# COMPARISON OF CRACK CLOSURE MEASUREMENTS USING MOIRÉ INTERFEROMETRY AND RESULTS FROM A DUGDALE-TYPE CRACK CLOSURE MODEL

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## ABSTRACT

Moiré interferometry has been used to give accurate measurements of crack shape during a fatigue load cycle. By taking data at several points in the load cycle the crack opening and closure behaviour were determined to sub-micron accuracy. These results were compared with an analytical model based on the Dugdale approach, modified to include a wake of plastically deformed material pasted to the crack faces. This material represents the result of crack tip plasticity at earlier positions of the crack tip. The crack shape predicted by the model correlates very well with the experimental results. The results obtained from this model should enable the calculation of accurate lifing predictions based on a crack tip plastic strain range model.

## KEYWORDS

Fatigue, crack closure, Dugdale model, moiré interferometry, R-ratio effects

## INTRODUCTION

Elber's (1970) discovery that cracks remain closed for part of a tensile fatigue cycle has led several researchers (e.g. Schmidt and Paris, 1973) to consider whether this closure could account for the R-ratio effect. Several analytical models and numerical analyses have been developed since to predict the closure behaviour of cracks during a fatigue cycle, and hence to infer the effective range of stress intensity factor $\Delta K_{eff}$ (e.g. Newman, 1981). The development of such models is important because they can be used to determine whether crack propagation is described adequately by a simple parameter such as $\Delta K_{eff}$, or if more complex quantities, such as the amount of cyclic plasticity, are required. This paper describes an experimental method of measuring crack opening and compares the results with the predictions of a simple strip-yield model.

A three point bend specimen made from Ti-6Al-4V with a single edge crack was used in this investigation. Moiré interferometry was used to find whole field measurements of in-plane displacements to sub-micron accuracy.

Moiré has many advantages over other measurement techniques:

i) it is less time consuming and more direct than the stereo-imaging or replica techniques;

ii) back-face strain gauges and potential drop methods give descriptions of the bulk behaviour of the crack that make correlations with a model difficult to interpret;

iii) gauges close to the crack only give the displacements of discrete points.

The disadvantage of moiré interferometry is that only surface displacements are known. However, some authors believe that closure at the surface governs crack growth rates, the increased level of closure acting as an anchor on each side of the crack. Also if the crack front is growing at a constant shape then all points through the thickness are progressing at a constant rate. The assumption that the surface will give information from which the crack propagation rate can be determined therefore seems reasonable.

Having developed an experimental method of determining the crack shape, an analytical model was constructed that could be compared with the experimental results. This was based on Dugdale's model, modified by pasting onto the crack faces a wedge of plastically deformed material, similar to that contrived by Newman (1981). The wedge of material represents the residual displacement left by previous cycles of loading, at earlier positions of the crack front.

## EXPERIMENTAL WORK

### *Specimen Preparation*

The specimen was machined from a titanium alloy (Ti-6Al-4V) and the geometry is shown in fig 1. A notch was machined to accelerate and localize crack initiation. The crack was then grown in a four point bend rig by fatigue loading under load control in a hydraulic testing machine. The load was applied sinusoidally between 0 and 2kN. After approximately 11000 cycles the combined length of the notch and the crack was measured using a long range microscope and found to be 0.85mm. An aluminium cross-line diffraction grating was bonded to the specimen using the method described by Post (1987). The grating had 1200 lines per mm in both directions. The specimen was then put into a loading fixture in a moiré interferometry rig and a single load cycle of 0 to 2kN was applied to split the grating to the crack tip.

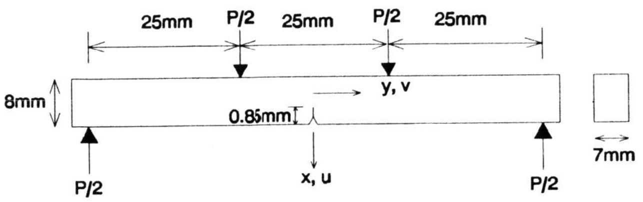

Fig. 1 - Specimen geometry and application of loads

### *Moiré Interferometry*

The principle of moiré interferometry is described in detail by Post (1987). The experimental set up used is outlined in fig 2. A collimated laser beam is split and then recombined in two ways to achieve two orthogonal displacements. u-displacements can be studied by interference between beam $B_1$ and the beam reflected from mirror $M_3$. v-displacements are created from the interference of beams $A_1$ and $A_2$, which are reflected by mirrors $M_1$ and $M_2$ respectively. These 'virtual' gratings are created by the interference of beams have spacings of 2400 lines per mm. When using the 4-beam rig the unwanted pair of beams for each displacement direction is eliminated by the use of a selective filter.

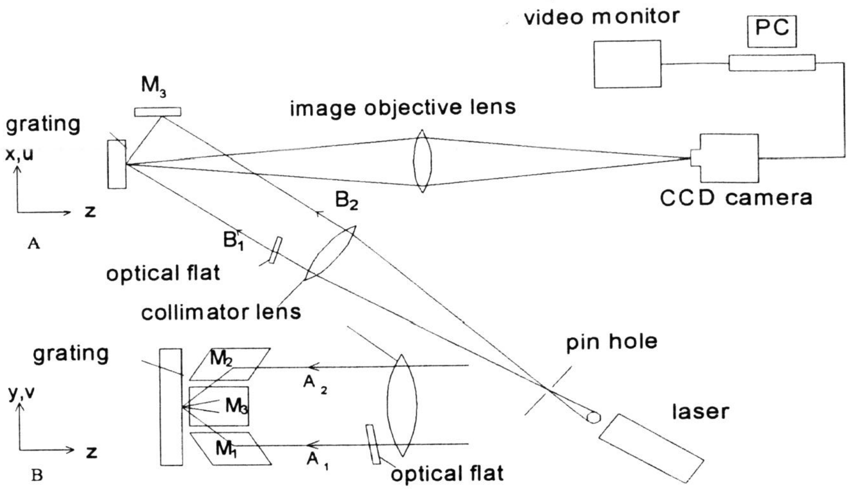

Fig.2 - Diagram of the moiré interferometry rig showing A - the interference of beams to produce the u-displacement field and B - the interference of beams to find the v-displacement field.

Before taking any measurements, a null field is created at zero load. This is done by tilting mirrors $M_1$, $M_2$ and $M_3$. It should be noted that the crack shape measured by the interferometer is therefore relative to the crack shape at zero load. A load is then applied to the specimen and the mismatch between the actual grating on the deformed specimen and the virtual grating created by the laser beam interference causes moiré fringe patterns. These fringe patterns or interferograms can then be captured by a CCD camera by focusing the emergent beam using an objective lens. To obtain each displacement map five interferograms were taken. High magnifications were used to determine the crack shape. The camera was focused on an area about 2mm square and 256 x 256 data points were captured over this area, so the distance between pixels was about 8.7μm. The displacement sensitivity for the system is dependent upon the grating frequencies and is 0.417μm per fringe order.

The specimen was loaded and unloaded in steps. At each load step interferograms were taken, which gave whole field displacements at nineteen positions in the load cycle.

### *Analysis of the Interferograms*

To determine the crack shape the v-interferograms were analysed which gave information about the separation of the crack faces. First the interferogram at maximum load was considered and it was assumed that the crack was fully open. From this interferogram the position of the crack tip was found and this information was then used to analyse the interferograms for all the other load conditions.

An automated moiré interferometry technique was employed which uses phase stepping techniques and creates a smooth displacement map of the whole field. The principles of this technique are reported elsewhere (Poon *et al.* 1993). As a continuous displacement map is produced the accuracy is greater than the 0.417μm obtained per fringe order. The method is therefore more sophisticated than the traditional method of simply counting round the number of fringes (Gray and MacKenzie, 1990). This is particularly important at low loads when the crack is more likely to be closed but the displacements are small and therefore the number of fringes is low. It should be noted, however, that at lower loads as there are fewer fringes the accuracy of the data is lower than that at higher loads. For each load condition the automated moiré interferometry technique was used. A number of positions along the crack face were considered and the separation of the crack faces could easily be read from the displacement field determined from the interferogram.

## ANALYTICAL WORK

An analytical model was developed based on the Dugdale approach in which the plastic zone is represented by a thin layer of elastic-perfectly plastic material. The Dugdale model has the limitation that it was developed to represent plane stress conditions. Wilks (1994) adapted the model for plane strain and found that the plastic zone sizes are smaller. However, the Dugdale model is not appropriate for use under plane strain conditions because the yield criterion is exceeded in areas close to the crack tip. This is not surprising as the model does not represent the correct mode of plastic deformation known to occur under plane strain conditions. A model such as that developed by Kanninen and Atkinson (1980) is more appropriate for plane strain conditions because they used dislocations on inclined slip planes. A Dugdale type model is, however, simple to formulate and may well be the most appropriate when describing displacements at the surface of the specimen where there is no stress component normal to the free surface[1].

The model developed is similar in concept to those of Budiansky and Hutchinson (1978) and Newman (1981). Space precludes a comprehensive description here, and full details will be reported elsewhere, but an outline description will be given. Displacement discontinuity boundary elements of strength $b_i$ are used to model both the crack opening and yield in the strip ahead of the crack. Following the approach of Newman, additional material is pasted onto the faces of the crack in order to reproduce the stretched material produced when the crack was smaller, through which the crack has grown. A self-similarity argument is used to show that, if the crack has been growing under constant amplitude remote loading, the thickness of this material must decrease linearly away from the current crack tip. The boundary conditions on the crack are as follows: either the crack is open and the faces are traction free, or the crack is closed and there is a compressive stress between the faces:

$$\begin{array}{lll} \sigma_{yy} = 0 & b \geq 0 & \textit{crack open} \\ \sigma_{yy} \leq 0 & b = 0 & \textit{crack closed} \end{array} \qquad (1)$$

where b is the Burgers vector or displacement associated with the plastic flow.

Similarly, ahead of the crack, three conditions can occur in any given incremental load step: no yield, yield in tension, or yield in compression:

$$\begin{array}{lll} \Delta b_i = 0, & |\sigma_{yy}| \leq \sigma_{YIELD} & \textit{no yield} \\ \Delta b_i \geq 0, & \sigma_{yy} = \sigma_{YIELD} & \textit{tensile yield} \\ -\Delta b_i \geq 0, & \sigma_{yy} = -\sigma_{YIELD} & \textit{compressive yield} \end{array} \qquad (2)$$

It will be apparent from (2) that an incremental formulation of the problem is appropriate. A difficulty encountered in obtaining a solution is to determine which of these boundary conditions apply at any particular point. A quadratic programming approach, essentially similar to a linear programming method originally suggested by Kalker (1971) for contact problems, is adopted. An object function is chosen which is always zero or positive and which is zero when (1) is satisfied along the crack and (2) ahead of the crack. This object function is then minimised using standard quadratic programming techniques, subject to constraints on the variables, including the required relationships between stress and boundary element strength. In this way, the closure and yield zones are automatically determined at each load step.

Only one parameter, the thickness of the wedge of additional material attached to the crack faces, needs to be determined by iteration. This is set so that the thickness of the material adjacent to the crack tip is equal to the residual crack tip stretch at minimum load. Implementation of the scheme on a PC provides a quick and easy means of predicting crack shape, yield zone size, and the position of the crack closure point.

## RESULTS

The crack shapes predicted by the model found experimentally are shown in figs 3, 4 and 5 for applied loads of 0.4kN on loading, 2.0kN maximum load and 0.4kN on unloading. It can be seen that the model tends to underestimate the level of opening. The crack tip opening displacement is plotted in fig 4 and shows a good correlation between theory and experiment. Figure 6 compares crack opening and closing loads predicted by the model with those found experimentally. From the results the effect of the crack peeling open observed by many authors can clearly be seen. Both the analytical model and the experimental data show the opening and closing points to be identical at loads less than 20% of the maximum applied load. The model and actual cracks were slightly more open on unloading than loading at higher loads and by 50% of the maximum load the cracks were fully open.

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[1] It is important to recognise that conditions at the free surface differ from those assumed in plane stress analyses since there are significant stress *gradients* in the direction normal to the surface. The stress state here is, therefore, fully three-dimensional. In contrast, conditions remote from the surface may be correctly modelled using a plane strain analysis.

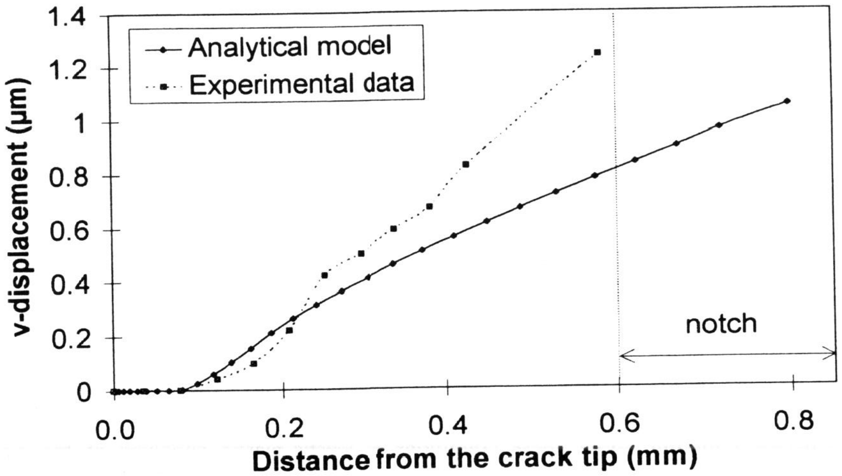

Fig 3. - Comparison of crack shapes found experimentally and predicted analytically at 0.4kN on loading.

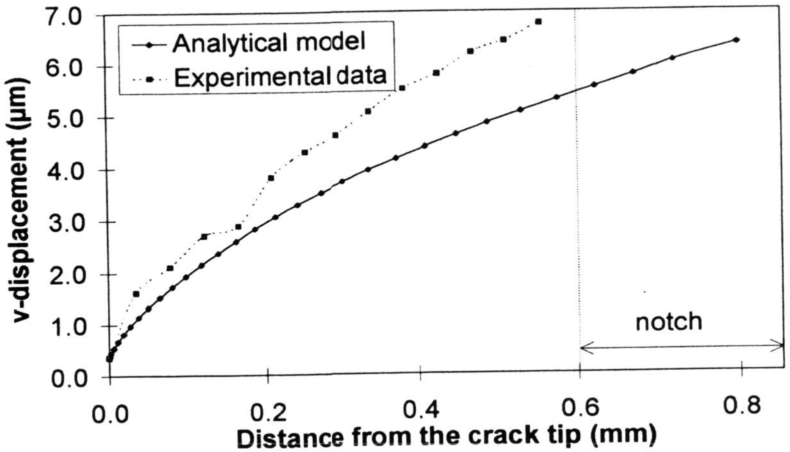

Fig 4. - Comparison of crack shapes found experimentally and predicted analytically at 2.0kN (maximum load).

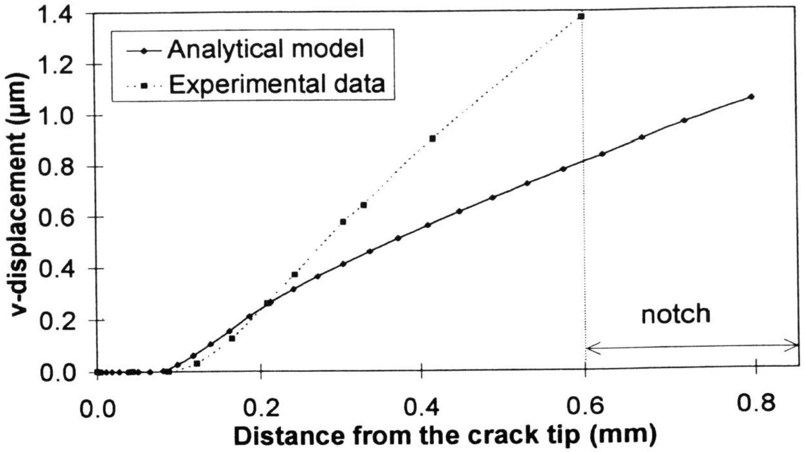

Fig 5. - Comparison of crack shapes found experimentally and predicted analytically at 0.4kN on unloading.

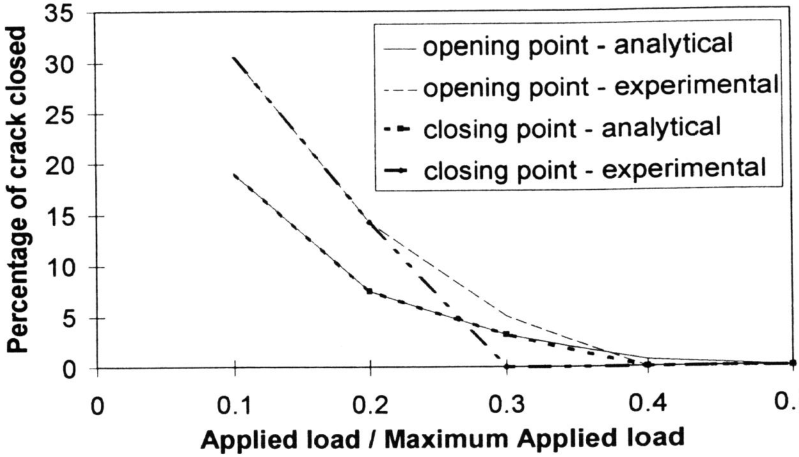

Fig 6. - Percentage of the crack that is open and closed on loading and unloading, predicted by the analytical model and found experimentally.

## CONCLUSIONS

Moiré interferometry has been shown to be an excellent tool in the determination of crack closure. The great disadvantage with the method is the use of aluminium gratings that are brittle and lead to poor quality interferograms close to the crack after small numbers of cycles. The authors are currently developing a method of writing gratings into a thin layer of photosensitive polymer on the surface of the specimens. If this is successful complex cycles could be analysed as well as enabling a greater range of R-ratios to be investigated. Preliminary work by Wilks (1994) has suggested that if the technique is developed good quality gratings could be produced in this manner.

Good correlation was obtained between actual and predicted crack shapes, crack opening and closure loads and crack tip opening displacement.

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