THREE DIMENSIONAL STRESS ANALYSIS OF CRACK PATCHING

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ABSTRACT
A fully three-dimensional, geometrically linear, finite-element stress analysis is presented for a centre-cracked plate repaired by a strip-like patch bonded to one side of the plate, over the crack, and subjected to a remote uniaxial tension, normal to the crack. It is shown that the root-mean-square of the stress intensity factor across the crack front increases monotonically with increasing crack length and approaches asymptotically a finite limiting value, rather than increasing indefinitely as would be the case for an un-repaired crack. This limiting value is found to be significantly higher than for the corresponding two-sided repair, but it is well approximated by an analytical expression derived recently. This analytical expression can therefore be used for repair design. It is also shown that there is a substantial variation of the stress intensity factor across the thickness, for material properties and thicknesses representative of repairs to aircraft skins, but this variation can also be accurately predicted by relatively simple analytical expressions.

KEYWORDS
Bonded repair, Plate theory, Stress analysis, Stress intensity factor.

1. INTRODUCTION

In the past two decades, bonded repair technology has proved a cost-effective means of repairing cracks in aircraft structures (*e.g.* Baker and Jones, 1988, Rose *et al.*, 1995). Bonded repairs fall into two geometric categories, either one-sided repairs or symmetric two-sided repairs. In the case of the two-sided repair, no bending in the reinforcement occurs over the repaired region; however, in the one-sided repair, bending does occur. The most important feature of bonded repairs is the asymptotic behaviour of the crack extension force as the crack length increases. This means that the stress intensity factor after the repair becomes constant as the crack length exceeds a characteristic length, which depends on the geometry and material properties of a repair.

This limiting value can be estimated analytically by a two-stage analysis involving energy-balance considerations (Rose 1988). The resulting formulae provide convenient (and conservative) estimates for the key design parameters and they have now been incorporated into an Engineering Standard for the design of bonded repairs (Davis, 1994).

Recently, Arendt and Sun (1994) have reported an analysis of crack-patching efficiency for the case of a one-sided repair to an un-supported cracked plate, in which it was found that

(i) the energy release rate (and hence the stress intensity factor) failed to reach a limit, with increasing crack length;

(ii)the analytical expressions given by Rose (1988) under-predicted the energy release rate over a range of adhesive thickness by an average of 176 %.

Arendt and Sun (1994) modelled the deformation of the cracked plate and of the reinforcement using Mindlin plate theory, with the two plates being coupled by distributed shear and tension springs, modelling the adhesive layer. The aim of the present work is to re-examine this configuration of one-sided repair in more detail, using a fully three-dimensional finite-element stress analysis. It will be shown that the results are consistent with, and well approximated by, suitably derived analytical expressions.

2. PROBLEM FORMULATION

Referring to Fig. 1, the problem being considered is that of a centre-cracked plate, with crack length $2a$, repaired by a bonded reinforcement in the form of a strip of height $2B$, running across the full width $2W$. A remote uniform tensile stress σ^{∞} is applied to the plate normal to the crack. The problem is to determine the stress intensity factor K in this repaired plate, as a function of the crack length $2a$ and of the relevant parameters of the patched system, notably the Young's modulus E and thickness t, subscripts (or superscripts) P, R, A will be used to distinguish properties pertaining respectively to the plate, the reinforcement and the adhesive layer, as in earlier work (Rose 1988).

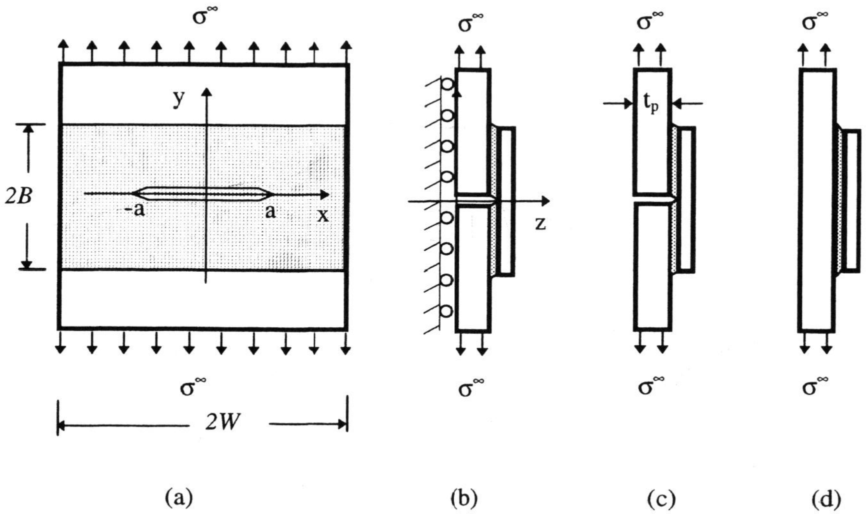

Fig.1 Repair configurations and coordinates (a) Plan view, (b) Cross section along centre line (x=0) with no bending deflection allowed, representing two-sided repair, (c) Cross section of an one-sided repair along the centre line, and (d) Cross section at $x\rightarrow\infty$ for one-sided repair.

In the absence of the bonded reinforcement, K for a centre-cracked panel would increase indefinitely with increasing crack-length, in accordance with the well-known formula

$$K = \sigma^{\infty}\sqrt{\pi a}\, F(a/W)\,, \qquad (1)$$

where F(*a/W*) represents a non-dimensional finite-width correction factor which is documented in handbooks (*e.g.* Murakami 1987). The reinforcement introduces a characteristic length related to the load-transfer length from the plate to the reinforcement, such that for sufficiently long cracks, K no longer increases with crack length, but instead K approaches a constant limiting value which can be estimated analytically (Rose 1988, Wang *et al.* 1996).

For the case of a reinforcement bonded to one face of the cracked plate, as in Fig. 1, the load-path eccentricity gives rise to secondary bending, and therefore K varies across the thickness, i.e. K varies with z, in the notation of Fig. 2, if the crack front is assumed to be straight and normal to the plate's faces. To fully characterise K one must therefore determine (i) an appropriate thickness-averaged value, and (ii) a measure of the thickness variation. It will be assumed here that

$$K(z) = K_m + \frac{2\zeta}{t_P} K_b\,, \qquad \zeta = z - t_p/2 \qquad (2)$$

where, in accordance with conventional plate theory (Hui and Zehnder, 1993), K_m denotes the contribution due to in-plane membrane loading (generalised plane stress), whereas K_b quantifies the bending contribution which varies linearly across the thickness, where ζ is the distance measured from the plate's mid-surface. This assumption will be

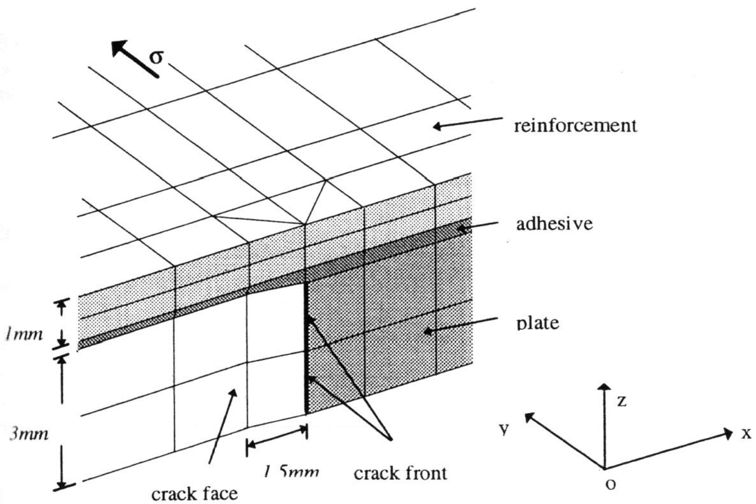

Fig.2 Finite element mesh for a cracked plate repaired with reinforcement (a) plan view and (b) near crack tip.

shown to be confirmed by the 3D finite-element analysis presented later. The significant point to note here is that for comparison with analytical estimates, the appropriate average for K is the root-mean-square value defined by

$$K_{rms} = \sqrt{\frac{1}{t_p} \int_0^{t_p} K^2(z)dz} \tag{3}$$

This arises because the analytical approach provides an estimate for the crack extension force (or energy release rate G_∞) for self-similar crack growth, and the stress intensity factor derived from that *via* the usual Irwin relation corresponds to the rms value defined in (3), *i.e.*

$$G_\infty = K_{rms}^2 / E \tag{4}$$

It is readily verified from (2) and (3) that $K_{rms} \neq K_m$, but instead

$$K_{rms}^2 = K_m^2 + \frac{1}{3} K_b^2 \tag{5}$$

In the present work, attention will be focussed on determining the variation of K_{rms} with crack length a for a one-sided repair, but the thickness variation at a given crack length is also briefly examined.

To facilitate comparison with the earlier work of Arendt and Sun (1994), the reinforcement is assumed to be isotropic (rather than orthotropic, as would be more appropriate for the uni-directional fibre composite patches used in practice, Baker and Jones 1988), and the same values that were used by Arendt and Sun (1994) for the elastic constants and the thicknesses will be used here, as summarised in Table 1. These are representative of values for an aluminium plate repaired by a boron/epoxy patch, using an epoxy-based structural adhesive.

Table 1: Dimensions and material properties of a typical repair

Layer	Young's modulus (GPa)	Poisson's ratio	Thickness (mm)	Height (mm)	Width (mm)
Plate	71.0	0.33	3.0	500	500
Reinforcement	207	0.3	1.02	100	500
Adhesive	1.89	0.35	0.203	100	500

3. FINITE ELEMENT ANALYSIS

A fully three-dimensional geometrically linear stress analysis has been performed for the configuration shown in Fig.1, using a commercially available software package PAFEC. Because of the symmetry with respect to both the x and y-axes, only one quadrant needs to be modelled, with the standard boundary conditions for symmetry being applied along x=0 and y=0.

The three constituents in Fig. 1, the plate, the adhesive layer and the reinforcing patch, are assumed to deform elastically only, and are each modelled by 20-noded isoparametric brick elements. The two main issues which need to be addressed are (i) how to deal with a relatively thin adhesive layer, which is typically an order of magnitude thinner than the plate or the patch (see Table 1); and (ii) how to determine the stress intensity factor at various positions across the crack front.

The technique of reduced integration is now well established for dealing with the problem of large aspect ratios for elements in modelling thin layers, and it has been used in the present work, as in several previous studies (*e.g.* Sun *et al.* 1996). To determine K, isoparametric wedge-shaped elements have been used around the crack tip node, as indicated in Fig. 2, with the mid-point nodes shifted to quarter-point locations to capture the characteristic crack-tip singularity, which is also a standard technique.

To validate the adequacy of the FE mesh, calculations were first carried out for an un-patched plate. The results were found to agree to within 2% with the handbook values for that case (Murakami 1987), for half-crack lengths ranging from *10mm* to *170mm*, as shown in Fig.3.

For the repaired plate, the results for a two-sided repair were obtained by using a symmetry restraint on the plane z=0, as indicated in Fig. 1(b). The stress intensity factor was determined for 18 different values of half-crack length ranging from *2mm* to *170mm*. These calculations were then repeated for the case of one-sided repair by removing the symmetry constraint on z=0.

Two layers of elements were used in modelling the plate, to capture the z-variation of K with sufficient accuracy, K being determined from the crack opening displacement at the near-tip nodes in accordance with the standard asymptotic relation

$$u_y(x \to a^-, y = 0^+, z) = \frac{4K(z)}{E} \sqrt{\frac{a-x}{2\pi}} \tag{6}$$

The results are shown in Figs. 3-5.

4. RESULTS AND DISCUSSION

Fig. 3 shows that K_{rms} for an one-sided repair significantly exceeds the value for the corresponding two-sided repair, indicating that the secondary bending induced by the load-path eccentricity has a significantly detrimental effect on the efficiency of bonded reinforcements.

However, the results also show that in both cases, K does not increase indefinitely, but increases monotonically towards an asymptotic value. An analytical estimate has been derived for this upper bound by Wang *et al.* (1996), following the general approach presented by Rose (1988). This analytical estimate is shown as a straight line in Fig.3 . It can be seen that this estimate provides an excellent approximation to K for sufficiently long cracks, and it has the desirable feature, from the viewpoint of parametric analyses for repair design, of representing a *conservative* estimate for K after repair, for all crack lengths.

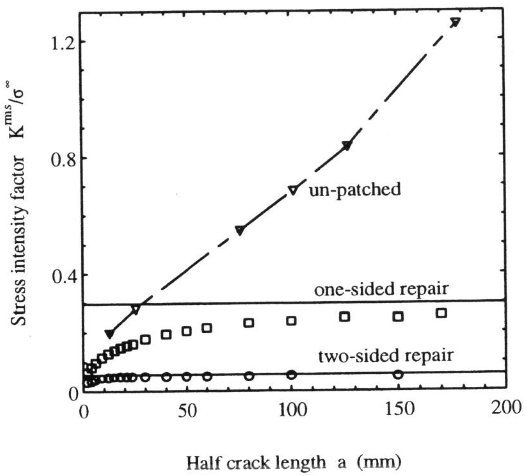

Fig. 3 Stress intensity factors versus crack length for two sided repairs

Fig. 4 shows how the results obtained by Arendt and Sun (1994) compare with those

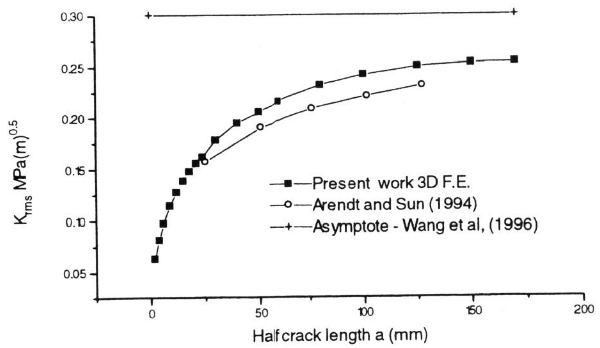

Fig.4 Comparison of work carried out by Arendt and Sun, and the present work ($\sigma^{\infty}=1$).

obtained in the present work using 3D FEA. It can be seen that the two sets of results agree to within 5-10%, with the results of Arendt and Sun, based on a coupled Mindlin-

plate theory, being consistently lower than the 3D FEA results. This general trend of plate theory leading to somewhat lower values for K than 3D FEA has also been reported in the more recent work of Sun *et al.* (1996) and it would appear to be an intrinsic feature of the approximation involved in a plate-theory formulation. The observation by Arendt and Sun (1994) that the energy release rate (or K_{rms}) failed to reach a limiting value can be seen from Fig. 4 to be attributable to the limited range of crack length examined in their work.

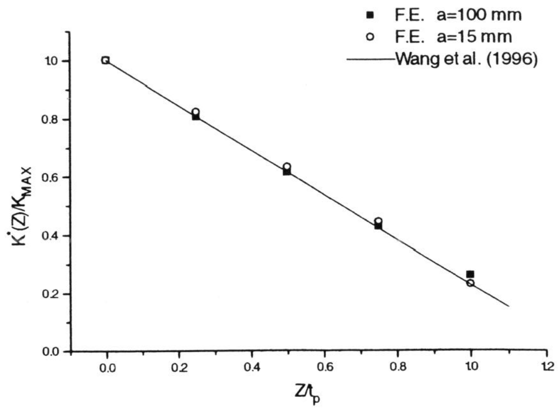

Fig. 5 Linear distribution of stress intensity factor along crack front. FE data points are shown, whereas the solid line represents an analytical prediction by Wang *et al.* (1996).

Fig. 5 shows the through-thickness variation of *K(z)* for two crack lengths, compared with an analytical prediction based on plate-theory calculation of the stress variation which would prevail in a patched, but un-cracked, plate (Wang *et al.* 1996). It is known that the stress singularity at the intersection of a crack front with a free surface differs from the standard inverse-square-root singularity (except for $v=0$), and therefore cannot be characterised by K. Nevertheless, the plate-theory prediction of a linear variation of K across the thickness, as in Eq.(2), appears to be quite adequate for all practical purposes (Hui and Zehnder 1993). It can be seen from Fig. 5 that the present 3D FEA results for a patched plate are consistent with a linear variation, and are well approximated by the analytical expression derived by Wang *et al.* (1996). This thickness variation can be seen from Fig.5 to be quite substantial, contrary to the claim by Arendt and Sun (1994) that it is negligible.

6. CONCLUSION

A fully three-dimensional geometrically linear, stress analysis of crack patching has shown that the stress intensity factor (K_{rms}) approaches a limiting value with increasing crack length for the case of one-sided repair, as in the previously well-documented case of two-sided repairs. However, the limiting value of K_{rms} is higher by a factor of 6 for the one-sided repair, relative to the corresponding two-sided repair, for the choice of parameters used in the present work, which is representative of repair to an aluminium wing skin by a boron/epoxy patch. This higher limiting value is accurately predicted by an analytical expression derived recently by Wang *et al.* (1996). The FE results presented here have also shown a significant variation of K through the thickness, which can also be well predicted by the analytical work of Wang *et al.* (1996).

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