STRANGE MULTIPLE CRACK PATHS IN DUCTILE FRACTURE

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ABSTRACT

Problems are investigated in ductile fracture where many cracks form, the natural (unprescribed) paths of which are not always in expected directions. Experiments agree with the predictions of rigid-plastic fracture mechanics.

KEYWORDS Ductile Fracture, Crack Paths, Multiple Cracking, Fracture Mechanics.

INTRODUCTION

By 'ductile fracture' is meant fracture where extensive plastic flow precedes crack initiation, and also accompanies crack propagation. In such circumstances, rigid-plastic fracture mechanics (where elasticity is neglected) has proved to be an acceptable line of attack to predict loads, displacements and energies before and after crack initiation and propagation, both where the crack paths are prescribed and where they are not (Atkins, 1993).

Problems to be discussed in this paper concern multiple axial cracking in the flaring-out of tubes; multiple radial cracking in perforation/hole flanging of plates and sheets; multiple radial fractures in the biaxial expansion of holes in plates and sheets; converging cracks forming detached tongues of material in tearing strips of material; and diverging buckled pieces formed when plate or sheet is loaded compressively in-plane. In some of these problems the crack *directions* are not known *ab initio*; in many of them, the *number* of cracks is not known *ab initio*. We shall explore how calculations may be performed. These sorts of problems are of technological importance in crashworthiness studies, packaging and so on.

MECHANICS

Before crack initiation, we have

$$Xdu = d\Gamma \tag{1}$$

where X is the applied load, u the load-point displacement and Γ is the plastic work. This is the basic equation of rigid-plasticity. Unlike traditional fracture mechanics, there need not necessarily be a starter crack, and Γ is spread throughout the body/structure and may extend to the boundaries. Γ does not relate merely to some plastic zone at a starter crack tip. Stress and strain fields making up Γ are likely to be non-uniform but, after some loading, cracks will be initiated at locations where material damage has reached a critical value. The critical value will depend upon microstructure and will be governed by path-dependent hydrostatic stress-effective strain relations, such as those of McClintock (e.g. 1966 & 1968), Rice & Tracey (1969) in continuum plasticity or Oyane (1972), Gurson (1977), or Rousselier (1979) in porous plasticity. Sometimes, particularly with sheets, necking will occur before crack initiation, which localises the deformation and the paths of subsequent cracking are often formed by the paths of the necks.

In metal forming, crack initiation sets limits on processes such as bulk forging, drawing etc; in sheet forming, preceding necking is often the limit. Chapter 5 of Atkins & Mai (1985 & 1988) gives many illustrations of fracture limits in metal forming and how they may be predicted using Equ (1). Most of the old-established empirical criteria for cracking in bulk plasticity turn out to be versions of the damage mechanics criteria now popular in elastoplastic fracture mechanics calculations, i.e. dependent on both stress state and strain state. In some ductile fracture processes, cracking is required, as in the opening of beverage and food cans, guillotining and cropping etc. Furthermore, even when fracture is undesirable, the extent of crack propagation for a given energy input is important, as in ship grounding or for energy-absorbing devices.

During crack propagation, we have

$$Xdu = d\Gamma + RdA \tag{2}$$

where R is fracture toughness and A is crack area. Γ in Equ (2) is different from that in Equ (1) because of the existence of one or more lengthening cracks. R is the 'specific essential work of fracture' as described by Cotterell, Mai and co-workers, (e.g. 1982) and is a crack tip term uncoupled from the remote plasticity described by Γ. Equ (2) in integrated form, is the basis of the Cotterell-Mai methods of determining R in modes I, II, and III (1977, 1982, 1984). Such independent estimates of toughness, along with traditional yielding values, enables Equ (2) to be used predictively for propagation forces, work done and so on. For example, consider the opening of a full aperture can, Fig 1a. The crack path is prescribed owing to the scoring of grooves around the circular lid of the

can. Γ concerns plastic bending and unbending of the whole lid, which is removed in practice at almost constant radius. Since the plastic work done is (work/volume) (volume), the plastic work (and associated force) increases to reach a maximum across the whole diameter and thereafter decreases. In contrast, the fracture work required is very high at the beginning and end, and least across the full diameter. This comes about because the crack area increment is least across the diameter (being parallel then to the direction of bending) but otherwise inclined to the direction of pull. The large force at the end of pulling explains why we have to 'waggle' the tops of cans finally to remove them: the direction of pull is at right angles to the direction of crack propagation. Calculations using Equ (2) show that

$$X \sin\theta = C_2 + C_1 \sin^2\theta$$

where $C_1 = Yt^2 r/\rho$ and $C_2 = 2Rt'$, with Y the rigid-plastic yield strength, t the lid thickness, t', the groove thickness, r the radius of the can lid and ρ the pulling-off radius of bending. Figure 1b shows that the simple analysis seems to work satisfactorily. The Rt' intercept of 11 N gives R = 55kJ/m^2 for the lacquered and softened 5052 H19 aluminium alloy, having $\sigma_y \approx$ 260MPa, t = 0.25 mm, t' = 0.1 mm and r = 70 mm. The slope corresponds with $\rho \approx$ 22 mm which accords with experiments.

A connexion between Equs (1) and (2) may be made if it is argued that crack propagation is a process of continous re-initiation of cracks, Atkins & Mai, 1987. The fracture work per area of crack (R) obtained from pre-cracked testpieces may be converted to a plastic work per volume by dividing by the process zone height (h), i.e. RdA is dissipated within a volume hdA. In an uncracked plate which eventually fractures in some location where the effective von Mises strain is $\bar{\varepsilon}_f$, the local plastic work per volume is $Y\bar{\varepsilon}_f$ (or $\sigma_o \bar{\varepsilon}_f^{n+1}/(n+1)$ if there is workhardening represented by $\bar{\sigma} = \sigma_o \bar{\varepsilon}^n$). Whence

$\bar{\varepsilon}_f = R/hY$ *or* $\left[(n+1)R/h\sigma_o\right]^{1/(n+1)}$. $\bar{\varepsilon}_f$ may be broken down into the ε_{1f} ... components using the plasticity flow rules and loading paths up to fracture. The method works particularly well when h is well defined, as in the necked-down zones of thin sheets. We have successfully predicted the fracture forming limit diagrams of *uncracked* sheets over a wide range of strain biaxialities, using the R measured by Cotterell-Mai methods in *pre-cracked* sheets of the same material, Atkins & Mai (1987). Since such failure diagrams can also be predicted by McClintock or other damage mechanics analyses (which relate to void growth) it follows that there is an R-to-void growth connexion too (see Chapter 5 of Atkins & Mai (1985 & 1988)).

MULTIPLE CRACKING IN TUBE EXPANSION AND IN PERFORATION OF PLATES BY CONICAL TOOLS

Tubes may be successfully inverted or everted by compression on to a female doughnut-shaped die (e.g. Al-Hassani et al, 1972). When things go wrong, however, a series of stably-propagating axial cracks is formed, Fig 2. The perceived wisdom for the onset of cracking in tube expansion is, presumably, the attainment of a critical hoop strain, or critical COD at the tip of each crack. But it says nothing about the *number* of cracks. A related problem is perforation of a sheet by a conical-headed projectile; if there is a starter hole, that process becomes hole-flanging (Johnson et al, 1973) . Once again, under certain conditions a series of radial cracks is formed resulting in petalling , Fig 3a, the number of which depends on the cone angle. How may we predict the number of cracks in such problems? It turns out that the global energetics of crack propagation, i.e. Equ (2), provides the answer, when coupled to the criterion of a critical amount of damage at each fracture site (which, for simplicity, may be represented by a critical hoop strain $\varepsilon_{\theta f}$ at fracture in these cases). We contrast the work increments for

(i) plasticity alone, Equ (1) and (ii) plasticity plus crack propagation, Equ (2). It is convenient to write

$$d\Gamma = WdV + VdW \tag{3}$$

where $W(=Y\bar{\varepsilon})$ is the average plastic work/volume over the whole volume V being plastically deformed. Thus we have

$$\text{(i)} \quad X_{before}du = WdV + VdW \tag{4}$$

$$\text{and (ii)} \quad X_{after}du = WdV + VdW + N.RdA \tag{5}$$

where N is the number of cracks. When the end of a tube is initially flared out, both terms in Equ (4) exist as there are incremental changes in dV and dW since there is new material coming over the curved die from the tube *and* the leading edge of the flare experiences bigger plastic strains as it grows to larger diameters. During fracture dW is zero in Equ (5) since there is no further plastic flow in the leading edge of the flare after cracks begin to propagate, and the plastic-deformation field becomes steady. WdV still exists, since new material continues to enter the deformation zone, to be brought up to the steady-state value of W. Hence

$$\text{(i)} \quad X_{before}du = WdV + VdW \tag{6}$$

$$\text{and (ii)} \quad X_{after}du = WdV + N.RdA \tag{7}$$

At first $X_{before} < X_{after}$; vice-versa during cracking. The transition from flow to flow and fracture, occurs when the work increments change relative magnitude i.e. when

$$WdV + N.RdA = WdV + VdW$$

or

$$N = (V/R)(dW/dA) \tag{8}$$

in steady state propagation. It may be shown (Atkins, 1987) that

$$N \approx (8\pi/\sqrt{3})(Y/R)r_o\varepsilon_{\theta f} \tag{9}$$

where r_o is the radius of the tube. (Other, slightly different expressions are possible, depending upon the assumptions made; Atkins, 1989). Equivalently,

$$N \approx (4\pi/\sqrt{3})(Y/R)(u_f^2/b) \tag{10}$$

where u_f is the axial displacement of the tube at the onset of cracking and b is the radius of the toroidal die. For the 50 mm diameter, 1.6 mm wall thickness mild steel and aluminium alloy tubes studied by Reddy and Reid (1986), compressed on to a die of radius b = 12 mm, n≈ 8-12 for both as-received and annealed tubes. This is about the prediction of Equ (10) using

$(Y/R) \approx 10^{-3}\ m^{-1}$, and $u_f = 4\ mm$.

Various features of Equ (9) & (10) are worth highlighting:

(a) Equ (9) which applies when $\varepsilon_{\theta f}$ is small, suggests that N is independent of b. The results in Reddy & Reid (1986) support this, e.g. Tables 2 and 3 in that paper.

(b) N depends upon the product of (Y/R) and $\varepsilon_{\theta f}$. In very ductile solids, (Y/R) will be small, but the strain to fracture (however defined) will be correspondingly high. Vice versa, in less ductile solids (Y/R) will increase but the fracture strain decrease. Thus, according to this rigid-plastic analysis, the number of cracks may not change much between same size tubes of metal in the annealed and work-hardened states. The results reported by Reddy & Reid (1986) support this conclusion.

If, in experiments, small saw-cuts at various orientations are used, extremely interesting behaviour is observed, depending on whether the number of starter saw cuts is smaller or greater than the value of N given by Equs (9) or (10). For fewer cuts, all starter cracks propagate and some then bifurcate to bring the final number of propagating cracks up into the characteristic range; for a greater number of cuts, some starter cuts never develop into cracks, which brings down the number of cracks propagating into the characteristic range. It seems significant that all saw cuts are presumably 'ready' to propagate but some did not and that others bifurcated.

The prediction of the number of radial cracks around perforations made by conical penetrators may be performed in the same way. We start with the hole-flanging plasticity analysis of Fig 3b (Johnson & Mellor, 1973). The distribution of strain within, and the profile along, the deformed lip is calculable. In particular the hoop strain at the tip of the lip is given by

$$\varepsilon_{\theta tip} = \ell n\left(\frac{b - H\sin\alpha}{a_o}\right) \quad (11)$$

where $$H = \frac{b}{\sin\alpha}\left[1 - \{1 - [1 - (\frac{a_o}{b})^{3/2}]\sin\alpha\}^{2/3}\right] \quad (12)$$

Thus $$\varepsilon_{\theta tip} = \ell n\left[\frac{b}{a_o}\{1 - [1 - (\frac{a_o}{b})^{3/2}]\sin\alpha\}^{2/3}\right] \quad (13)$$

The mean plastic work/volume for the whole lip is some $W = Y\,\varepsilon_{\theta tip}/2$ and the volume $V = \pi(b^2 - a_o^2)h_o$. We use Equ (8) again for the number of radial cracks n, viz:

$$N = (V/R)(dW/dA)$$

where $A = h_{tip}\, dH$. Thus

$$N = \frac{\pi(b^2 - a_o^2)}{R}\,\frac{Y}{2h_{tip}}\,\frac{d\varepsilon_{\theta tip}}{dH}$$

$$= \frac{\pi(b^2 - a_o^2)}{R}\,\frac{Y(b - H\sin\alpha)^{1/2}}{2h_o a_o^{1/2}} \cdot \frac{d\varepsilon_{\theta tip}/db}{dH/db} \quad (14)$$

This expression is clearly more cumbersome to use than Equs 9 or 10.

MULTIPLE NECKING AND FRACTURE

The case of petalling in cone penetration is almost certainly to do with preceding radial neck formation at the tip of the developing conical flange around the hole. When perforating sheets with no starter hole, the tip of the penetrator has first to emerge from the distal side before radial tensile cracks can form, so effectively there is a starter hole. In thick plates with no starter hole cracking may occur on the distal side in bending. The lip is thinnest at the leading edge, so the strains are greatest there and necking is expected to start at the free edge. The number of necks must relate to displacement compatibility of adjacent elemental rings of material, as well as the requirement for a smaller increment of work in the different deformation mode. Onat and Prager's slipline field model of plane strain neck formation (1954) predicts an actively deforming shrinking region for the neck, with shoulders at $\tan^{-1}(1/2) \approx 27°$ to the axis of deformation widening out to the necking thickness t_n. Necking commences in a region of length equal to t_n and the extension e of the necked region is $e = (t_n - x)$ by geometry, where x is the current dimension of the plastically deforming zone. A ring of radius r expanding uniformly to $(r + \Delta r)$ extends by $2\pi\Delta r$. If, instead, N necks form, each of starting length t_n, it might be argued that the total length $(2\pi r - Nt_n)$ between necks remains at the same length. Each neck extends by $e = (t_n - x) = t_n - (t_n - \Delta t) = \Delta t$. For hoop displacement compatibility

$$(2\pi r - Nt_n) + N(t_n + \Delta t) = 2\pi(r + \Delta r)$$

i.e. $$N = 2\pi(dr/dt). \quad (15)$$

dr/dt is a measure of the 'side profile' of the neck, i.e. the rate of thinning. Continuity of thickness says that elemental rings will, in fact, thin down as they expand at constant Δr, which modifies the prediction for N. Even so, Equ (15) provides an acceptable estimate for N.

Rather like the saw-cuts that may not propagate in the tube expansion problem, not all such necks may become cracks. A fascinating problem which displays this feature is that of the biaxial ductile expansion of circular holes in plates (Arndt, 1996), Fig 4a. Biaxial expansion is achieved using hydraulic bulging with a second ductile sheet beneath to act as a hydraulic seal. Arndt finds (a) that the number of necks depends on the size of the starting hole; (b) that not all necks become cracks; (c) that necks initiate not at the edge of the hole (as in hole-flanging) but at some radial distance into the sheet. The latter is explained by appreciating that around the hole, the stress and strain rate is uniaxial tension (so that the radial strain is compressive) but far away from the hole the strain state is biaxial tension (with tensile radial strain). Plane strain necking occurs for those elements for which both the increment of radial strain is zero and the hoop strain is equal to the strain hardening index n. Once necks initiate at the 'in-board' location, they continue to thin down at that location, simultaneously spreading both outwards towards the rim of the disc and inwards to the bore. Eventually, at the original location, the critical fracture strain is reached, and further deformation produces radial cracks, with biggest opening at the original location, but not, initially, extending to the bore. Of importance we note that not all necks become cracks, Fig 4b. Arndt (1996) is investigating this in terms of competing work increments for (a) cracking over N* cracks within necks versus (b) continued necking over all N necks formed initially.

This sort of behaviour is seen in high rate deformation. Experiments with focussed blasts on plates showed that before tears formed, localisation occurred at approximately 24 locations around a pre-existing hole but as the deformation progressed, only 8 developed into long tears (Bammann et al, 1992).

CONVERGING & DIVERGING CRACKS

When attempts are made to tear a parallel strip from a large sheet by pulling and bending up the strip, the two tears invariably converge to a point, producing a 'gothic window' tongue of material. Fig 5 shows an experiment on a brown paper envelope. It occurs with all types of materials. The phenomenon is irritatingly familiar in packaging where tear paths are not always in the direction intended, e.g. creamer cups where the top itself tears to a point rather than at the adhesive joint around the periphery which is intended to give a 'full aperture opening; in the stripping of wall paper; in food packaging of all sorts; in getting into blister packs; and so on. When adhesive tape becomes detached from its roll along a diagonal path, it is one half of a converging pair of tear paths.

In contrast, if a sheet of material is loaded by an in-plane compressive load which tears the material, two diverging cracks are often formed, with material piling up concertina-fashion as the tear progresses, Fig 5. The diverging tear problem is familiar when a piece of paper, stuck to a notice board with a drawing pin, is suddenly pulled down with the pin in place. It is also a daily observation at the bottom of newspaper pages where, on the printing press, conveyor belt 'teeth' have indented and torn the newsprint when moving it along. Like the converging tear problem, the effect can be produced in a variety of different types of material.

There are large-scale engineering examples of both phenomena. The top of the fuselage of a Boeing 737 became detached over Hawaii in 1988: a series of pointed-tongues occurred along the lower half of the fuselage where the upper half finally ripped off. When the Exxon Valdez was dry-docked after the accident in Prince William Sound, Alaska, in 1989, a hole from which the oil had spilled was a concertina-type diverging tear (see Atkins, 1994).

Why can such different patterns of cracking behaviour occur in the same material? Why are these natural paths of cracking not straight? It is found that minimum energy arguments provide the answer.

Let us first consider the tearing/bending strip problem. A strip of initial width w is rolled up to a constant radius ρ, Fig 6 (Atkins, 1991). The tear takes its own (as yet unknown) path. At some distance x measured from the beginning of the tear, the current width of the tear is $2y$ and the distances measured along the unknown tear path are given by s. For an increment dx, the incremental volume of material bent to radius ρ is $2y\,tdx$, so $d\Gamma = \sigma_y(t/4\rho)2ytdx$. Over the same increment dx, the fracture work required is given by $2Rtds$ for both tears together. Hence Equ (2) becomes

$$Md\theta = Y(t/4\rho)2ytdx + 2Rtds \tag{16}$$

where M is the rolling-up external moment and θ is the angle of rotation. We note that $d\theta$ and dx are related by $\rho d\theta = dx$; also $ds = dx\sqrt{1+p^2}$ where $p = dy/dx$ If we minimize this work rate , two different paths are predicted, viz: a parallel path, where tearing is self-similar in the direction of the original tab sides (that is, $y = w$ = constant), and a catenary-shaped path with equation

$$y = w + \frac{8R\rho}{Y}t\left[1 - \cosh\left(\frac{Ytx}{8R\rho}\right)\right] \tag{17}$$

The pair of cosh curves converge to a point, and if $x = L$, when $y = 0$, the aspect ratio (length : initial width) is given by:

$$\frac{L}{W} = \frac{8R\rho}{wYt}\cosh^{-1}\left[1 + \left(\frac{Ytw}{8R\rho}\right)\right] \tag{18}$$

Equation (18) well describes the 'gothic window' shapes of torn tongues. No explanation was given as to why a converged path was invariably followed rather than the equally-possible parallel path. More recent work by Muscat-Fenech and the author (1994 a, b,c) has shown that it is the result of the local lift-off of the flat sheet near the tear front, and the associated contra-curvature thereby produced. Very small curvature is required to tip the balance, and Equs (17) and (18) adequately describe experimental data, without the inclusion of the lift-off curvature.

That the crack paths converge makes sense physically, in that the incremental tear length is least for a parallel path (dx); any outwards- or inwards-pointing tear would have a longer incremental length $ds = \sqrt{dx^2 + dy^2}$. Were converging bending-tearing tongues to diverge, not only is the crack path length longer, but more and more material would be rolled up compared with a reference parallel rolled-up tear. So no one is "adding more to more", and greater work is inevitably required. When crack paths converge, the fracture work increment is still greater than a parallel tear, but now the plastic bending work is smaller as less and less material is being rolled up. So an increasing function is being added to a decreasing function, which leads to an optimum shape. Hence the pair of catenary curves meeting at a point. The argument for diverging tears is not so obvious, but the clue is found in the deformation of the propagating strip. In the rolling-up case all the material is bent; in the buckled concertina case, deformation is mainly concentrated in the plastic hinges of the buckles.

Wierzbicki (1995) has given a model for tearing along a *prescribed* parallel path with material buckling up into a concertina, Fig 7. The plate is of width b and thickness t. Because of the periodic nature of the simultaneous tearing and folding failure process, it suffices to consider one section of the plate length $2H$, where H is an unknown wavelength. The plastic work Γ comprises bending work in the system of hinge lines and in-plane membrane work of stretching and compression. The tearing work along the (prescribed) parallel path is simply $2Rt\,dx$, since ds = dx in this case. Equation (2) may be applied and the external mean force obtained by minimising the energy with respect to H and ξ which locates the intersection of the hinge lines, and is another unknown of the problem.

The parallel tear analysis has been adopted by Liu (1996) to a concertina tear running at an arbitrary angle λ and the solution minimised. It is found that the optimum tear path is indeed divergent, and good agreement is found with experiment. Of significance in the result is that the fold wavelengths H increase as the tear lengthens. Since the wavelength increases with tear length in a diverging concertina, the average tear length increment ($H/\cos\lambda$) on which the mean force is calculated, increases and each wave requires more fracture work. Consequently the mean force from fold to fold (operating over longer distances H) can still be kept low even with the additional lengths of hinge line in a diverging tear because the major part of the fold remains flat and is underformed. Were the concertina to converge, instead of diverge, the wavelengths would decrease and, at a given concertina length, some of the previously underformed 'flat' material would have been hinged, leading to a greater work requirement. It is the fact that the concertina mode of deformation takes place in discrete regions which makes a diverging tear energetically favourable.

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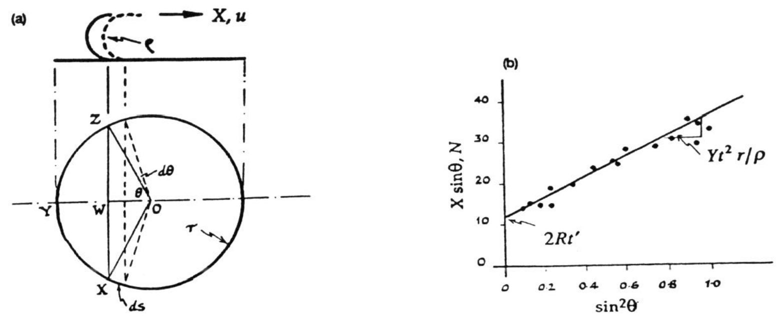


Fig 1 The opening of a "full aperture" can by force X applied parallel to the top surface. Model assumes plastic bending at constant ratio ρ throughout. (a) Geometry of opening; (b) Experimental forces plotted in a way suggested by model.

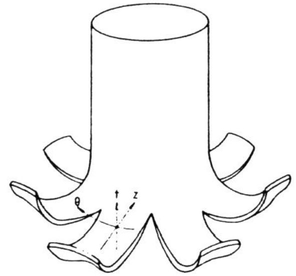
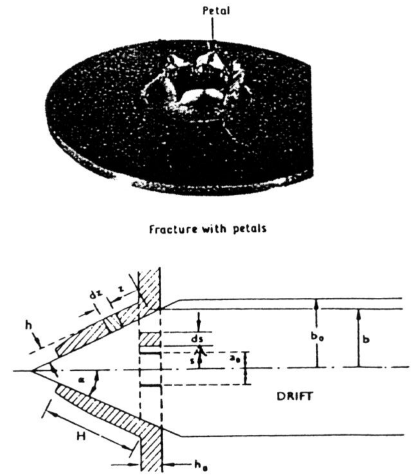


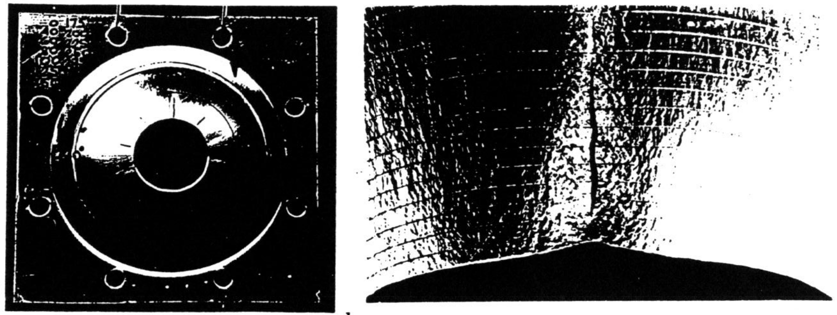
Fig 2 Axial splitting of flared-out ductile metal tubes.

a



b

Fig 3(a) Petal formation in hole flanging with conical drifts; (b) Model based on ring expansion.



a b

Fig 4(a) Radial neck formation during biaxial hole expansion; (b) Detail of radial crack (not all necks become cracks).

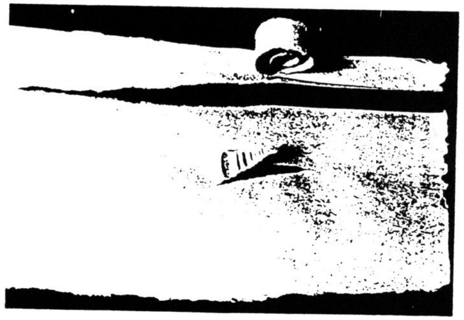


Fig 5 Converging and diverging tears in the same brown paper envelope.

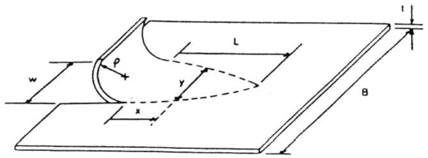


Fig 6 Model for rolled-up converging tear.

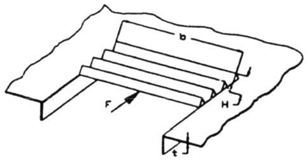


Fig 7 Wierzbicki's Model for concertina tearing along a prescribed path.