

## STATUS OF RESEARCH AIMED AT PREDICTING STRUCTURAL INTEGRITY

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### ABSTRACT

Considerable research has been performed throughout the world in measuring the fracture toughness of metals. This capability fulfills the need encountered when selecting materials, thermal-mechanical treatments, welding procedures, etc., but needs to be expanded to include the ability to predict the fracture process of structural components containing cracks. The Idaho National Engineering Laboratory and the Massachusetts Institute of Technology have been collaborating for a number of years in developing capabilities for using fracture toughness results to predict structural integrity. Because of the high cost of fabricating and testing structural components, these studies have been limited to predicting the fracture process in specimens containing surface cracks. This paper summarizes the present status of the experimental studies of using fracture toughness data to predict crack growth initiation in specimens (structural components) containing surface cracks. These results are limited to homogeneous base materials.

### KEYWORDS

Surface cracks, fracture toughness, structural integrity.

### INTRODUCTION

The concern addressed in this paper is to identify the ability and limitations of using a single fracture toughness parameter ( $K$ ,  $J$ , or  $\delta$ ) that uniquely quantifies the displacement, strain, and stress fields at the crack tip to predict structural integrity. In predicting the fracture process (crack growth initiation, stable crack growth, and catastrophic failure) of structural components, it is necessary to have some measurement of the fracture toughness and the ability to transfer these measurements to the structural component. ASTM test standards exist for measuring plane strain fracture toughness ( $K_{Ic}$ ) per E399 (ASTM, 1990) and the critical value of  $J$  ( $J_{Ic}$ ) near the onset of stable crack extension per E813 (ASTM, 1989a). The critical value for crack tip opening

displacement ( $\delta$ ) measured per E1290 (ASTM, 1989b) may be substituted for  $J$ . The former ( $K_{Ic}$ ) is limited to linear-elastic behavior whereas the latter is used for linear-elastic and elastic-plastic conditions.

Three potentially significant differences exist between test specimens used in ASTM E399, E813, and E1290 and structural components (see Table 1). The difference in size can often lead to an elastic-plastic or fully plastic condition in the test specimen while the structure may exhibit a linear-elastic behavior. Also, there may be a significant difference between the size of the crack in the test specimen and in the structural component. This difference can lead to a nonconservative estimate of the structural lifetime because of statistical effects. The longer crack is more likely to intercept an embrittled region (lower fracture toughness) than the shorter crack. For elastic conditions, it is possible to relate  $K$  and  $J$  using Equation (1),

$$J = \frac{K^2(1 - \nu^2)}{E} \quad 1$$

where:  $\nu$  = Poisson's ratio, and  $E$  = Young's modulus.

Table 1. Comparison between specimens and structures.

Configuration	Specimen Size	Crack Configuration	Constraint <sup>a</sup>
Test specimens	Small flat plates	Straight, through-thickness, crack lengths are generally short	High
Structural components	Large flat plates or cylindrical sections	Curved, part-through thickness, crack lengths may be large	Variable

<sup>a</sup>Constraint is defined as the ratio of the hydrostatic stress to the equivalent von Mises stress.

Differences between a through crack and a part-through crack are the former is frequently typified as a two-dimensional crack problem with the crack driving force reasonably constant along the straight crack front. Crack growth initiation occurs whenever the crack driving force at any location along the crack front equals the fracture toughness. Since the crack driving force is generally constant along the straight crack front, it follows that initiation of crack growth occurs at the same "instant." The surface crack is a three-dimensional crack (see Figure 1), and the crack driving force varies around the perimeter of the crack front. Currently, the normal approach is to assume that when the crack driving force at any location around the crack perimeter equals the fracture toughness of the material, then crack growth initiation occurs. Another difference is that the test specimen is normally removed from a plate, whereas the structure can be a plate, a cylindrical section, etc.

For test specimens that are configured to obtain a maximum constraint, the measured fracture toughness is generally the lowest value that may be obtained. For structural components that exhibit a lower constraint (and generally a higher fracture toughness) due to shallow cracks, etc., the use of the lowest value fracture toughness may be too

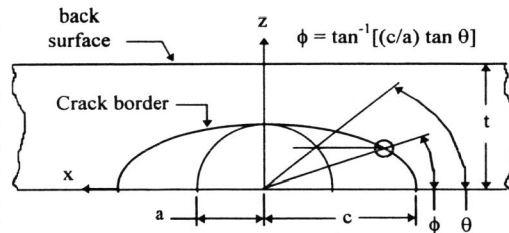


Figure 1. Schematic showing surface crack geometry.

conservative, i.e., requiring unnecessary work stoppage or repairs. This paper summarizes experimental research that provides answers to many questions of transferability.

## APPROACH

Fracture toughness measurements provided in ASTM E 399, E 813, and E 1290 are based on conditions associated with initiation of crack growth. For linear-elastic fracture mechanics (LEFM) conditions, crack growth initiation is often but not always synonymous with catastrophic failure. Therefore, an approach based on crack growth initiation appears to be useful for structures that exhibit linear-elastic behavior. An alternative approach is to use a procedure based on crack arrest. E 1221 (ASTM, 1988) provides test methods for measuring plane-strain crack-arrest fracture toughness. However, it is much more difficult to transfer this concept to a structure because it is necessary to quantify the compliance of the component as it plays a key role in the crack arrest process. Therefore, crack-arrest fracture toughness will be ignored in this paper.

For elastic-plastic [nonlinear elastic fracture mechanics (NLEFM)] and fully-plastic conditions, it is generally observed that substantial stable crack growth may occur after initiation of crack growth. Therefore, to more accurately predict the fracture process for these conditions, it is necessary to have test results that provide information on stable crack growth. The  $J$ - $\Delta a$  results obtained using E 1152 (ASTM, 1987) provide information on stable crack growth. But from an analytical viewpoint, the wake of a growing crack includes cold-worked, plastically deformed material, where Hutchinson-Rice-Rosengren (HRR) solutions are no longer valid, i.e., no longer unique (personal communication with F. McClintock, September 11, 1996). The limit of crack growth for  $J$  validity has been studied by Xia et al. (1995) who concluded that no approach can be based on a single parameter resistance curve. To evaluate if this is a practical problem, however, requires experimental verification. Dadkhah and Kobayashi (1994) and May and Kobayashi (1995) performed experiments in which they observed that  $J$  no longer provided the HRR fields at the crack tip when  $\Delta a$  exceeded some amount of crack growth. This strongly suggests that  $J$  no longer represents the crack tip stress fields when crack growth,  $\Delta a$ , is more than two to three times crack-tip opening displacement (CTOD). Based on these statements, the following discussion is limited to using fracture toughness results obtained per existing ASTM Test Standards to predict conditions for initiation of crack growth in specimens containing surface cracks. These specimens have been fabricated from homogeneous materials.

Because of the complexity of the issues being considered, this paper will examine linear-elastic fracture mechanics (LEFM) and nonlinear elastic fracture mechanics (NLEFM) as two separate topics. A major need fulfilled in the following is that the materials used to measure the fracture toughness were removed from the same piece of material used to fabricate specimens containing surface cracks.

## LEFM CONDITIONS

The ability to predict the maximum flaw size that may be allowed in a structural component is based on knowing the applied stress, the fracture toughness (measured per E399), and having an applicable equation. For these tests, the maximum applied stress intensity factor ( $K_{max}$ ) was calculated using the failure load, the actual crack size, and the Newman-Raju (1978) equations:

$$K = (\sigma_t + H\sigma_b) \sqrt{\frac{\pi a}{Q}} F \quad 2$$

where:  $\sigma_t$  = applied tensile load,  
 $\sigma_b$  = applied bending load,  
 $H$  = correction factor for bending load,  
 $a$  = crack depth,  
 $c$  = one-half the crack length at free surface,  
 $Q$  = geometry and plastic zone correction term,  
 $2b$  = specimen width, and  
 where:  $t$  = thickness

$$F = \left[ M_1 + M_2 \left( \frac{a}{t} \right)^2 + M_3 \left( \frac{a}{t} \right)^4 \right] f_\phi g f_w \quad 3$$

$$\begin{aligned} M_1 &= 1.13 - 0.09 a/c \\ M_2 &= -0.54 + 0.89/(0.2 + a/c) \\ M_3 &= 0.5 - 1.0/(0.65 + a/c) + 14(1.0 - a/c)^{24} \\ g &= 1 + [0.1 + 0.35(a/t)^2](1 - \sin \phi)^2 \\ f_w &= [\sec \pi c/(2b - a/t)]^{1/2} \\ f_\phi &= [(a/c)^2 \cos^2 \phi + \sin^2 \phi]^{1/4} \\ H &= H_1 + (H_2 - H_1) \sin^p \phi \\ p &= 0.2 + a/c + 0.6 a/t \\ H_1 &= 1 - 0.34 a/t - 0.11(a/c)(a/t) \\ H_2 &= 1 + G_1(a/t) + G_2(a/t)^2 \\ G_1 &= -1.22 - 0.12 a/c \\ G_2 &= 0.55 - 1.05(a/c)^{0.75} + 0.47(a/c)^{1.5} \end{aligned}$$

The ability to predict crack growth initiation was quantified by calculating the ratio of the calculated maximum applied stress intensity ( $K_{max}$ ) to  $K_{Ic}$ .

### Results

The following results were obtained from three materials consisting of Ti-15-3, a monolithic SiC, and D6-aC (a high-strength steel). The Ti-15-3 was heat-treated to a yield strength ( $\sigma_{ys}$ ) = 1,452 MPa, with resulting plane strain fracture toughness ( $K_{Ic}$ ) = 41.4 MPa $\sqrt{m}$ . The specimens containing surface cracks had a fatigue precrack starter notch fabricated by electric discharge machining (EDM), and the specimens were then load cycled either in tension or in bending to grow the desired fatigue precrack. The fatigue precracks had a crack depth-to-thickness ratio ( $a/t$ ) ranging from 0.05 to 0.94 and crack depth-to-length ratios ( $a/2c$ ) ranging from 0.01 to 0.47. The specimens were tested by monotonic loading in either tension or in bending. The test results consisted of load versus acoustic emission, load versus crack mouth opening displacement, and load versus displacement (quantified using moiré interferometry). The specimens containing surface cracks failed catastrophically with little or no stable crack growth. Reuter et al. (1994) discussed these results and noted that  $K_{max}/K_{Ic}$  ranged from 1.02 to 1.64 for specimens loaded in tension, and ranged from 1.02 to 2.07 for specimens loaded in bending.

Standardized test procedures are not available for measuring the plane strain fracture toughness of ceramics. But, the procedures provided in ASTM E 399 were used, except that the SiC specimens contained EDM notches as opposed to the required fatigue precracks. The defects in the specimens containing surface cracks were also made with EDM and had the same notch root radius. It was assumed that comparisons between the EDM-notched specimens would be as valid as comparisons between specimens containing fatigue precracks. The specimens were tested by monotonic loading in bending. The test results were the same as those collected for Ti-15-3. The specimens failed catastrophically with no stable crack growth. Reuter et al. (1994) presented these results and noted that  $K_{max}/K_{Ic}$  ranged from 0.99 to 1.41 for one series of SiC specimens and ranged from 0.94 (1.00)<sup>a</sup> to 1.93 (1.39) for a second series of SiC specimens.

The tests described above were performed at a single facility and it was desired to broaden the scope of material to include a high-strength steel and multiple test facilities. Therefore, an International Cooperative Test Program was organized to perform tests on specimens fabricated from D6-aC, a high-strength steel (Reuter et al., 1996a). The material was heat-treated to  $\sigma_{ys}$  = 1,587 MPa, with resultant  $K_{Ic}$  = 54 MPa $\sqrt{m}$ . The surface crack configuration had a  $a/2c$  ratio ranging from 0.08 to 0.60 (0.56) for tensile loading and from 0.08 to 0.51 for bending loads, and the  $a/t$  ratio ranged from 0.23 (0.28) to 0.89 (0.83) for tensile loads and from 0.22 to 1.0 (0.84) for bending loads. Initially, the test results were the same as those collected for Ti-15-3. Many of the surface-cracked specimens exhibited substantial crack growth prior to attainment of the maximum load. Use of the initial precrack size and shape and the maximum load at failure to calculate  $K_{max}$  for comparison with  $K_{Ic}$  was inappropriate. Therefore, the test plan was modified to detect the onset of crack growth initiation using d.c. potential drop and acoustic emission. A change of 5% in d.c. potential drop was defined as crack growth initiation. The  $K_{max}/K_{Ic}$  ratio ranged from 0.84 (0.98) to 1.56 for tensile loading and from 1.13 to 1.8 for bending loads.

### Discussion

In a vast majority of these 99 tests,  $K_{max}/K_{Ic} \geq 1.0$ .<sup>b</sup> This illustrates that the use of Equation (2) and the measured  $K_{Ic}$  result in conservative estimates of failure for specimens (structures) containing surface cracks. But a number of instances (11 out of 34 metal specimens tested in bending) were observed in which considerable conservatism occurred ( $K_{max}/K_{Ic} > 1.52$ ) when  $K_{max}$  occurred at the free surface ( $\phi = 0$  degrees). These results are acceptable for many applications, but it might be necessary to better understand the conditions controlling fracture. The primary questions of interest are (1) What parameter other than the calculated crack driving force ( $K_{app}$ ) is responsible for initiation of crack growth? and (2) Is the local K or modification (average, specific locations, etc.) responsible for initiation of crack growth?

Test results of the SiC specimens (Reuter et al., 1994) showed that 10 specimens had  $K_{max}$  occurring at the free surface and that the ratio  $K_{max}/K_{Ic}$  ranged from 0.99 to 1.47 (1.28), suggesting that the substantial conservatism was not observed in this material. For the Ti-15-3 specimens tested (Reuter et al., 1994), 10 specimens had  $K_{max}$  occurring at the free surface and the ratio  $K_{max}/K_{Ic}$  ranged from 1.18 to 2.07 (1.83), suggesting that substantial conservatism was observed in this material. Reuter et al. (1994) examined the use of  $K_{max}$ ,  $K_{average}$  (root-mean square of K along the crack front),  $K_{ave}$  (average of local K values calculated at all locations along the crack front),

a. Items in parentheses denote the value of the next closest neighbor.

b. This ratio was greater than 1.0 for 97 of 99 specimens tested.

and  $K_\phi$  (at a specific location). It was concluded that  $K_{max}$  was the most conservative single parameter fracture criterion if attainment of  $K_{Ic}$  is considered a sufficient condition for fracture. The use of  $K_{average}$  and  $K_{\phi=30^\circ}$  was based on results of Sommers and Aurich (1991) for elastic-plastic conditions where it was observed that the maximum crack driving force ( $J$ ) occurred at  $\phi = 30$  degrees. This suggested that the maximum CTOD ( $\delta$ ) also occurred at  $\phi = 30$  degrees. Reuter and Lloyd (1990) showed that  $\delta$  was not a maximum at  $\phi = 30$  degrees for specimens tested in tension that exhibited elastic-plastic behavior. They observed that the relative magnitude of  $\delta$  followed the calculated relation for  $K$  at applied stresses where crack growth initiation was detected.

As noted earlier, many tests of D6-aC steel specimens containing surface cracks exhibited substantial crack growth. A combination of electric potential change (DCP) and acoustic emission (AE) monitoring were used to detect initiation of crack growth, and the applied load associated with it. The load was reduced after initiation was detected, and cyclic loading was applied to decorate the location and extent of crack growth. Reuter et al. (1996b) examined several specimens loaded in tension and observed that crack growth initiation occurred at  $\phi = 90$  degrees, with the majority of crack growth within  $\pm 15$  degrees of  $\phi = 90$  degrees, and with no crack growth at  $\phi = 30$  degrees (see Figure 2). Several of these specimens, loaded in tension, were loaded multiple times, which consisted of (1) fatigue precracking, (2) monotonic loading to obtain stable crack growth, (3) decreased load, (4) cyclic loading to outline the region of the stable crack growth, and (5) unloading. Steps 2, 3, 4, and 5 were repeated until the crack penetrated the back surface (see Figure 1 and Figure 2).

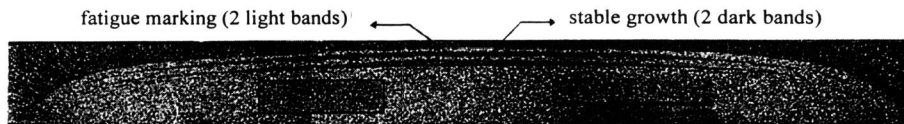


Figure 2. D6-aC Specimen #26 - Fracture surface showing two load sequences.

For three specimens with thickness ( $t$ ) = 6.4 mm,  $K_{init}$  increased for each cycle of monotonic loading, i.e., with increasing  $a/t$  (see Figure 3). This suggests (1) a loading history effect, i.e., plastic zone development, (2) a constraint effect due to the proximity of the crack tip to the back surface, or (3) the Newman-Raju (1978) stress intensity distribution limit  $a/t < 0.80$  for accurate results. At this time, none of the three possible explanations have been ruled out.

Chao and Reuter (1996) examined several specimens loaded in bending and observed that initiation did not occur at the free surface even though  $K_{max}$  was calculated to occur at  $\phi = 0$  degrees (see Figure 4). Of 19 specimens tested in bending, the crack growth initiation sites were located for 15 specimens. For these specimens, the crack initiation sites occurred at  $\phi$  ranging from 45 to 9 degrees, with an average of  $\phi = 21$  degrees (close to 30 degrees). The reason for  $\phi = 21$  degrees is not yet understood.

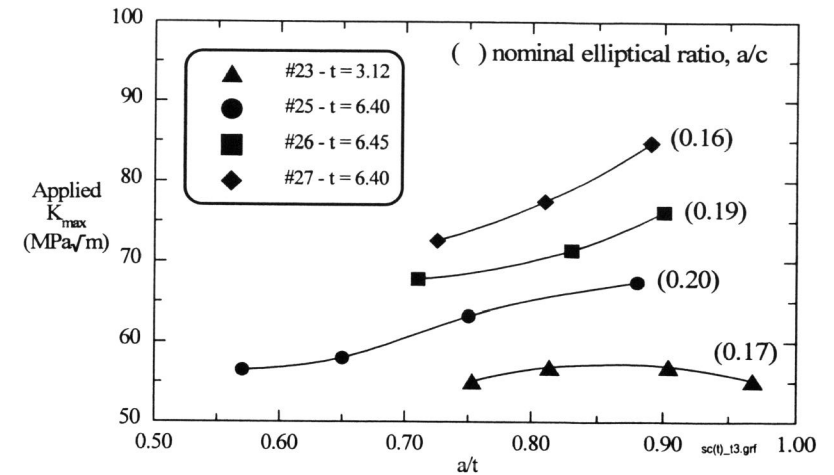


Figure 3. Crack initiation stress intensity factors for varying crack depths.

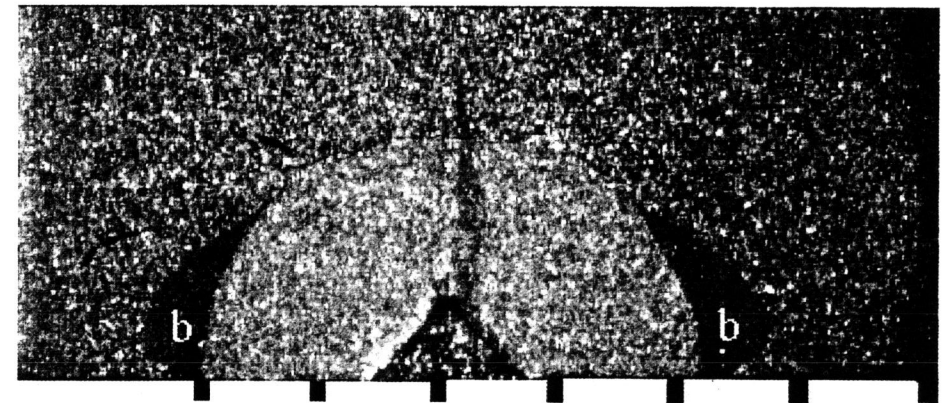


Figure 4. D6-aC Specimen #10 - Fracture surface showing stable growth near surface at "b." (mm scale bars)

## ELASTIC-PLASTIC CONDITIONS

For conditions in which  $K$  is no longer applicable, it is necessary to use either  $J$  or  $\delta$  as the critical fracture toughness parameter. The boundary separating LEFM from elastic-plastic fracture mechanics is not well defined.

### Results

The following results were obtained by testing specimens fabricated from ASTM A710 steel at a temperature (22°C) corresponding to about midway in the ductile-brittle transition region. Reuter et al. (1991) performed fracture toughness tests of C(T) (compact tension), SE(B) (single-edge notch bend), M(T) (center-cracked plate), and SC(T) (surface cracked plates loaded in tension) specimens. Both multiple-specimen techniques and single-specimen techniques were used in the test procedure. For multiple specimens, several replicate specimens were loaded, each to a different value of load or displacement, and the specimens unloaded. Each of these specimens were sectioned, polished, and examined metallographically to measure the extent of crack growth and the corresponding value of  $\delta$ . These results were used to obtain a plot of  $\delta$  versus  $\Delta a$  and then extrapolating to  $\Delta a = 0$  to estimate  $\delta$  for crack growth initiation. For the single-specimen evaluation, a specimen was loaded until crack growth was detected, the load reduced, and either the specimen experienced cyclic loading or it was loaded in liquid nitrogen to cause cleavage fracture, either of which will decorate the magnitude of stable crack growth. The two fracture surfaces were then examined using microtopography to measure  $\delta$  at crack growth initiation.

In addition, the constraint (hydrostatic stress normalized by dividing by the equivalent stress based on the von Mises yielding criteria) was calculated for each specimen configuration, and a relation was observed between  $\delta$  (crack growth initiation) and constraint. This relation was expanded and more completely developed in Hancock et al. (1993).

In Reuter and Lloyd (1990), a series of tensile tests were performed in which measurements of  $\delta$  and crack tip opening angle were made as a function of load at several locations around the perimeter of the surface cracks. These specimens were loaded in tension and had a  $a/2c$  ratio of 0.1 or 0.5. For specimens with  $a/2c = 0.1$ , crack growth initiation occurred at  $\phi = 90$  degrees and disappeared at  $\phi = 0$  degrees. As the applied stress ( $\sigma$ ) to  $\sigma_{ys}$  ratio approached 0.96,  $\delta_{pl}$  (plastic component of  $\delta$ ) followed the prediction of Newman and Raju (1978). As  $\sigma/\sigma_{ys} > 0.96$ , there was a larger increase in  $\delta_{pl}$  near the free surface than occurred elsewhere.

For  $a/2c = 0.5$ , crack growth initiation appeared to occur at  $\phi = 90$  degrees and disappeared at  $\phi = 0$  degrees. (Some crack growth was visible around much of the surface crack perimeter, but none at  $\phi = 0$  degrees.) As  $\sigma/\sigma_{ys}$  approached 1.22,  $\delta_{pl}$  followed the prediction of Reference 9. When  $\sigma/\sigma_{ys} > 1.3$ , a substantial increase in  $\delta/\delta_{el}$  ( $\delta_{el}$  is an elastic component of  $\delta$ ) occurs, particularly at  $\phi = 90$  degrees.

### Discussion

The results in Reuter et al. (1991) suggest that the use of  $\delta$  measured per E 1290 may provide too severe conservatism when predicting the condition for crack growth initiation of a structural

component containing a shallow crack. If the relation between  $\delta$  and constraint is known, it is possible to use a more realistic value for  $\delta$ . The corresponding value of  $\delta$  for the surface crack specimen was 1.4 times the value obtained in the more highly constrained SE(B) specimen (see Figure 5).

From a metallurgical point of view, the relationship between  $\delta$  or  $J$  and constraint is unique because of the microstructure, trace elements, and thermal mechanical procedures used to develop the material to its final form. Therefore, depending on the consequences of failure, it is strongly suggested that specimens/crack depth that provide the same constraint expected in the structure be tested to verify the magnitude of  $\delta$  responsible for crack growth initiation.

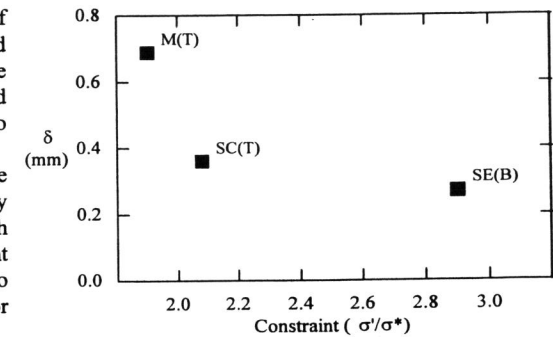


Figure 5. Crack tip opening displacement (at initiation) for different amounts of constraint. The substantial changes in  $\delta$  as a function of  $\sigma/\sigma_{ys}$  observed in Reuter and Lloyd (1990) is probably due to loss of constraint. Parks and Wang (1988) predict loss of HRR field dominance to occur at about  $\sigma/\sigma_{ys} = 0.85$  and  $1.04$  for  $a/2c = 0.1$  and  $0.5$ , respectively. But constraint was still high up to the load at which crack growth initiation occurred, and it was observed that crack growth initiation occurred at  $\phi = 90$  degrees.

## CONCLUSIONS

Actual structural components were not available for comparing the ability for using fracture toughness data to predict structural integrity, but specimens containing surface cracks were used instead. This is a logical step as the surface cracks used in the specimens encompass many of the configurations found in structural components.

These results were not applicable to evaluate crack size effects associated with differences between the specimen size and the structural size because the surface cracked specimens were limited in size.

For LEFM conditions, crack growth initiation did not occur when  $K_{max} = K_{Ic}$ . The use of  $K_{Ic}$  and Equation (2) resulted in conservative predictions of crack growth initiation. This was true for surface-cracked specimens loaded in tension or in bending. For specimens tested in bending, the conservatism may be substantial when  $K_{max}$  occurs at the free surface. This was especially true for the metallic specimens, and the magnitude of the conservatism was reduced considerably for the monolithic ceramics. The above conclusions are limited to conditions in which crack growth initiation is detected. If stable crack growth occurs, then the use of the maximum applied stress and the original crack size used in conjunction with Equation (2) to calculate  $K_{max}$  has no relevance to  $K_{Ic}$ .

At this time, it is not possible to answer if an average  $K$  (a  $K$  at a specific location or an additional parameter with a local  $K$  value) is responsible for initiation of crack growth for surface-

cracked specimens. There is no basis to suggest that structural components will behave differently from the surface-cracked specimens tested.

For NLEFM conditions, it is possible to predict crack growth initiation for specimens containing surface cracks if the relationship between  $\delta$  and constraint has been quantified. This relation is known to vary as a function of material type and is expected to vary as a function of heat-to-heat variations within a given material. Therefore, once this relation is known for the specific material of interest, it is then necessary to quantify the constraint for the specific crack in the structural component. It is then possible to identify the critical value of  $\delta$  associated with the specific constraint for predicting structural integrity.

The above conclusions are limited to flat plate in tension or in bending and do not address cylinders containing axial or circumferentially oriented cracks on the ID or OD surfaces.

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