Analysis of Dynamic Crack Propagation

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Abstract

Extending the dimensional analysis of Mott on dynamic crack propagation, we have derived the model equations of dynamic fracture of brittle materials which take account of the fracture surface roughness. This equation predicts the oscillation of the crack velocity when the fracture surface roughness increases, in qualitative agreement with the recent experiment of Fineberg et al.[Phys. Rev. B45 (1992) 5146].

I. Introduction

Although numerous works have been done on dynamic crack propagation in brittle materials for many years, the mechanism that govern the dynamics of cracks are not well understood. One of the basic difficulties is that cracks do not attain the limiting velocity predicted by linear elastic theory [1]. Another difficulty has been how to explain characteristic sequence of the fracture surface known as "mirror, mist and hackle". Namely, an initially smooth and mirror-like fracture surface begins to appear misty and then evolves into a rough hackled region [2,3]. Recently Fineberg et. al. performed the refined experiment of dynamic crack propagation in brittle plastic, PMMA [4]. Improving the resolution on the measurements of the crack velocity , they have revealed the existence of the critical velocity at which the velocity of a crack begins to oscillate. They have also found the strong correlation between the oscillation of the crack velocity and the fracture surface roughness. This experiment is extended further and the cause of the oscillation is found to be the attempted local crack branching [5]. Besides, the fracture surface roughness, w, defined as the rms deviation of the fracture surface height from it's mean value, is observed as $w \propto L^{\zeta}$; $\zeta \approx 0.7$, where L is the measurement scale. The value of $\zeta \approx 0.7$ for the "roughness exponent", ζ has been conjectured to be universal in brittle two-dimensional materials [6]. The experimental results described above have also been found by the computational molecular dynamics for two dimensional triangular solids with mode I loading [7]. In the molecular dynamic (MD) simulation, in which Lennard-Jone's 12:6 potential is used for interatomic forces, the origin of the erratic velocity oscillation is observed to be associated with the stair-step branching and connecting of regions of failure at and preceding the crack tip. This oscillating zigzag motion of the crack tip is found in both of the MD simulation with and without dislocation emission. Beside, the width w is observed to scale as $w \propto L^{\zeta}$; $\zeta \approx 0.81$.

Taking account of these results on dynamic crack propagation in mind, various theoretical attempts have been made. For example, Langer [8] has studied models of crack propagation with different dissipation mechanisms, such as velocity dependent friction and Kelvin viscoelasticity. In these models the dissipative mechanism acts in the bulk of the material and steady state crack propagation is realized. Recently Langer and Nakanishi [9] have introduced a new model of crack propagation in which novel viscous dissipation acts only on the fracture surface. The stress field at the crack tip was found to be nonsingular even without applying the usual Barenblatt condition. The basic idea in these theoretical analysis is the recognition that the essential dynamic degrees of freedom are neglected in the conventional dynamic fracture mechanics. In other words the idea, that it might be possible to determine the direction and perhaps even the speed ofcrack extension by examining only the singular stress fields in the neighborhood of a geometrically sharp crack tip, is questioned.

In section II, we will analyze the experimental results of Fineberg et al.[4] in terms of the extended Mott's analysis [10], from which the model equation for the velocity of a crack is derived. The model equation predicts that the crack velocity oscillates if the fracture surface becomes rough and the the excess fracture energy obeys the equation derived by the Hamiltonian which corresponds to Ginzburg-Landau Hamiltonian of the second order phase transition. The direction of further development is given in section III.

II. Analysis of Dynamic Crack Propagation

We consider propagation of a centrally located through crack in an infinitely large elastic plate subjected to a time-independent uniaxial tension perpendicular to the plane of a crack. The energy balance equation of a crack is

$$\frac{dU_{ext}}{dt}=\frac{dU_S}{dt}+\frac{dU_K}{dt}+\frac{dD}{dt} \tag{1}$$

where t is time and U_{ext} is the work done by the external loads. U_S is the elastic component of the stored energy. U_K is the kinetic energy and D is the sum of all the irreversible energies such as surface energy, plastic work and viscous dissipation.

In order to estimate the effect of kinetic energy, Mott [11] assumed that the stress and displacement fields for dynamic problem are the same as those for elastostatic problem with the same crack length. The kinetic energy is found to be

$$U_K=\frac{k\rho a^2\dot{a}^2\sigma^2}{2E^2} \tag{2}$$

where k is a constant which depends only on the Poisson's ratio and ρ is the mass per unit area of the plate. The thickness of the plate is taken to be a unit length. E is Young's modulus and σ is the applied stress. The dot in Eq.(2) represents derivative with respect to time.

From the elastic solution of the plate under constant stress at infinity, for the change in the quantity $U_{ext}-U_S$ due to the existence of a crack of length 2a, we may write

$$U_{ext}-U_S=\frac{\pi\sigma^2 a^2}{E} \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), we find

$$\frac{1}{\bar{c}^2}(\dot{a}^2+a\ddot{a})-1+\frac{R}{G_0}\frac{a_0}{a}=0 \tag{4}$$

where crack resistance R and the energy release rate G_0 at $a=a_0$ are, respectively, defined as

$$R=\frac{\partial D}{\partial a}, \qquad G_0=\frac{2\pi\sigma^2 a_0}{E} \tag{5}$$

$\bar{c}$ in Eq.(4) is defined by the velocity of longitudinal wave, c_1, and a constant k as

$$\bar{c}=\sqrt{\frac{2\pi}{k}}\,c_1 \quad \left(c_1=\sqrt{\frac{E}{\rho}}\right). \tag{6}$$

Analyzing the lumped mass spring model of a crack, the equation which is equivalent to Eq.(4) is derived by Williams [12].

Let us define the following normalized quantities,

$$\alpha=\frac{a}{a_0},\ \tau=\frac{\bar{c}t}{a_0},\ v=\frac{d\alpha}{d\tau} \tag{7}$$

and rewrite Eq.(4) as

$$\alpha\frac{dv}{d\tau}=1-v^2-\frac{R}{\alpha G_0} \tag{8}$$

We obtain the following solution of Eq. (8) for the case $R=G_0$, which is the Griffith's critical condition for fracture,

$$v=1-\frac{1}{\alpha} \tag{9}$$

This solution is often used to explain various different experiments. Assuming that the solution of Eq. (8), still holds for the case $R\neq G_0$, we rewrite Eq. (8) as follows,

$$\frac{dv}{dt}=(1-v)^2(v-\psi) \tag{10}$$

$$\psi=\frac{R}{G_0}-1 \tag{11}$$

Notice here that the following equation can be derived from $v=1-\alpha^{-1}$

$$\frac{dv}{dt}=(1-v)^2 v \tag{12}$$

It is reasonable to replace G_0 in Eq. (11) by the surface energy $R=2\gamma_0$ since the fracture surface is smooth initially as observed in the experiment [4]. Thus we find that ψ remains to be zero as far as the fracture surface is in the mirror state. As the fracture surface evolves from mirror to mist or hackle, ψ becomes positive since it requires more energy than $2\gamma_0$ and the crack resistance R increases. When the fracture surface remains to be in a mirror state, the crack is accelerated according to Eq. (12). The crack is decelerated when the condition, $v < \psi$, is satisfied, which is realized by the rough fracture surface.

Taking account of the existence of the critical crack velocity, v_c, we find that the behavior of ψ closely resembles with that of the order parameter, widely discussed in the second order phase transition [13]. Instead of the temperature in the phase transition, we use the crack velocity for dynamic fracture as a control parameter. Making use of Ginzburg-Landau Hamiltonian [14], we find the equation for ψ as follows,

$$\frac{d\psi}{d\tau}=\{b_0(v-v_c)^2\,\mathrm{sgn}(v-v_c)\}\,\psi-b_1\psi^3 \tag{13}$$

where $\mathrm{sgn}(v-v_c)$ is the sign function. The term { } in R.H.S. of Eq. (13) is chosen referring to the experimental result [4]

Eqs. (10) and (13) are the phenomenological equations for dynamic fracture. The terminal velocity of the crack are derived as follows. The steady state solution, ψ_0, of ψ can be obtained from Eq. (13),

$$\psi_0=\begin{cases}\sqrt{\dfrac{b_0}{b_1}(v-v_c)} & v\geq v_c\\[2ex] 0 & v<v_c\end{cases} \tag{14}$$

The crack progresses when the condition, R=G is satisfied, where the quantity G is the energy release rate. Thus ψ defined by Eq. (11) represents the excess energy release rate for fracture and it corresponds to the fracture surface roughness. The velocity dependence of the fracture surface roughness is experimentally measured and Eq. (14) is in accord with it. Substituting the value of Ψ_0 for the case $v \geq v_c$ into Eq. (10) and setting $(dv/d\tau)=0$, we find the terminal velocity, v_t, of the crack as

$$v_t = \frac{\sqrt{b_0/b_1}}{\sqrt{b_0/b_1 - 1}} v_c \qquad (15)$$

where we assumed the condition

$$\frac{b_0}{b_1} > 1 \qquad (16)$$

The quantities, v and Ψ, could oscillate depending on the choice of the parameters as is shown in Fig. 1, which is obtained by numerically solving Eqs. (10) and (13) . Fig. 1 corresponds to oscillation of the crack velocity and the fracture surface roughness observed in the experiment. Thus, the phenomenological equations (10) and (13) qualitatively agree with the experimental results as discussed. We will not get involved in details of the solutions of Eqs. (10) and (13) since we did not theoretically find the expression for b_0 and b_1 at this stage.

Fig. 1. Numerical result of Eqs. (10) and (13) for the case $b_0 = 40$, $b_1 = 3.7$ and $v_c = 0.2$

The analysis given so far is only based on analogy of Eqs. (10) and (13) with the second order phase transition motivated by the experimental observation of the existence of a critical velocity [4]. We now give plausible argument on the physics associated with Eqs. (10) and (13).

The importance of the craze on dynamic crack propagation in PMMA has been known [15]. When a crack propagates at low velocity the length of the craze extending ahead of the crack is small. As the velocity of a crack increases the craze grow to a larger extent. Cotterell noted this growth of the craze as a self generating process [15]. If the length of the craze ahead of the crack grow sufficiently large, the brittle plastic, PMMA, responds elastoplastically. Then a crack velocity decreases and the stress field at the tip of a crack increases. Thus more energy will be expended in this phase of crack propagation. As the stress at the crack tip increases furthermore, the material restores it's brittleness and the velocity of a crack increases. The transition velocity of the response of the material

for dynamic crack propagation discussed above can be regarded as the critical velocity which appears in Eq. (13).

III. The Direction of Further Development

What we could conclude at this stage is to point out that the conventional theory of dynamic fracture is based on "erroneous belief" that the crack velocity is only determined by the global energy balance equation, extending the case of the static fracture. This is also recognized in the recent theoretical development on dynamic fracture as discussed in section I. According to author's opinion, however, the recent experiments suggest that the erratic zigzag motion of the crack tip does not depend on such detailed dissipative conditions as studied by theoretical physicists [8,9]. Although the present results shown in section II look too much artificial, the basic theoretical structure would remains in the way as it stands. Extension of the present work will be presented in the conference.

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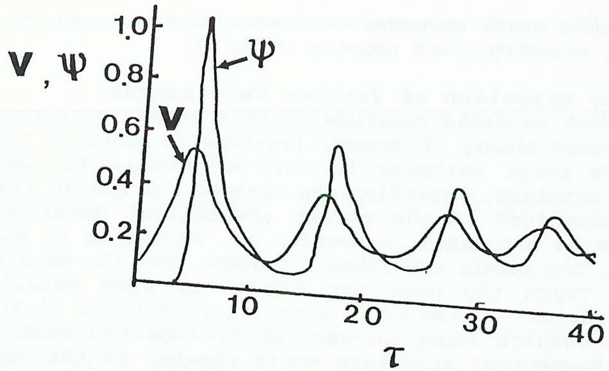

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