the dynamic crack is required; this criterion should predict the crack initiation (and crack arrest) and the subsequent propagation of the crack (crack growth criterion)<sup>1</sup>. This criterion is typically taken to be an extension of Griffith's energy balance ideas for equilibrium cracks to the dynamic problem by including kinetic energy terms and considering the energy rate balance. Furthermore, the energy expended per unit area of crack extension,  $\Gamma$ , is taken to be independent of the rate of crack extension. The result of this assumption is that the details of the fracture process are ignored and only the net energy consumption in the fracture process is taken into account; this is, in fact, the main attraction of the Griffith type fracture criterion even in the quasi-static formulation. This essentially decouples the dynamic fracture criterion from the elastodynamic problem used to calculate the mechanical fields in the body; thus, the elastodynamic problem is solved first for arbitrary velocity of crack,  $\nu$ , and then the appropriate velocity is determined by applying the dynamic fracture criterion (energy rate balance).

The main result of the elastodynamic analysis of the running crack problem is that the stress field in the vicinity of the crack tip is square-root singular; the Cartesian components of the stress tensor (in polar coordinates r,  $\theta$ ) are given as

$$\sigma_{\alpha\beta}(r,\theta) = \frac{K_I^{dyn}(t,\nu)}{\sqrt{2\pi r}} f_{\alpha\beta}(\theta,\nu) \qquad \alpha,\beta = 1,2 \qquad \text{as } r \to 0$$
 (1)

where  $f_{\alpha\beta}(\theta, \nu)$  is a known function of the angle  $\theta$  and the crack speed  $\nu$ . The amplitude of the singularity is then characterized by the single parameter,  $K_I^{dyn}(t,\nu)$ , called the *dynamic stress intensity factor*. Analyses have focused primarily on the determination of admissible dynamic stress intensity factor variations for different geometries and loading configurations, assuming

While the crack growth criterion should predicit the direction as well as the speed of crack extension, generally only rectilinear crack propagation is considered.

constant velocity of crack propagation. The correct variation of the dynamic stress intensity factor is selected from the admissible solutions when the velocity of the crack is determined by imposing the energy rate balance criterion as discussed next.

Instead of dealing with the energy flux, in analogy with quasi-static fracture, the *dynamic energy* release rate G(t,v) is defined as the energy released per unit crack extension. Note that the dynamic energy release rate is not a time-rate quantity, but is the energy release per unit crack area. The use of the dynamic energy release rate as the fracture criterion eliminates the need to know the details of the fracture process and its evolution. If the stress field near the crack tip is completely governed by the dynamic stress intensity factor, then the dynamic energy release rate can be expressed in terms of the dynamic stress intensity factor as:

$$G(t,v) = \frac{1-\mu^2}{E} A_I(v) \left[ K_I^{dyn}(t,v) \right]^2$$
 (2)

where 
$$A_l(v) = \frac{v^2 \alpha_d}{(1-\mu)C_s^2 D}$$
,  $D = 4\alpha_d \alpha_s - \left(1+\alpha_s^2\right)^2$ ,  $\alpha_d = \sqrt{1-v^2/C_d^2}$ ,  $\alpha_s = \sqrt{1-v^2/C_s^2}$  and  $C_d$ ,  $C_s$ 

are the dilatational and distortional wave speeds. The dynamic crack growth criterion is obtained simply by equating<sup>2</sup> the energy release rate to the energy required for fracture,  $\Gamma$ :

$$\frac{1-\mu^2}{E}A_I(v)\left[K_I^{dyn}(t,v)\right]^2 = \Gamma. \tag{3}$$

This represents the crack tip equation of motion and indicates a relationship between the crack-driving force, characterized by the dynamic energy release rate, and the crack speed,  $\nu$ . Clearly, as  $\nu \to C_R$ , the Rayleigh surface wave speed,  $A_I(\nu) \to O(1/(C_R - \nu))$ ; this is the main argument for considering  $C_R$  to be the limiting crack speed.

This criterion has also been expressed in terms of the dynamic stress intensity factor as

$$K_I^{dyn}(t, \nu) = K_{ID}(\nu) \tag{4}$$

where  $K_{ID}(\nu)$  is a material property called *dynamic fracture toughness*, to be characterized through experiments and is the dynamic analog of the plane-strain fracture toughness in quasistatic fracture. However, the dynamic fracture toughness incorporates much more than simply the fracture energy of the surface; all energy dissipated in the fracture process including that due to plastic work or other fracture processes is included as the fracture energy.

The elastodynamic theory as summarized above appears to work well in certain situations. In nominally brittle materials, Ravi-Chandar and Knauss (1984a) compared experimentally determined stress intensity factor variations for the pressurized semi-infinite crack problem with the theoretical predictions of Freund (1972) and found good agreement as long as the load levels were small and crack speeds were below about  $0.2\,C_p$ . In metallic materials, where the fracture process

is dominated by significant plastic deformation in the neighborhood of the crack, experimental determination of the dynamic fracture toughness has been quite successful (Rosakis and Zehnder, 1985) and theoretical models (Freund, 1990) incorporating plasticity effects in the crack tip vicinity appear to predict the experimental trend quite accurately. However, there remain some unresolved issues in the dynamic fracture of nominally brittle materials, three of which will be discussed here. First, the limiting crack speed obtained from the energy balance approach to brittle fracture is the Rayleigh wave speed  $C_R$  as indicated above; however, cracks are seldom observed to grow speeds greater than about  $0.5\,C_R$  (Ravi-Chandar and Knauss, 1984c). Second, the existence of a unique relation between the crack speed and the instantaneous stress intensity factor has been questioned, from different perspectives - lack of singular field dominance to specimen dependence. Finally, running cracks exhibit spectacular branching patterns that are not predictable by the elastodynamic fracture theory.

The main reason for the breakdown of the classical elastodynamic fracture theory in these instances is the insistence that the elastic (or the elastic-plastic) continuum dictate the energy dissipation process in the fracture process zone; i.e. the insistence that the crack tip processes **must** consume energy at whatever rate it is delivered to it by the surrounding continuum simply by altering its speed. In fact, the dynamics of the fracture processes - the dynamics of the multiple crack problem alluded to in the introduction - cannot be neglected in formulating the dynamic crack problem and this is considered to be the key in determining the crack speed, limiting speed and promoting branching (Knauss and Ravi-Chandar, 1985). That the fracture process zone plays a key role can be inferred simply from a cursory examination of the fracture surface: whereas the classical theory assumes a constant dissipation per unit extension, the fracture surface roughness always evolves with time indicating a continuous change in the dissipation per unit extension. The failure processes in the crack tip region and its evolution will be examined in detail in the next section, providing a model for dynamic crack growth.

There are other questions concerning the dominance of the square-root singular field (Ma and Freund, 1986, Ravi-Chandar and Knauss, 1987, Krishnaswamy and Rosakis, 1991), particularly over a size much larger in comparison to the fracture process zone, (Ravi-Chandar and Knauss, 1987), the influence of the transient, higher order terms in the near crack-tip stress and deformation fields both as they affect the fracture process and the experimental measurement of the dynamic stress intensity factor typically using optical methods such as dynamic photoelasticity, caustics or coherent gradient sensing technique. While these are important issues in performing and interpreting dynamic fracture experiments, these are not considered here; the present focus is primarily on an examination of the processes that occur in the fracture process zone with the goal of developing a modeling strategy for dynamic fracture.

## MICROMECHANISMS OF DYNAMIC FRACTURE

The dynamic fracture process zone evolution in polymethylmethacrylate (PMMA) is examined in this section. Dynamic crack growth was induced in large PMMA specimens by an electromagnetic loading device (Ravi-Chandar and Knauss, 1982). This loading corresponds to a pressure loaded semi-infinite crack in an infinite medium for about 150  $\mu$ s, as shown in Figure 1. The crack

<sup>&</sup>lt;sup>2</sup> It is assumed that dissipation occurs only at the point of the crack tip; otherwise, this equivalence is incorrect.

speed was determined, through high speed photography, to be nearly constant at about  $0.5\,C_R$ ; since the photographs were taken at a rate of 100 kHz, fluctuations at frequencies greater than 50 kHz cannot be identified. The dynamic stress intensity factor,  $K_t^{dyn}(t,v)$ , was measured using the method of caustics and found to increase slowly along the crack from 1.03 MPa√m at crack initiation to 1.2 MPa/m as the crack extended about 25 mm. The details of these variations can be found in Ravi-Chandar and Yang (1996). An examination of the fracture surface of this specimen reveals a dramatic variation in the surface features which provide insight into the mechanism of dynamic fracture. To the naked eye, the fracture surface appears to be rather smooth and specularly reflecting in the early stages of the growth, but after a few mm of growth, the surface becomes very rough with surface features on the scale of a few tens of microns. A few mm further down, the crack surface presents a periodic variation in roughness. A complete examination of the fracture surface has been presented by Ravi-Chandar and Yang, (1996); only the specularly reflecting surface is considered in the present paper in detail. Figure 2a shows the fracture surface from a PMMA specimen during the early stages of crack growth. Clearly, this region exhibits a rather flat surface tiled with conic markings. Figure 2b shows a high magnification view of one of the conic markings. The focus of the conic is clearly visible and radial lines emanate from the focus; Smekal (1953) suggested that in the enhanced stress field of the main crack, inhomogenieties triggered the initiation of a secondary (micro) crack ahead of the primary crack front; the secondary fracture may not be in the same plane as the main crack front and when these two fronts intersect in space and time, the ligament separating the two cracks breaks up leaving a conic marking on the fracture surface. Leeuwerik (1962) observed a bowl shaped cavity with a diameter of about 0.3 µm at the foci of the conic markings, but the nature of these flaws has not been identified satisfactorily. In an attempt to identify the origin of the conic markings, the specimens were observed under an Atomic Force Microscope (AFM), providing surface topograpy at the nanometer scale. While it was easy to locate the AFM probe tip in the region of the conic markings, it has not yet been possible to locate the probe tip on the focal point of one of the conic marks; this effort continues with better viewing and positioning equipment for the AFM. The origin of these microcrack nuclei is hence still uncertain, but a simple estimate of the stress levels that may cause cavitation are discussed later. However, within each

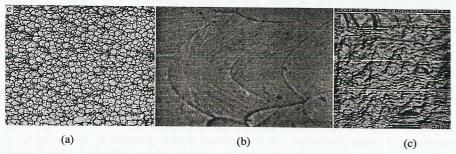


Figure 2. Surface morphology of the PMMA fracture surface. (a) and (b) are optical micrographs, while (c) is an atomic force microscope scan converted to gray scale based on height. The horizontal dimension in (a) is 770  $\mu$ m, in (b) is 50  $\mu$ m and in (c) is 0.5  $\mu$ m.

conic mark, the surface exhibits quite a remarkable structure. Figure 2c shows an image, with the level differences converted to a gray scale; this image covers a 0.5  $\mu m$  x 0.5  $\mu m$  square region from within one of the conic markings in Figure 2b, but not near the focal point. At first glance, the surface appears to be tiled with a pattern similar to observations at the microscale; clearly, the surface is rough (two features, one at the scale of 30 nm and another at a scale larger than 100 nm can be seen); height differences also appear in two scales - a local roughness in the 300 to 400 nm range and a global roughness in the 1000 to 1500 nm range. Since the surface features are of the order of molecular dimensions, it is suggested that these surface elevations and depressions are the result of polymer ligaments stretched out and broken by the process region of the microcrack during its growth; closer examination of these features at this and lower scales is required, but this first look demonstrates that even at the nanometer scale dynamic fracture is not a smooth continuous process. In molecular dynamics simulations of fracture by Abraham et al., (1994) and Nakano et al., (1995), the formation, growth and coalescence of "cracks" off the main crack are seen on a scale that is at least an order of magnitude smaller than the nanometer scale of the AFM observations. The similarity in the rough surface appearance at the micro, nano scales demonstrated above and the atomic scale roughness obtained molecular dynamics simulations suggests the possibility for scaling the results from MD simulations to the structural level, with some effort.

An estimate of the conditions of the formation of these conic markings and their foci can be obtained by an evaluation of their geometry and the stress state. The density and spacing of the conic markings increase along the crack path from about 350 per mm<sup>2</sup> to about 1500 - 2600 per mm<sup>2</sup> prior to the appearance of the periodic bands on the fracture surface. Note that the speed of the main crack which is now the ensemble average of all the microcracks is nearly constant along the crack path and only the stress amplitude increases. These observations suggest that the nucleation of a microcrack is a stress induced event; at larger stress levels, more microcracks are nucleated. The focal length is also an indication of how far ahead of the main crack the microcrack was nucleated; this nucleation distance  $d_n$  is equal to twice the focal length. From measurements of the conic markings in Figure 2, we observe that the average spacing between nuclei decreases from 53  $\mu m$  to 20  $\mu m$  in the region where periodic banding begins to appear. Furthermore, the average nucleation distance  $d_n$  increases from about 5.5  $\mu$ m to 8.5  $\mu$ m over this same crack path. It is clear that the conic markings on the surface indicate an increase in the number of nuclei activated into growing along the crack path and an increase in the nucleation distance at which the secondary microcracks begin to grow; thus as the stress level increases, more flaws are nucleated and they are nucleated farther ahead of the main crack. These flaws grow, interact, and coalesce leading to the overall growth of the "ensemble crack". This appears to be the primary mechanism of crack growth in PMMA. Ravi-Chandar and Yang, (1996) also explore the fracture surface features in other polymers and show some similarity between PMMA and other polymers.

As indicated earlier, the stress field near the crack tip in these specimens was determined using the method of caustics; the dynamic stress intensity factor at crack initiation was 1.03 MPa√m and gradually increased to about 1.2 MPa√m as the density of flaws increased from about 350 per mm² to about 2600 per mm². The yield stress (determined from quasi-static experiments) for PMMA is approximately 100 MPa and is likely to be higher at the high strain rates encountered at the dynamically growing crack tip; in an effort to estimate the size of the plastic zone, it was

assumed that PMMA obeys a power law hardening behavior of the type  $\epsilon = \sigma^n$ , with n=3. The size of the plastic zone under plane strain conditions appropriate to the interior of the specimen is then given by<sup>3</sup>

$$r_p = \frac{1}{6\pi} \left( \frac{n-1}{n+1} \right) \left( \frac{K_I}{\sigma_y} \right)^2. \tag{5}$$

Hence, the plastic zone size increases from about 2.8 µm to 3.8 µm as the stress intensity factor increases from 1.03 to 1.2 MPa/m. Note that the plastic zone is very small, and in particular smaller than the average nucleation distance  $d_n$  which increases from about 5.5 µm to 8.5 µm over this same crack path; thus conditions of small-scale yielding seem to be appropriate. Assuming then that the linear elastodynamic stress field estimates are appropriate at these distances, the stress state at the nucleation of the flaws can be determined; note that the stress state is one of unequal triaxial tension. The triaxial stress state at the nucleation distance of 5.5 µm, corresponding to a stress intensity factor of 1.03 MPa $\sqrt{m}$ , and a crack velocity of about 0.25 $C_R$  can then be calculated as:  $\sigma_{11} = 189$  MPa,  $\sigma_{22} = 175$  MPa and  $\sigma_{33} = 125$  MPa; similarly at the nucleation distance of 8.5 μm, corresponding to a stress intensity factor of 1.2 MPa√m, and a velocity of about  $0.25C_R$ , the triaxial stress state is given by:  $\sigma_{11} = 172$  MPa,  $\sigma_{22} = 164$  MPa and  $\sigma_{33} = 118$  MPa; note that these stresses are much higher that typical values quoted for crazing in PMMA under uniaxial tension, which is in the range of 60 to 100 MPa. It appears that at such high triaxial tensile stress levels, extensive cavitation occurs and these cavities act as the nuclei for further development of the dynamic fracture process. The energy dissipation in these nucleation and growth processes and the dynamics of evolution of this cluster of interacting micropores and microcracks is quite complex; this interaction is what leads to crack surface roughness, periodicity in fracture surface roughness, attempted and successful branching and is instrumental in limiting of the crack speed to levels much below the Rayleigh wave speed allowable by the continuum. It is, however, quite difficult to incorporate this interaction into models of dynamic fracture, derived purely from continuum considerations.

# FRACTURE PROCESS ZONE MODEL FOR A BRITTLE CRACK

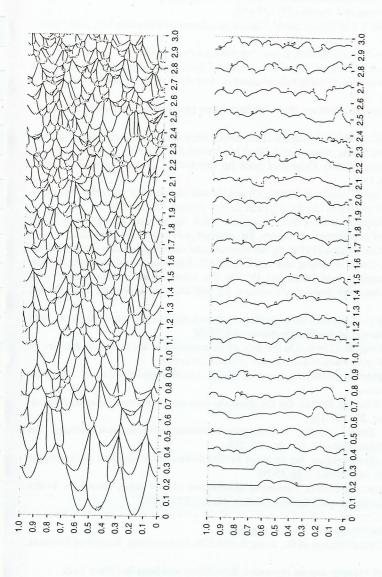
The above discussion of the mechanisms contributing to the growth of a brittle crack suggests that the dynamics of the fracture process zone is crucial to the modeling of brittle fracture; i.e., one must look for a departure from the traditional continuum point of view that incorporates the evolution of the fracture process itself. It appears that there are many alternative paths: an atomistic point of view, such as in MD simulations (Abraham *et al*, 1994 and Nakano *et al*, 1995); a phenomenological lattice point of view, using simple spring-mass elements (Marder and Gross, 1995); a micromechanical point of view, considering the kinetics of microcrack nucleation, growth and coalescence (Zhurkov 1965, Curran *et al*, 1973, and Ravi-Chandar and Yang, 1996);

and a macroscopic view, incorporating a phenomenological representation of the fracture process (Johnson, 1992, Xu and Needleman, 1994, Yang and Ravi-Chandar, 1996).

In molecular dynamics simulations, typically, a large number of atoms (about a million or so) are arranged usually in a perfect two-dimensional crystalline arrangement and the interaction of the atoms is calculated numerically using an interaction model; while Abraham et al., (1994) used an idealized Lennard-Jones solid, Nakano et al., (1995) considered a porous silica and a silicon nitride in their simulations and included more complex interactions between the atoms. Since the numerical computation is time consuming and expensive, the simulation is performed only over the size scale of a few nanometers spatially and only over a few picoseconds temporally. The results of these MD simulations are remarkable in that they exhibit many of the features not observed in the continuum model: the average crack tip speed reaches a limiting value of about 57% of the Rayleigh wave speed, the instantaneous crack tip exhibits erratic oscillations due to crack path deflections, and crack surface exhibits significant roughness, caused by secondary cracks forming away from the main crack at an angle and linking. These results are, however, somewhat paradoxical. The simulation is based on a regular arrangement of the atoms and hence should be predictive of cleavage fracture along special crystallographic planes; but experiments indicate that the crack surface in single crystals is smooth and the crack speed reaches a significant fraction of the Rayleigh wave speed without exhibiting branching while noncrystalline materials exhibit a crack surface roughening, limiting crack speed of about 50% of the Rayleigh wave speed and crack branching. Thus, the simulation presents features that are observed at a much larger scale in experiments with noncrystalline materials! A lattice dynamics model of fast crack propagation has been presented by Marder and Gross (1995). Here, the growth of a crack in a spring mass idealized lattice is considered. In this model, they demonstrate that steady-state crack growth is not possible at slow speeds and further that fast fracture is path unstable at very high speeds. In a sense, the lattice dynamics model reproduces some of the features of the MD simulations, although the length and time scales here are much larger. The paradox of the MD simulations remains in that while the lattice dynamics should predict crystalline behavior, it exhibits features of crack growth that are similar to noncrystalline materials.

Nucleation and growth models for crack growth were considered first by Zhurkov (1965) for thermally activated crack processes such as creep. Curran et al., (1977) applied these ideas to the stress induced nucleation and growth processes in the dynamic spalling problem. The description of dynamic crack growth provided in this paper appears to be well suited to the nucleation and growth model. In this model, dynamic crack growth is due to the stress induced nucleation of micropores which then become microcracks and grow and coalesce with each other. A simple simulation of this model was demonstrated by Ravi-Chandar and Yang (1996). In this simulation, an increasing density of nuclei was used to simulate stress induced micropore nucleation observed in experiments; nucleation criterion was based on a critical distance from an activating crack front again measured experimentally; finally the crack growth criterion was derived from maintaining a constant rate of energy dissipation. The crack surface features generated by this nucleation and growth model and the corresponding crack front evolution are shown in Figure 3. This simulation shows the development of a tortuous crack front, the generation of a large fracture process zone which is the ensemble of all active microcracks, and a constant speed of propagation of the "ensemble crack front". The speed of the ensemble crack front is also obtained from this simula-

<sup>&</sup>lt;sup>3</sup> This is a quasi-static estimate; under dynamic conditions, since the yield stress is likely to be higher, and also due to the shortening of the zone caused by inertia effects, r<sub>y</sub> will typically be smaller than this estimate. Hence we have an upper bound estimate of the plastic zone size.



through the nucleation

tion and is shown in Figure 4; while the average speed is constant, the instantaneous speed exhibits significant fluctuations. These high frequency oscillations are similar to the oscillations observed by Fineberg et al., (1991); it is suggested that the experimental observations are sensitive to the microcracking evolution within the fracture process zone. The microcrack nucleation and growth model of fast fracture also provides possible mechanisms for the formation of the periodic banding and crack branching. First, in the ensemble microcracking process, some islands are left unbroken; these ligaments break at a later time and lead to the stress waves that are typically observed in the high speed photographs. If one attributes a critical stretch to the ligament at break, then a periodic behavior could be established with the period determined by the critical stretch divided by the crack speed. A second possibility is based on a runaway growth of microcracks. As the nuclei spacing decreases and the nucleation distance  $d_n$  increases, the cascading effect occurs where a whole region of microcracks are activated into growing, but this will drain the energy supply very quickly thus slowing or stopping the process until the energy supply catches up. This process is then repeated thus generating a periodic process and banding on the crack surface. Perhaps a combination of these two ideas would be needed to explain the periodic banding. The model at present lacks a tie-in to the stress field generated near the crack by the applied loading. Such a connection would provide the basis for the determination of the distribution of the nuclei, the evolution of the nucleation and growth criterion in terms of the applied loading; thus a bridge between the micro and macroscales is required.

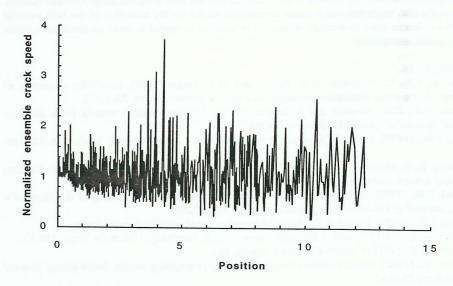


Figure 4. Ensemble crack speed variation along the crack path. High frequency fluctuations are due to the growth of a large number of microcracks in the process zone; the ensemble average appears to be nearly constant.

Macroscopic or continuum level modeling of the fracture process zone has also been attempted. Johnson (1992) considered a damage mechanics approach by considering the region near the crack tip to be of an elastic-softening type material, where the damage is triggered by the dilatational strain. This model was implemented in a very fine scale finite element simulation of the pressurized semi-infinite crack problem using the same conditions that were attained in the experiments of Ravi-Chandar and Knauss (1984a-d). Identifying the spread of the damage away from the main crack line as crack surface roughening, the finite element simulation indicated that the crack tip indeed did not accelerate to the Rayleigh wave speed, but instead, had a tendency to limit itself at a much lower speed; the simulations also indicated a rapid growth of the total process region and exhibited a tendency to branch much like the experimental observations. Xu and Needleman (1994) also used a finite element method to simulate dynamic crack growth; they modeled the cracking process by considering a cohesive law governing the separation of the surfaces both under tension and shear. While the development of a process zone could not be characterized in these simulations easily, they exhibited a crack branching pattern at high loading rates and crack speed. However, there appears to be some mesh dependence on the branching phenomenon which is not unlike the observations of other discrete simulations such as the molecular dynamics and lattice dynamics simulations. Yang and Ravi-Chandar (1996) also considered a cohesive zone model for dynamic crack growth; they used an elastic-softening material model for the cohesive zone similar to the model of Xu and Needleman; crack growth was confined to rectilinear growth in order to examine the role of the process zone in dynamic crack growth. The results indicated that significant non-steady growth of the crack and process zone occurred under dynamic loading. The results also suggested that a better constitutive model for the evolution of the bulk material into the cohesive zone or fracture process zone material is needed in order to simulate dynamic crack growth completely.

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