DYNAMIC INTERFACIAL CRACKING OF THERMOMECHANICALLY LOADED TWO-PHASE COMPOUNDS

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ABSTRACT

The dynamic interface crack growth along a curvilinear interface of a two-phase compound consisting of two brittle solids with different thermoelastic material properties and subjected to mechanical crack surface loads and superimposed thermal strains acting along the ligament is investigated. By applying the linear theory of plane thermoelasticity and by assuming a small interface curvature as well as by restricting to almost steady state conditions with reference to a running interface coordinate system, the associated boundary value problems have been transformed mathematically to vectorial Hilbert problems thereby adopting Stroh's method of generalized complex potentials. The curvature of the interface was handled by applying the conformal mapping technique as well as methods of the first order perturbation analysis. Further, the parameters of the eigenvalues and of the eigenvectors of the Hilbert problems can physically be interpreted as elastodynamic interface mechanics parameters reading as $(\beta, \nu_p, \beta_{Hf}, \mu_{Hf})$ and depending on the velocity of a running interface crack tip.
Finally, based on a physically reasonable stress intensity vector definition, explicit integral formulae for the stress intensity vectors of the **p**-problem (mechanically loaded crack surfaces) as well as of the **q**-problem (thermally strained ligament) are obtained by applying an appropriate eigenvector plane approach, and by involving Rice's scaling procedure for stress intensity factors. Some numerical examples are given.

KEYWORDS

Dynamic interface crack growth, Hilbert-problems, elastodynamic interface parameters, dynamic mixed-mode stress intensity factors.

INTRODUCTION

Experimental investigations of fracture phenomena in fiber reinforced composites show the existence of different failure mechanisms in the low- and high-fiber concentration ranges of such composite structures known as matrix and interface cracks, respectively. Further, the appearance of branched crack systems consisting of a combination of curved matrix and interface cracks has also been observed several times. Thus, an important problem concerning the failure behaviour of mechanically and/or thermally loaded two-phase compounds consists in the prediction of the prospective paths of extending cracks as functions of the geometrical configuration as well as on the applied thermomechanical load distribution belonging to a given composite structure. The quasistatic extension of straight and curved interface cracks, respectively, as well as the crack path prediction of extending thermal cracks in self-stressed multi-phase solids have been investigated in the past by several authors (Erdogan, 1965; England, 1966; Rice and Sih, 1965; Herrmann, 1983, 1985, 1994; Herrmann and Grebner, 1984, 1985; Herrmann and Dong, 1994). From a mechanical view point, the formation of interface cracks and a prospective subsequent unstable crack propagation can be described by a set of distinct parameters of the interface mechanics. Especially, after the onset of unstable crack propagation, the material inertia cannot be neglected if the crack tip velocity exceeds about one half of the Rayleigh wave velocity, v_R, of the more compliant bimaterial component. In this case a non-negligible portion of the initial elastic energy stored in a multiphase compound is converted into kinetic energy of the cracked solid.

Therefore, there exists a strong interest for the theoretical modelling of the general situation of a dynamic crack propagation of straight and curved interface cracks for the sake of an understanding of the interactions of the crack-tip velocity, and of the mechanical crack surface loads superimposed by the self-stresses originating from the applied thermal strains as well as from the curvature of the interface. A comprehensive review concerning the state-of-the-art of the analysis of the quasi-static and steady-state dynamic interface crack propagation of elastically isotropic and anisotropic dissimilar materials has been given by Noe (1994). The investigations on rapid crack propagation were triggered by Willis (1971) and Yang et al (1991) and were continued by Deng (1993) and Noe and Herrmann (1993, 1994) and Herrmann and Noe (1995). Further, in the latter papers, the most general situation of a rapid crack extension along curvilinear interface contours subjected to thermomechanical loads was investigated thereby also considering the related shadow-optical method of caustics for an experimental determination of stress intensity factors.

STATEMENT OF THE PROBLEM

In this paper the analysis of a dynamic interface crack growth in the material interface of an elastically anisotropic dissimilar two-phase compound with mechanically stressed crack surfaces and under constant cooling or heating ΔT has been performed. Figure 1 shows the corresponding material model for a curvilinear interface crack propagating along the interface with the velocity **v**.

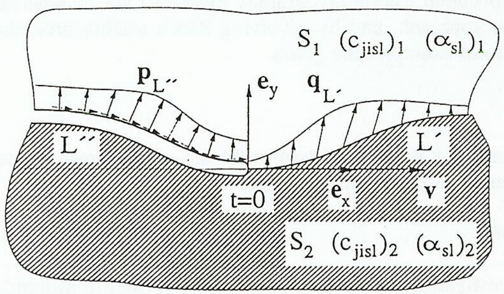

Fig. 1 Propagating interface crack

The associated mixed boundary value problem of the dynamic fracture mechanics has been solved by applying Stroh's method of generalized complex potentials as well as Volterra's dislocation method. Starting from the basic equations of linear thermoelasticity and by considering the boundary and continuity conditions

$$\{\sigma_{nn} - i\sigma_{nt}\}_j = \begin{cases} [g_n(s) - ig_t(s)]_- & ; \; s \in L'' \\ [g_n(s) - ig_t(s)]_+ & ; \; s \in L' \end{cases} \quad ;(j=1,2) \tag{1}$$

$$\{u_t + iu_n\}_1 = \{u_t + iu_n\}_2 \quad ; \quad s \in L' \tag{2}$$

along the curvilinear path $L = L'' \cup L'$ as well as by assuming zero stresses at infinity the elastic problem can be analyzed by the Lamé-Navier equations (LNE)

$$c_{jisl} u^{(el)}_{s,lj} = \rho \ddot{u}^{el}_i = \rho v^2 \left(\cos^2(\alpha)\, u^{el}_{i,11} + \sin(2\alpha)\, u^{el}_{i,12} + \sin^2(\alpha)\, u^{el}_{i,22} \right) \tag{3}$$

formulated in a non-rotational crack tip coordinate system $(\mathbf{e}_x,\mathbf{e}_y)$, running with the velocity **v**. For crack tip positions near to the non-moving basis $(\mathbf{e}_X,\mathbf{e}_Y)$, and thus for small angles α, time independent LNE are obtained which are the same as for the case of a straight crack extension, namely

$$C_{jisl}(v) u^{el}_{s,lj} = \left(c_{jisl} - \rho v^2 \delta_{1j} \delta_{1l} \delta_{is} \right) u^{el}_{s,lj} = 0_i \tag{4}$$

By introducing the displacement vector approach according to Stroh (1962)

$$\mathbf{u}^{el}_{kj} = \mathbf{a}_{kj} f_{kj}(z_{kj}) \quad , \quad z_{kj} = x + i\beta_{kj} y \quad , \quad (k,j = 1,2) \tag{5}$$

the solution of the LNE (4) is reduced to the solution of the following eigenvalue problem

$$\left(\mathbf{QA} + (\mathbf{R} + \mathbf{R}^T)\mathbf{AP} + \mathbf{TAP}^2 \right)_j \mathbf{f}_j''(\mathbf{z}_j) = 0 \tag{6}$$

where, according to Ting's (1986) notation, the matrices

$$\mathbf{A}_j = (\mathbf{a}_1, \mathbf{a}_2)_j \quad , \quad \mathbf{P}_j = \mathrm{diag}\,(p_1, p_2)_j = \mathrm{diag}\,(i\beta_1, i\beta_2)_j \tag{7}$$

are composed of the eigenvectors and eigenvalues of the LNE (4).
The potential vectors

$$\mathbf{f}_j(\mathbf{z}_j) = \left(f_1(z_1), f_2(z_2) \right)_j^T \quad , \quad \mathbf{z}_j = (z_1, z_2)_j^T \tag{8}$$

are determined from an associated Hilbert problem.

FORMULATION OF A HILBERT-PROBLEM

By applying the conformal mapping technique the curvilinear interface contour L is mapped onto the arc L_Ω of an auxiliary ζ-plane according to the conformal mapping function

$$z = \omega(\zeta) = \zeta + if(\zeta) \quad , \quad f(\zeta = 0) = f'(\zeta = 0) = 0 \quad , \quad \zeta = \xi + i\eta \tag{9}$$

Then for slightly curved interfaces the associated vectorial Hilbert problem

$$\mathbf{F}_{\Omega 1}(\xi)\mathbf{B}_1 \tilde{\mathbf{f}}'^{+}_1(\xi) \;+\; \mathbf{F}_{\Omega 2}(\xi)\mathbf{B}_2 \tilde{\mathbf{f}}'^{-}_2(\xi) = \mathbf{p}_{L_\Omega''}(\xi) \quad , \quad \xi \in L_\Omega'' \tag{10}$$

$$\mathbf{H}_\Omega(\xi)\mathbf{F}_{\Omega 1}(\xi)\mathbf{B}_1 \tilde{\mathbf{f}}'^{+}_1(\xi) + \overline{\mathbf{H}}_\Omega(\xi)\mathbf{F}_{\Omega 2}(\xi)\mathbf{B}_2 \tilde{\mathbf{f}}'^{-}_2(\xi) = \mathbf{q}_{L_\Omega'}(\xi) \quad , \quad \xi \in L_\Omega' \tag{11}$$

is obtained for the determination of the desired potential vectors $\mathbf{f}_j(\mathbf{z}_j)$.

Further, by neglecting terms of the order $0\left(f'^2(\xi)\right)$ the following definitions hold true

$$\mathbf{F}_{\Omega j}(\xi) = \left(\mathbf{I} - f'(\xi)\rho v^2 \mathbf{A}\mathbf{B}^{-1}\right)_j \quad , \quad \mathbf{B}_j = (\mathbf{R}^T\mathbf{A} + \mathbf{T}\mathbf{A}\mathbf{P})_j \tag{12}$$

Besides, the non-constant coupling matrix $\mathbf{H}_\Omega(\xi)$, whose algebraic structure determines most of the mechanical interface parameters, is defined by

$$\mathbf{H}_\Omega(\xi) = \mathbf{H} + i f'(\xi) v^2 \mathbf{H}_f \tag{13}$$

and consists of the respective skew-Hermitian matrices

$$\mathbf{H} = -\overline{\mathbf{H}}^T \quad ; \quad \mathbf{H} = \mathbf{A}_1\mathbf{B}_1^{-1} - \overline{\mathbf{A}}_2\overline{\mathbf{B}}_2^{-1} \tag{14}$$

and

$$\mathbf{H}_f = -\overline{\mathbf{H}}_f^T \quad ; \quad \mathbf{H}_f = -i\left[\rho_1(\mathbf{A}_1\mathbf{B}_1^{-1})^2 - \rho_2(\overline{\mathbf{A}}_2\overline{\mathbf{B}}_2^{-1})^2\right] \tag{15}$$

The load vectors

$$\mathbf{p}_{\Omega L_{\Omega'}}(\xi) = (\sigma_{xy}, \sigma_{yy})^T_{\Omega L_{\Omega'}} \quad , \quad \mathbf{q}_{\Omega L_{\Omega'}}(\xi) = (\mathbf{u}^{th}_{2,\xi} - \mathbf{u}^{th}_{1,\xi})^T_{L_{\Omega'}} \tag{16}$$

denote the crack surface load and the thermal distortion jump.
Finally, the solution of the Hilbert problem (10), (11) ist gained by superimposing the mechanical and the thermal distortion problem, hereafter called the **p**- and the **q**-problem, respectively. Furthermore, the methods of linear algebra are applied and the solutions of the **p**- and the **q**-problem, respectively, are determined by using the so-called associated eigenvector planes. A detailed outline of this technique is given in Noe and Herrmann (1993).

DETERMINATION OF STRESS INTENSITY VECTORS AND NUMERICAL RESULTS

Integral representations of the ligament stress fields for the **p**- and the **q**-problem, respectively, which are related to the solutions of the Hilbert-problem, and supplemented by an appropriate stress intensity vector definition lead to explicit integral formulae for the stress intensity factors $K_{R\Omega p} = \left(K_{R\Omega p II}, K_{R\Omega p I}\right)^T$ and $K_{R\Omega q} = \left(K_{R\Omega q II}, K_{R\Omega q I}\right)^T$ as derived by Herrmann and Noe (1995). According to these results a physically reasonable definition of a stress intensity vector at the tip of a rapid propagating interface crack reads as follows

$$\mathbf{K}_{R\Omega}(\hat{r}) = \sqrt{2\pi}\lim_{r\to 0}\left[\mathbf{V}_q\mathbf{X}_R(\hat{r})\left(\mathbf{X}^+_{\Omega L_{\Omega'}}(r)\right)^{-1}\mathbf{V}_q^{-1}\mathbf{p}_{\Omega L_{\Omega'}}(r)\right] \quad , \quad \mathbf{K}_{R\Omega} = (K_{R\Omega II}, K_{R\Omega I})^T$$

$$\mathbf{X}^+_{\Omega L_{\Omega'}}\left(\mathbf{X}^+_{\Omega L_{\Omega'}}(r)\right)^{-1} = r^{1/2 - f'\alpha_{HF}}\,\mathrm{diag}\left[(r/\hat{r})^{-i\varepsilon}, (r/\hat{r})^{+i\varepsilon}\right] \quad , \quad \left[K_{R\Omega II,I}\right] = \mathrm{Nmm}^{-3/2} \tag{17}$$

where r denotes the distance from the crack tip and $\hat{r}$ is a characteristic scaling length introduced by Rice (1988) for stress intensity factors at interface crack tips. Further, the classical bimaterial constant ε which is now velocity dependent causes the well-known phenomenon of the oscillatory singularity at an interface crack tip, whereas the additional bimaterial constant α_{Hf} arises only if a rapid interface crack propagation along a curved interface occurs.
By assuming that the crack surface loading acts along the interval [-a,0] and by taking the associated ligament stress vector $\mathbf{p}_{\Omega p L_{\Omega''}}$ as well as the definition (17), where only the first order terms are taken into account, an explicit formula for the stress intensity factors in case of the **p**-problem has been obtained

$$\begin{pmatrix} K_{R\Omega p II} \\ K_{R\Omega p I} \end{pmatrix} = \sqrt{\frac{2}{\pi}}\cosh(\pi\varepsilon)\int_0^{-a}\begin{pmatrix} R_e Y_p & -f'\,\mathrm{Re}\,Y_p + v_p\,\mathrm{Im}\,Y_p \\ \left(1 + \frac{\mu_{Hf}}{v_p}\right)f'\,\mathrm{Re}\,Y_p - \frac{1}{v_p}\mathrm{Im}\,Y_p & \mathrm{Re}\,Y_p \end{pmatrix}\begin{pmatrix} p_t \\ p_n \end{pmatrix}(-dt) \tag{18}$$

$$Y_p = \left(\frac{-at}{a+t}\right)^{-1/2 - f'(t)\alpha_{Hf}}\left(-\frac{t}{\hat{r}}\frac{a}{a+t}\right)^{-i\varepsilon} \quad , \quad \text{a: interface crack length}$$

Figures 2 and 3 show numerical simulations of the influences of the curvature and the crack-tip velocity on the magnitude as well as on the mixed-mode phase angle of the stress intensity vector, $\mathbf{K}_{R\Omega p}$, in case of crack surfaces under normal and shear loading. Thereby these quantities have been simulated for curvilinear interface contours up to a maximum slope of $|f'_{max}| = 0.5$, where the assumed contours are modelled by third order polynomials. The magnitude of $K_{R\Omega p}$ has been normalized by the value at zero-velocity and for a straight interface, $K_{R\Omega p0}$. Further, an elastically mismatched bimaterial with the bimaterial constant, $\varepsilon_0 = \varepsilon(v=0) = 0.096$ and the Rayleigh-wave velocities $v_{R1} = 988\,\mathrm{m/s}$ and $v_{R2} = 2914\,\mathrm{m/s}$ has been chosen, where the numerical simulations have been carried out in the velocity interval $0 < v < 0.91 v_{R1}$.
Besides, the characteristic length $\hat{r} = 1.0\,\mathrm{mm}$ has been selected and from the graphs given in the Figs.2 and 3 for a mixed-mode loading situation with the magnitude $p_t = p_n = -1{,}0\,\mathrm{N/mm^2}$, acting along the interval [-a,0], a = 5.0 mm, it can be clearly recognized that the magnitude $\left(K_{R\Omega p}/K_{R\Omega p0}\right)$ varies with increasing crack-tip velocity v. However, significant changes can only be observed for velocities exceeding about one half the minimum Rayleigh-wave velocity v_{R1}. In addition, the rapid increase of the ratio $\left(K_{R\Omega p}/K_{R\Omega p0}\right)$ for velocities higher than $v = 0.8 v_{R1}$ points to the limit of the validity of the present first order theory. Furthermore, one should remind in this connection that experimentally measured crack tip velocities are usually limited to about $0.8 v_{R1}$, Tippur and Rosakis (1991). The mixed-mode phase angle ω_R varies only slightly with the crack-tip velocity where in view of the experimental interface fracture mechanics a velocity-independent ω_R possibly leads to a reduction of experimentally recorded parameters to only one if the mixed mode phase angle ω_{R0} of the static case has been measured once.
Moreover, for a thorough assessment of the thermally strained ligament case, and by assuming the thermal strains to act along the finite interval [0,b] the corresponding explicit formula for the stress intensity factors in case of the **q**-problem has been attained

$$\begin{pmatrix} K_{R\Omega q II} \\ K_{R\Omega q I} \end{pmatrix} = \sqrt{\frac{2}{\pi}}\cosh^2(\pi\varepsilon)\int_0^{b}\begin{pmatrix} R_e Y_q & 0 \\ f'\frac{\mu_{Hf}}{v_p}\mathrm{Re}\,Y_q - \frac{1}{v_p}\mathrm{Im}\,Y_q & f'\frac{1}{v_p^2}\mathrm{Re}\,Y_q \end{pmatrix}\begin{pmatrix} \Delta\alpha_{11} \\ \Delta\alpha_{22} \end{pmatrix}\frac{\Delta T}{D_1}dt \tag{19}$$

$$Y_q = \left(\frac{bt}{b-t}\right)^{-1/2 - f'(t)\alpha_{Hf}}\left(\frac{t}{\hat{r}}\frac{b}{b-t}\right)^{-i\varepsilon} \quad , \quad \text{b: ligament length}$$

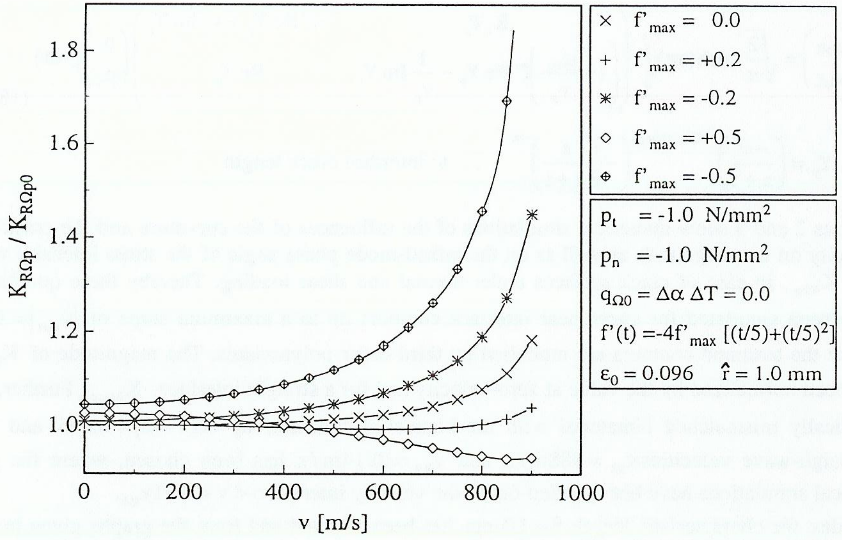

Fig.2. Influence of the curvature and the crack-tip velocity on the magnitude of the stress intensity vector $K_{R\Omega p}$ in case of crack surfaces under normal and shear loading

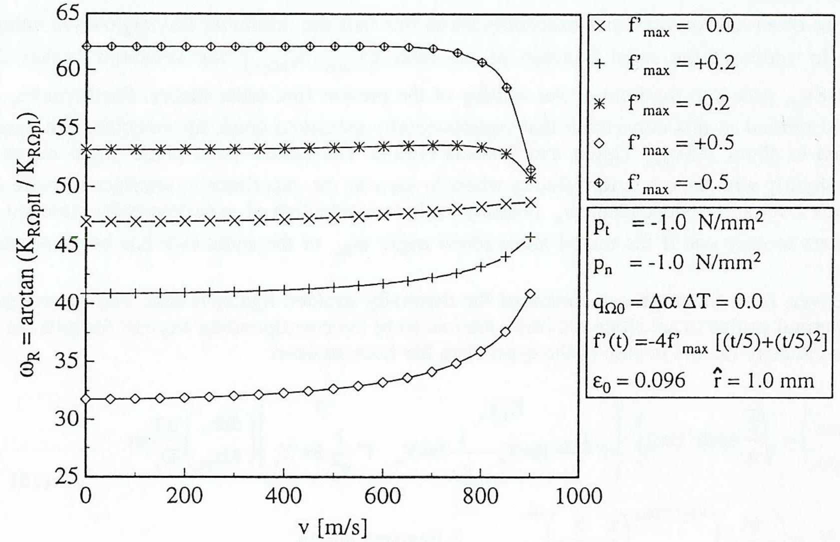

Fig.3. Influence of the curvature and the crack-tip velocity on the mixed-mode phase angle of the stress intensity vector $\mathbf{K}_{R\Omega p}$ in case of crack surfaces under normal and shear loading

where $\Delta\alpha_{11} = \left[(\alpha_{11})_2 - (\alpha_{11})_1\right]$ and $\Delta\alpha_{22} = \left[(\alpha_{22})_2 - (\alpha_{11})_1\right]$ denote the difference of the thermal expansion tensors and $\Delta T = T - T_o$ means the applied cooling or heating. Numerical simulations of the influences of the curvature and the crack tip velocity on the magnitude as well as on the mixed-mode phase angle of the stress intensity vector $K_{R\Omega q}$ have been performed in case of thermal strains acting along the ligament. Thereby the thermal strains operate along the interval [0, b] of the length b = 5,0 mm, whereas the bimaterial properties, the interface contours and the characteristic length $\hat{r}$ are the same as for the **p**-problem. Detailed results can be found in Herrmann et al. (1995).

ELASTODYNAMIC INTERFACE MECHANICS PARAMETERS AND GENERALIZED DUNDURS DIAGRAM

The analysis of formula (18) makes apparently clear, that the invidual influence of each one of the interface fracture mechanics features of interest, namely mechanical loading, interface, crack-tip velocity and interface curvature, and their interactions on the stress intensity factors is rather complicated. Since multi-valued relations between the mechanical features and the mechanical parameters exist, the influence of distinct mechanical parameters can hardly be concluded from formula (18). A similar conclusion can be drawn concerning formula (19) and the corresponding relations between the thermomechanical parameters and the associated thermomechanical features.

Furthermore, a detailed discussion of the elastodynamic parameters for uncracked and cracked bimaterial interfaces has been given in Noe and Herrmann (1996). Here, because of space limitations in Fig. 4 the structure of the generalized Dundur's diagram for the dynamic case is given. Thereby the generalized Dundur's diagram is framed by the polygon ACEF where the associated part of the Dundur's diagram for the static case is enclosed by the polygon ABDF. The subdomain ABEF contains static Dundur's parameters for $v_1 \geq v_2$.

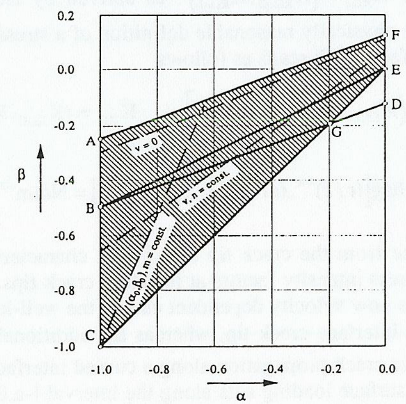

Fig. 4. Generalized Dundur's diagram of dynamics

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