# SOME PROBLEMS ON MODELING OF INTERFACE FRACTURE

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## ABSTRACT

Modeling of an interface crack limit equilibrium and growth was performed by the author and his colleagues during the lasts years. The paper is mainly based on these results. An asymptotic approach for calculation of the stress state near the contour of a delamination of an arbitrary shape in plan at an interface of a two-layered structural element was developed. Using the method of matched asymptotic expansions enabled to construct an effective procedure for calculation of the stress intensity factors at the crack contour on the basis of the plate theory taking into account 3D effects near the crack contour caused by the influence of the external boundaries of the bi-material element. A bridged interface crack model was suggested for modeling and evaluation of the adhesion fracture resistance of an interface. Bond stresses were searched for by numerical solving a derived system of two singular integro-differential equations. An analysis of the limit force and energetic characteristics in the crack end region was performed in assumption of the linear elastic behavior of the bonds.

## KEYWORDS

Interface crack, layered plate, bond, adhesion fracture resistance.

## INTRODUCTION

Modeling of an interface crack limit equilibrium and growth implies both calculation of the stress state near the crack contour taking into account the geometry of a non-homogeneous body (structural element) and evaluation of the adhesion fracture resistance of the joint experimentally (e.g., by the "blister" test or similar ones) or through a model of bonding in the region close to the delamination contour (the end region in 2D case).

The elasticity problems on calculation of the stress intensity factors and deformation energy release rate at the delamination contour can be solved by several routine procedures of commonly used methods (e.g. FEM, BEM, methods of the complex potentials) in case of piecewise homogeneous isometric structural elements being considered as 2D or 3D elastic bodies. However, very often delaminations are occured at the interfaces between thin layers in composite panels or between a thin coating and structural element. Series of papers have been devoted to studying the delaminations in thin layered bodies in assumption that the delaminated part of a thin layer (or two layers) can be modeled as a thin plate (or two plates) with the conditions of rigid clamping along the crack contour (cf. Kachanov, 1976; Slepjan, 1981; Bazant, 1991; Bolotin, 1986; Williams, 1988; Parzevskii, 1990; Nilsson and Storakers, 1990; Chai, 1990; Suo and Hutchinson, 1990). This model is attractive since it allows, in particular, to analyze the delamination behavior by rather simple energetic methods. However, the applicability of such a model need to be considered specially. Indeed, a zone of 3D stress state exists along the contour of an interface crack when the delaminated layer (coating) is thin. As a result the conditions of the plate clamping along the crack contour in general differ from the conditions of the rigid clamping. These conditions will be considered following Goldstein and Konovalov (1996 a,b) where the clamping conditions have been determined on the basis of an asymptotic analysis of the full 3D elasticity problem on a delamination in a thin two layered structural element. Note, that the stress intensity factors and energy release rate at the delamination contour are strongly influenced by a difference of the clamping conditions from the rigid clamping ones.

If the local stress state near the contour of an interface crack is determined then to search for the conditions of the crack limit equilibrium and growth one should use a criterion of the adhesion fracture. The criterion of the critical value of the energy release rate suggested by Salganik (1963) is often used. According to this criterion the adhesion fracture resistance of an adhesive joint is characterized by the adhesion fracture energy. It seems to be promising to evaluate the adhesion fracture resistance through the parameters of interfacial bonding. Such estimates will give us an orientation on improving the adhesion fracture resistance by adjusting the bond deformation and rupture characteristics. The problem on searching for the dependences of the adhesion fracture resistance on bonding parameters will be considered following Goldstein and Perelmuter (1996 a,b) were a model of a bridged interface crack has been developed. Note, that the bridge crack models for a crack in homogeneous bodies are well known (cf. Cox and Marshall, 1994). A crack at the interface of two dissimilar half-planes is considered. An interaction of the crack surfaces is provided by bonds acting in its end regions. The bond stress is assumed to be proportional to the crack opening. The end region size is not assumed to be small as compared with the crack length. The problem on searching for the bond stresses is reduced to a system of two singular integro-differential equations. The numerical solution of the system enables us to analyze the sensitivity of the adhesion fracture resistance to the bonding parameters and end region size.

## 3D-PROBLEM ON A DELAMINATION IN TWO-LAYERED STRUCTURAL ELEMENT

### Stress-Crack Opening Relations

Let us consider an element which consists of two layers of the same thickness, H, with the elastic constants $\lambda_1$, $\mu_1$ and $\lambda_2$, $\mu_2$, respectively. Assume that a crack, G, of arbitrary shape is located at the interface, $x_3=0$. The upper, $x_3=H$, and lower, $x_3=-H$, boundaries of the element are free of loads. The external normal and shear loads are applied to the crack surfaces. Denote by $u_1$, $u_2$, $u_3=w$ the components of the relative displacements of the crack surfaces in the directions of the coordinate axes $x_1$, $x_2$, $x_3$, respectively, and by $\sigma$, $\tau_1$, $\tau_2$ the normal and shear stresses at the crack plane.

The operator relations connecting the stresses and crack opening can be written as follows



$$\begin{aligned}
\sigma &= -2\mu_o\xi[R_o^+(\xi)w - iQ_o(\xi)(\xi_1\xi^{-1}u_1 + \xi_2\xi^{-1}u_2)] \\
\tau_i &= -2\mu_o\xi_i[iQ_o(\xi)w + R_o^-(\xi)(\xi_1\xi^{-1}u_1 + \xi_2\xi^{-1}u_2)] \\
&\quad - \mu \,\mathrm{th}\, \xi H[(\xi^2 - \xi_i^2)\xi^{-1}u_i - \xi_1\xi_2\xi^{-1}u_j]
\end{aligned} \tag{1}$$

where (and later on) j=2 at i=1 at j=1 at i=2; $\xi=(\xi_1, \xi_2)$ is the parameter of the Fourier transformation on the coordinates $x=(x_1, x_2)$

$$\begin{aligned}
&f(\xi) = \frac{1}{2\pi}\int f(x)e^{i\xi x}dx, \quad f(x) = \frac{1}{2\pi}\int f(\xi)e^{-i\xi x}d\xi, \\
&dx = dx_1dx_2, \quad \xi x = \xi_1x_1 + \xi_2x_2;
\end{aligned} \tag{2}$$

$$\begin{aligned}
R_o^{\pm}(\xi) &= T_o^{-1}(\xi)(\lambda_o + 2\mu_o)(\lambda_o + \mu_o)(\mathrm{sh}\,\xi H\,\mathrm{ch}\,\xi H \mp \xi H) \\
Q_o(\xi) &= T_o^{-1}(\xi)(\lambda_o + \mu_o)[\mu_o\mathrm{sh}^2\,\xi H + (\lambda_o + \mu_o)(\xi H)^2] \\
T_o(\xi) &= (\lambda_o + 3\mu_o)(\lambda_o + \mu_o)[\mathrm{ch}^2\,\xi H + \mu_o^2 + (\lambda_o + \mu_o)^2(\xi H)^2]
\end{aligned} \tag{3}$$

$$\begin{aligned}
(\lambda_o + \mu_o)^{-1} &= (\lambda_1 + \mu_1)^{-1} + (\lambda_2 + \mu_2)^{-1} + \mu^{-1} - \mu_o^{-1} \\
\mu^{-1} &= \mu_1^{-1} + \mu_2^{-1} \\
\mu_o^{-1} &= \mu_1^{-1} - \mu_2^{-1} + 2\mu_2(\mu_2^{-1}(\xi H)^2 + (\lambda_2 + \mu_2)^{-1}\mathrm{sh}^2\xi H)(\mathrm{sh}^2\xi H - (\xi H)^2)^{-1}
\end{aligned} \tag{4}$$

### Two-Layered Element of Small Thickness

Assume that the thickness of the element, 2H, is small as compared with the characteristic scale of the delamination, l. The ratio $\varepsilon=(2H/l)$ is the small parameter of the problem. By the expansion of the coefficients in relations (1) and solution ($u_i$, w, $\sigma$, $\tau_i$) in powers of $\varepsilon$. Then the main term of the asymptotic expansion of relations (1) has the following form

$$\begin{aligned}
\sigma &= -\left(\frac{1}{D_o} - \frac{B_o}{C_o^2}\right)^{-1}\xi^4 w + \left(\frac{C_o}{D_oB_o} - \frac{1}{C_o}\right)^{-1} i\xi^2\sum_i \xi_i u_i \\
\tau_i &= -\left(\frac{C_o}{D_oB_o} - \frac{1}{C_o}\right)^{-1} i\xi_i\xi^2 w - \left[\left(\frac{1}{B_o} - \frac{D_o}{C_o^2}\right)^{-1}\xi_i^2 + a_o(\xi^2 - \xi_i^2)\right]u_i + \\
&\quad + \left(a_o - \left(\frac{1}{B_o} - \frac{D_o}{C_o^2}\right)^{-1}\right)\xi_1\xi_2 u_j
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
&D_o^{-1} = D_1^{-1} + D_2^{-1};\ B_o^{-1} = B_1^{-1} + B_2^{-1};\ C_o^{-1} = C_1^{-1} - C_2^{-1};\ a_o = a_1 - a_2; \\
&a_i = \mu_i H,\ D_i = E_i H^3/12(1 - \nu_i^2), \\
&C_i = E_i H^2/6(1 - \nu_i^2),\ B_i = E_i H/4(1 - \nu_i^2),\ i = 1,2
\end{aligned}$$

and $D_i$, $B_i$, $C_i$ are the bending, membrane and membrane-bending rigidities of the upper (1) and lower (2) layers, respectively.

Relations (5) can be treated as ones relating a gap between two plates through the crack region, G, clamped along the crack contour and stresses acting at the plates. It's evident that the solution of plate problem (5) will represent the solution of the full crack problem only at some distance from the crack contour. Near the crack contour one should consider a boundary layer problem.

### Boundary Layer Near the Crack Contour

To analyze the boundary layer let us consider relations (1) in the local coordinates directed along the normal and tangent to the crack contour at a given point, $x^{\Gamma}$, such that the coordinate along the normal will be the "fast" one: $z=(\varepsilon z_1, z_2)$ where $\tilde{z} = (z_1, z_2) = V(x - x^{\Gamma})$ and V is the rotation matrix. Denote by $\varsigma = (\varsigma_1/\varepsilon, \varsigma_2), (\varsigma_1, \varsigma_2) = V^{-1}(\xi_1, \xi_2)$ the parameters of the Fourier transform on the coordinate z. Then the main term of the asymptotic expansion of relations (1) in these local coordinates can be written as follows

$$\begin{aligned} \sigma &= -2\mu_o\varsigma_1(R_o(\varsigma_1)w - iQ_o(\varsigma_1)u_1) \\ \tau_1 &= -2\mu_o\varsigma_1(iQ_o(\varsigma_1)w + F_o(\varsigma_1)u_1) \\ \tau_2 &= -\mu_o\varsigma_1 \,\mathrm{th}(\varsigma_1 H_o)u_2, \quad H_o = H/\varepsilon \end{aligned} \qquad (6)$$

Relations (6) determine the operator of a 2D problem on the boundary layer in the plane normal to the crack contour and going through the point $x^{\Gamma}$. The 2D problem is the problem on a half-infinite crack in two-layered strip. Therefore, the full problem (1) was reduced to two asymptotic problems. The "outer" problem is determined by plate relations (5). The "inner" problem corresponds to 2D relations (6). Both problems should be solved at special matching conditions then they will give the asymptotic solution of the full 3D problem of a plane crack at the interface of two-layered structural element.

### Matching Conditions

In the inner problem one considers a two-layered strip with a half-infinite crack. Using relations (6) this elasticity problem can be reduced to the special Riman problem for a system of unknown functions. The Riman problem admits exact analytical solution. The asymptotics of that solution at the crack tip allows to define the stress intensity factors. The solution of the inner problem is determined by the loads applied to the half-infinite crack at infinity.

On the other hand, the asymptotics of the inner solution at infinity and the asymptotics of the outer (plate) problem near the crack contour (clamping contour) should coincide in appropriate coordinates. These matching procedure leads to the correct clamping conditions in the outer plate problem. In the simplest case neglecting the terms of order $o(1/\varepsilon)$ at $\varepsilon \to o$ one can obtain the clamping conditions in the following form of the relation between the angle of rotation, $\varphi$, and bending moment, M,

$$\varphi = M\delta / D_o \qquad (7)$$

where



$$\delta = 2\pi^{-1}(1 + \ln 2) + \alpha_1 + \alpha_2$$

$$\alpha_1 = \frac{1}{\pi}\int_o^\infty \frac{d}{dx}\left[\ln\left(\sqrt{\frac{\gamma_1\gamma_2}{(1 - d^2g^2)}} \tanh^2 x\right)\right]\frac{dx}{x}$$

$$\alpha_2 = \frac{1}{\pi}\int_o^\infty \frac{d}{dx}\left[\frac{1}{2\sqrt{1 + x^2d^2}} \ln\left(\frac{\gamma_1}{\gamma_2}\psi^2\right)\right]\frac{dx}{x}$$

$$\psi = xd / (1 + \sqrt{1 + xd^2})$$

$$\gamma_{1,2} = \frac{\sin t \cos t \pm t\chi}{\sin^2 t - t^2}, \quad \chi = \sqrt{1 - t^2q^2}$$

$$q = d(1 + gt^{-2}\sin^2 t)$$

$$d = \frac{(\mu_2 - \mu_1)(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}{(\lambda_1 + 2\mu_1)\mu_2(\lambda_2 + \mu_2) + (\lambda_2 + 2\mu_2)\mu_1(\lambda_1 + \mu_1)}$$

$$g = \left(1 + \frac{\lambda_1 - \lambda_2}{\mu_1 - \mu_2}\right)\frac{\mu_1\mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}$$

Condition (7) allows to search for the solution of the outer problem and, hence, the resulting force and moment acting at the plate contour. Using these values as the loads acting at infinity in the inner problem one can calculate undetermined coefficients in the solution of the inner problem. Finally, the stress intensity factors and energy release rate can be calculated from the asymptotics of the inner solution near the crack tip. More detailed description of the results is given by Goldstein and Konovalov (1996 a,b).

## MODEL OF A BRIDGED INTERFACE CRACK

Let us consider a crack of length 2l at the interface of two dissimilar half-planes at $|x| \leq l$, $y=o$. Denote by d the size of the end regions near the crack tips $((l-d) \leq |x| \leq l, y=o)$ where an interaction of the crack surfaces exists. The mechanisms of the interaction are changed in dependence on the crack scale, distance from the crack tip, nature of the adhesion and properties of the joint materials. Assume that transition from one mechanism to another occurs at each point of the end region when the crack opening in that point attains a certain value. The interaction of the crack surfaces can be modeled by effective bonds. Then, in general, an effective bond deformation curve can be represented as the envelope of deformation dependences related to different interaction mechanisms. Several interface bonding mechanisms were modeled by Goldstein *et al.* (1996). In particular, the deformation curves taking into account the bond pull-out and bond cross-linking as well as the presence of the long-range bonds at an interface polymer-metal were obtained. Let us suppose for simplicity that the adhesion bonds can be modeled as the linear elastic springs with a variable compliance depending on the distance of the bond on the crack tip. Note, that similar model of the bonds for a crack in a homogeneous body was used by Weitsman (1986) and Rose (1987).

Boundary conditions of the problem have the following form

$$\sigma_{yy}(x,o) = -\sigma_o\,,\quad \sigma_{xy}(x,o) = -\tau_o\,,\quad |x| \le \ell$$

$$\sigma_{yy}(x,o) = -\sigma_o + q_y(x)\,,\quad \sigma_{xy}(x,o) = -\tau_o + q_x(x)\,,\quad (\ell - d) \le |x| \le \ell \qquad (8)$$

where $q_y$, $q_x$ and $\sigma_o$ , $\tau_o$ are the components of the bond stresses and external tractions, respectively. Denote by u(x) the crack opening

$$u(x) = u_y(x) - iu_x(x)\,,\ i^2 = -1 \qquad (9)$$

where $u_y$, $x_x$ are the normal and shear components of the displacements of the crack surfaces. The crack opening can be represented as follows

$$u(x) = u^o(x) - u^q(x) \qquad (10)$$

where $u^o(x)$, $u^q(x)$ are the parts of the crack opening caused by the external loads and bond tractions, respectively. Assume that the relation between the components of the crack opening, $u^q_y$, $u^q_x$ has the following form

$$u^q_y(x) = c_y(x)q_y(x)\,,\quad u^q_x(x) = c_x(x)q_x(x) \qquad (11)$$

where $c_x(x)$ and $c_y(x)$ are the bond compliances along and transverse to the interface.

Using the expression for the interface crack opening under the action of the concentrated forces (cf. Slepjan, 1981) one can obtain a system of the singular integro-differential equations with the Cauchy kernels relative to the bond tractions in the end regions of the crack (Goldstein and Perelmuter, 1996 a,b). Numerical solution of this system allowed to analyze the stress intensity factors and energetic characteristics sensitivity to the bond parameters and end region size.

Note, that in the model under consideration a condition of the limit equilibrium of the crack can be written as follows

$$G_{tip}(d,\ell) = G_{bond}(d,\ell) \qquad (12)$$

where $G_{tip}(d,l)$ and $G_{bond}(d,l)$ are the deformation energy release rate and rate of the energy consumption on the bond deformation in the end region. This condition allows to search the equilibrium size of the end region. To determine the critical load one should to amplify the model by the conditions of the bond rupture and crack extension. Bond rupture can be related to the limit bond stretching or traction. The crack extension can be connected with attaining the critical value of the modulus of the stress intensity factor, $K=(K_I^2+K_{II}^2)^{1/2}$ where $K_I$, $K_{II}$ are the normal and transverse shear components of the stress intensity factor. The results of systematic parametric numerical calculations of force and energetic characteristics of the problem were given by Goldstein and Perelmuter (1996 b).

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