THREE LAYER TECHNIQUE FOR BONDED COMPOSITE PATCH

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ABSTRACT

This study introduces the two-dimensional finite element analysis involving the three layer technique to investigate the repair of cracked metallic structures using adhesively bonded composite patch. The three layer technique uses two-dimensional Mindlin plate elements with transverse shear deformation capability for all three layers; cracked plate, adhesive, and composite patch. The accuracy of the three layer technique to compute the stress intensity factor for the metallic crack and the strain energy release rate of debond at the adhesive interface is demonstrated by a comparison with available two-dimensional and three-dimensional models. The three layer technique provides an efficient and accurate alternative method to expensive three-dimensional finite element analysis. Further, the three layer technique is capable of investigating in-depth the adhesive effects on the bonded composite patch repairs.

INTRODUCTION

Today, the growing age of commercial and military aircraft fleets, combined with their large replacement costs, poses significant challenges to those responsible for providing reliable and safe operations in an era of fiscal constraints. To maintain the present aging aircraft's operation beyond their original design or lifetimes, adhesively bonded composite patch

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which has inherent advantage of high stiffness/strength to weight ratio are gaining more and more acceptance since their development in the late 1960's as an economical method to repair cracked aerospace metallic structures [3]. Bonded composite patch bridges the stresses between the cracked plate and composite patch. The redistribution of stresses lead to a reduced stress field in the vicinity of the crack, which reduces the stress intensity factor, retards the crack growth, and improves the fatigue life [1,3,4].

Several studies have been conducted to investigate the mechanics of bonded composite patch to repair the cracked metallic structure, especially to analyze the redistribution of stresses in the repaired structure, and to compute the stress intensity factor after repair [5,7,9]. Although, three-dimensional finite element analyses of the composite patch repair have been conducted [9], however due to the small thickness of the adhesive compared to the plate or the composite patch, the three-dimensional model becomes very expensive to perform even with a minimal number of elements across the thickness which in turn causes a very large aspect ratio in the finite element models. This high cost of three-dimensional analysis is a great contributor in directing the present effort toward developing a better two-dimensional model. This paper, therefore, introduces the three layer technique to model cracked metallic plate repaired with bonded composite patch, describes its modeling procedures, and shows its validity.

THREE LAYER MODEL

This technique utilizes two-dimensional finite element analysis and uses three layer of Mindlin plate elements to model cracked plate, adhesive, and composite patch. The three layer technique is different where the adhesive is modeled as an elastic continuum medium replacing the shear spring elements (non-continuum body) used in the existing finite element models [5,9]. The motivation behind modeling the adhesive as a continuum is to, first, provide an economical two-dimensional finite element model with minimal difference from the three-dimensional model. Second, it would capture the characteristics of adhesive realistically which would be required to model thermal effects, non-linear material behavior, progressive damage etc.

In the three layer technique, two-dimensional Mindlin plate elements with transverse shear deformation capability are used for all three layers; cracked plate, adhesive, and composite patch. In both the symmetric and unsymmetric repairs, the Mindlin plate assumption (i.e. linear displacement field along the plate thickness) is enforced for all three layers across the thickness,

$$u_x = \overline{u}_x + z\overline{\theta}_y \ , \tag{1}$$

$$u_y = \overline{u}_y + z\overline{\theta}_x \ , \tag{2}$$

$$u_z = \overline{u}_z \ , \tag{3}$$

where $\overline{u}_x$, $\overline{u}_y$, and $\overline{u}_z$ are the mid-plane displacement, and $\overline{\theta}_x$ and $\overline{\theta}_y$ are the rotations of the cross-section. Also, the geometric compatibility (i.e. continuos displacement field) is enforced at the plate-adhesive and adhesive-patch interface for bonded case but not where debond is modeled. Note that the three layer model is different from a single Mindlin plate in the sense that the three layer's rotations are independent of each other and only uses displacement constraints at the interfaces to enforce geometric compatibility.

FRACTURE MECHANICS

Cracks that exist in bonded patch repair are classified into two types; cohesive and adhesive cracks. First, the cohesive cracks are those which exist in the cracked plate and assumed to be through-the-thickness cracks. These cracks are short to medium in size such that using the patching technique reduces crack growth and insures the integrity of the structure without catastrophic failure. Second, the adhesive cracks are those which initiate at either the plate-adhesive or adhesive-patch interface causing a debond. These adhesive cracks or debond have different characteristics in unsymmetric and symmetric repair where the former encounter bending stresses which leads to peeling stresses. These peeling stresses are known to be more critical than the shear stresses in the sense that adhesive failure resistance is much weaker under peeling stresses compared to shearing stresses, and they cause a higher debond growth rate leading to an earlier patch failure if ignored.

First, for **cohesive** crack, the strain energy release rates for the opening (I) and sliding (II) modes, are computed by using the modified crack closure method [8,10],

$$G_I = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} F_y.\Delta v + \lim_{\Delta a \to 0} \frac{1}{2\Delta a} M_x.\Delta\theta_x \ , \tag{4}$$

$$G_{II} = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} T_x.\Delta u + \lim_{\Delta a \to 0} \frac{1}{2\Delta a} M_y.\Delta\theta_y \ , \tag{5}$$

where Δa is the virtual crack extension and it is equal to the length of the first element in front of the crack tip, Δu, Δv, $\Delta\theta_x$, and $\Delta\theta_y$ are the crack opening displacements and rotations at the first node in front of the crack tip, and T_x, T_y, M_x, and M_y are forces and moments required to close the crack. Note that in case of symmetric patch, rotations are restricted and become negligible. For linear elastic continuum, the stress intensity factors are computed using the strain energy release rates according to

$$K_I^2 + K_{II}^2 = (\frac{GE}{\beta}) \ , \tag{6}$$

where β is equal to unity for plane stress and $1-\nu^2$ for plane strain, and E is the Young's modulus which decouple if the applied load is known to be of mode I (opening) or mode II (sliding) type.

Second, for **adhesive** crack or **debond**, the strain energy release rate is required to predict the debond growth behavior. The technique adopted to calculate the debond's strain energy release rate is based on the modified crack closure method. Note that the adhesive crack has no physical dimension through the thickness and the strain energy release rate is defined as the strain energy per unit area in the debond plane. Considering two plates which are adhesively bonded, the adhesive crack's strain energy release rates at the interface in term of the plates' mid-plane section displacements and rotations are

$$G_I = \lim_{\Delta a \to 0} \frac{1}{2A\Delta a} N_z.(\Delta\overline{w}_z - \Delta w_z) \ , \tag{7}$$

$$G_{II} = \lim_{\Delta a \to 0} \frac{1}{2A\Delta a} F_n.(\Delta\overline{v}_n - \Delta v_n) + \lim_{\Delta a \to 0} \frac{1}{2A\Delta a} F_n.(\frac{\overline{h}}{2}\Delta\overline{\theta}_n + \frac{h}{2}\Delta\theta_n) \ , \tag{8}$$

$$G_{III} = \lim_{\Delta a \to 0} \frac{1}{2A\Delta a} T_t.(\Delta\overline{u}_t - \Delta u_t) + \lim_{\Delta a \to 0} \frac{1}{2A\Delta a} T_t.(\frac{\overline{h}}{2}\Delta\overline{\theta}_t + \frac{h}{2}\Delta\theta_t) \ , \tag{9}$$

where overbar indicates bottom plate quantities and no bar indicates upper plate quantities, $\Delta\bar{u}_t$, $\Delta\bar{v}_n$, $\Delta\bar{w}_z$, $\Delta\bar{\theta}_n$, and $\Delta\bar{\theta}_t$ are the debond opening displacements and rotations along the tangential, normal, and out-of-plane directions of lower plate, at the first node in front of the debond front, similarly Δu_t, Δv_n, Δw_z, $\Delta\theta_n$, and $\Delta\theta_t$ are the debond opening displacements and rotations along the tangential, normal, and out-of-plane directions, respectively, T_t, F_n, and N_z are the tangential, normal, and through-the-thickness reaction forces required to hold the crack closed, and A is the average area of local elements surrounding the nodal reaction forces. The total strain energy release rate is sum of G_I, G_{II}, and G_{III}.

RESULTS AND DISCUSSION

To verify the validity of the three layer technique for symmetric and unsymmetric repairs, the stress intensity factors and the strain energy release rates for cohesive and adhesive cracks are computed, respectively, and compared with those available in the literature.

Cohesive Crack

For both single sided repair, unsymmetric (see Figure 1a), and double sided repair, symmetric, the cracked plate, adhesive, and composite patch are modeled as continuum elastic medium using four noded shell elements available in the commercial finite element code ABAQUS. The material properties and dimensions of the single and double sided repair are given in Reference 9. The cohesive central crack total length is $2a = 50.0$ mm. A uniformly distributed stress in the y-direction, σ_y, equal to 0.689 MPa is applied. A comparison of the stress intensity factor between the three layer technique and the previous study [9] are shown in Table 1 where the stress intensity factor is normalized with respect to $\sigma_y\sqrt{a\pi}$. Note that the stress intensity factors are calculated assuming that the local stress field near the crack tip is in state of plane strain for single sided repair and plane stress for double sided repair as in Reference 9. Results (Table 1) from the present two-dimensional finite element analysis where adhesive is modeled as continuum layer are in better agreement with their counterparts from three-dimensional analysis than those from the previous two-dimensional finite element analysis where adhesive is modeled as discrete spring elements. This clearly shows the validity and advantage of the present three layer technique.

Adhesive Crack

For adhesive crack, the present three layer technique is used to analyze an elliptical debond which is documented in previous studies [2, 6]. A two-dimensional quarter model for an elliptical debond is shown in Figure 1b. The semi-major axis for the elliptical debond c^* is equal to 25.0 mm. The elliptical debond aspect ratio r which is the ratio of the semi-major axis c^* to the semi-minor axis b^*, is equal to 0.4 . The dimensions and material properties for the debond model are given in Reference 9. For the single sided repair, the comparison of total debond strain energy release rate is shown in Figure 2 which is normalized with respect to σh where h is the plate thickness. Note that, for the three layer technique, the total debond strain energy release rates are presented with and without the adhesive rotations. The reason behind including and excluding the adhesive rotation is to demonstrate the fundamental difference in modeling the adhesive layer between the three layer technique and previous study [9]. In this previous study and earlier study [5], shear spring element to model the adhesive is used which ignores the effects of adhesive rotations. On the other hand, the three layer technique models the adhesive as a continuum elastic medium which can account the adhesive rotation effects. Thus, total strain energy release rate in the case, where the adhesive rotations are excluded from the computation, is in a reasonable agreement with the previous study [9] as shown in Figure 2. On the other hand, inclusion of adhesive rotations, increases the strain energy release rate as expected. For the double sided repair where the bending effects do not exist and rotation effects are insignificant, the adhesive rotation is no longer an issue and it is shown in Figure 3.

Using the three layer model, a comparison between the debond strain energy release rate for both single sided and double sided repairs, where the same amount of patching material is used in both repairs, show the same trend with larger values for the single sided repair which indicate an additional contributions due to the bending (in unsymmetric repair) as expected (Figure 4). A similar comparison in the previous study [9] where the adhesive is modeled with spring elements showed, to the previous authors' [9] surprise, that the total debond strain energy release rate is larger in the double sided repair than that in the single sided repair. Thus, this indicates the advantage of modeling the adhesive as a continuum media in order to characterize the debond effects.

CONCLUSIONS

The two-dimensional finite element technique consisting of three layers is presented to analyze the repair of cracked metallic structure with an adhesively bonded composite patch. In this three layer approach, two-dimensional Mindlin plate elements with transverse shear capability are used for all three constituents; cracked plate, adhesive, and composite patch. The validity of the three layer technique for modeling cohesive crack (i.e. crack in plate) and adhesive crack (i.e. debond) is presented in this study.

For the cohesive crack, stress intensity factors calculated using the three layer model are in good agreement with previous study using two-dimensional and three-dimensional finite element analysis where adhesive is modeled by spring elements. For adhesive crack (debond), strain energy release rates from the present study show agreement with previous study for both single and double sided repairs. For the single sided repair, modeling the adhesive as continuum elastic medium in the three layer technique captures the effect of rotation on debond strain energy release rate. Hence, modeling the adhesive as continuum elastic medium provides an accurate and efficient representation of bonded patch repair.

Finally, the three layer technique provides an alternative economical two-dimensional finite element model with minimal difference from three-dimensional model. It also has the potential to capture the characteristics of adhesives realistically to study in-depth its role in the bonded patch repair.

ACKNOWLEDGMENT

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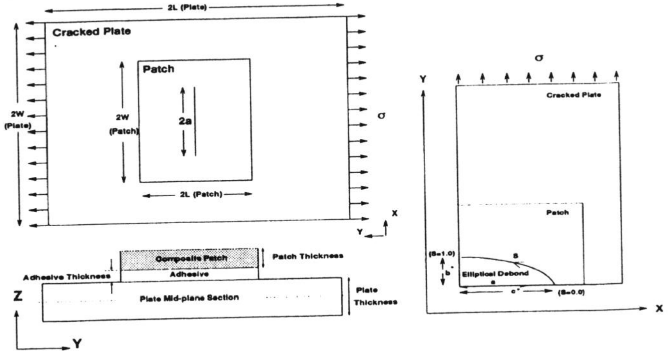
Table 1: Normalized Stress Intensity Factor at Mid-Plane for Single and Double Sided Patches.

Type of Patch	Three Layer Model 2-D Present Study	Sun et al [9] 2-D	Sun et al [9] 3-D
Single Sided	0.570	0.536	0.612
Double Sided	0.253	0.246	0.263



(Figure 1a)

(Figure 1b)

Figure 1: Single sided composite patch; (a) completely bonded; (b) with elliptical debond.

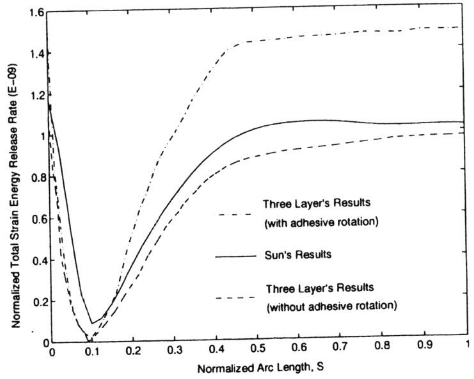


Figure 2: Comparison of total strain energy release rate at elliptical debond front in single sided repair.

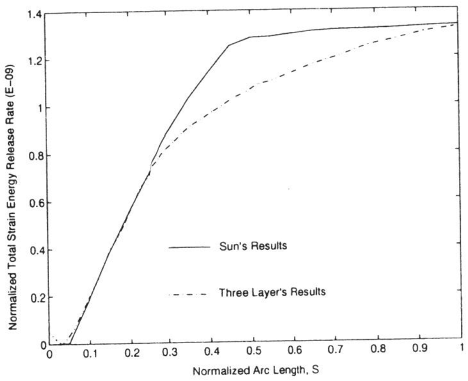


Figure 3: Comparison of total strain energy release rate at elliptical debond front in double sided repair.

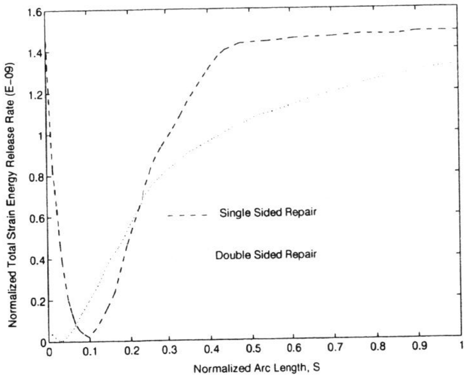


Figure 4: Comparison of total strain energy release rate for single and double sided repair with cohesive crack behind debond front.