THERMAL STRESS CLEAVING OF A THIN STRIP USING A POINT HEAT SOURCE

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ABSTRACT

A thermal stress field induced by localized temperature change in the vicinity of crack tip often causes crack extension. Taking advantage of this phenomenon, some glassy materials can be cleaved without any cutting tool but adequate controlling of a temperature distribution in the body. In the present paper, based on the plane thermoelasticity and linear fracture mechanics, a time dependent thermal stress intensity factor of a line crack pre–introduced from the edge of a thin plate was investigated when the ahead of the crack tip was heated locally by a continual point heat source. It was found that the most successful heating position for the thermal stress cleaving is strongly affected by the width of the strip but is almost independent from the crack length or other geometric conditions. The present calculation was compared with the experimental results and good agreement was found.

KEYWORDS

Stress Intensity Factor, Thermoelastic Problem, Brittle Material, Point Heat Source.

INTRODUCTION

Machining the brittle materials, such as glass, ceramics and silicon wafer, using the ordinary mechanical methods sometimes causes serious damage on the generated surfaces. In such a damaged surface, micro cracks and high degree of residual stress are commonly observed. On the other hand, by utilizing a thermal stress field induced by a local temperature rise in the vicinity of the main crack tip, not only highly strong surface without any micro cracks can be obtained but also a waste-less, noiseless and clean processing of the glassy materials can be achieved (Chryssolouris,1991; Imai *et al.*,1989).

In the present paper, dividing the semi–infinite strip using a continual point heat source shown in Fig.1 was investigated in the extent of plane theory of thermoelasticity.

THERMOELASTIC FIELD INDUCED BY AN ISOLATED HEAT SOURCE

The axisymmetric temperature distribution $T^0(r,t)$ and resulting thermal stress field $\sigma_r^0(r,t)$, $\sigma_\theta^0(r,t)$ due to a continual point heat source applied to a thin infinite elastic plate can be written in the following forms.

$$T^0(r,t) = \frac{Q}{4\pi\lambda}\int_0^t \frac{\exp\{-r^2/4\kappa(t-\tau)\}}{t-\tau}d\tau \quad (1)$$

$$\sigma_r(r,t) = -\alpha E\cdot\frac{1}{r^2}\int_0^r T^0(R,t)RdR \quad (2)$$

$$\sigma_\theta(r,t) = -\alpha E\cdot T^0(r,t) + \alpha E\cdot\frac{1}{r^2}\int_0^r T^0(R,t)RdR \quad (3)$$

Where, r is a distance from the point heat source, t is a heating time, Q is a magnitude of heat source per unit time and per unit thickness, λ is a thermal conductivity, E is a modulus of elasticity, κ is a thermal diffusivity and α is a linear expansion coefficient of the material. When the plate thickness is sufficiently thin, the temperature in the direction of plate thickness is regarded uniform and the resulting thermal stress field would be in the ideal plane stress state.

The upper and lower edges and the end of strip are assumed to be thermally insulated. In order to get the temperature field of the strip, Eq.(1) may be superposed periodically as shown in Fig.2. Thermally insulated boundaries can be achieved on the dashed lines due to symmetry.

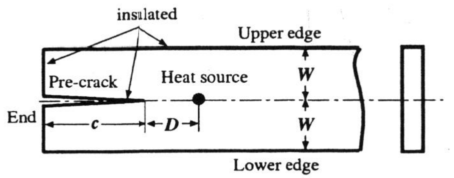


Fig.1 Cleaving of a semi–infinite strip using a point heat source

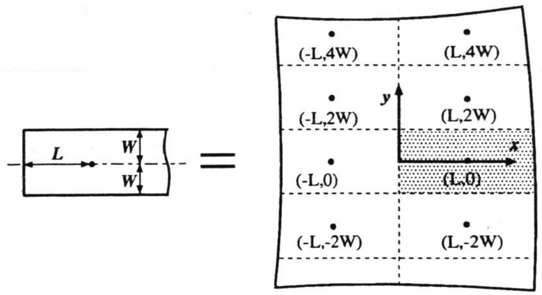


Fig.2 Method of getting the temperature field in the strip.

THERMOELASTIC FIELD INDUCED BY THE PERIODIC HEAT SOURCES

The thermoelastic field due to the infinitely periodic heat sources in Fig.2 can be obtained simply by superposing the thermoelastic fields due to the point heat sources acting at $(\pm L, \pm 2nW)$, $(n = 1,2,\cdots)$. The final expression for the field may be written in the cartesian coordinate system as,

$$T(x,y,t) = \frac{Q}{4\pi\lambda}\sum_{i=1}^{2}\left[\sum_{n=-\infty}^{\infty} E_1\left(\frac{r_{i,n}^2}{4\kappa t}\right)\right] \quad (4)$$

$$\sigma_x(x,y,t) = -\frac{\alpha EQ}{8\pi\lambda}\sum_{i=1}^{2}\left[\sum_{n=-\infty}^{\infty}\left\{E_1\left(\frac{r_{i,n}^2}{4\kappa t}\right) - \left(1-e^{-\frac{r_{i,n}^2}{4\kappa t}}\right)\cdot\left(1-2\frac{X_i^2}{r_{i,n}^2}\right)\cdot\frac{4\kappa t}{r_{i,n}^2}\right\}\right] \quad (5)$$

$$\sigma_y(x,y,t) = -\frac{\alpha EQ}{8\pi\lambda}\sum_{i=1}^{2}\left[\sum_{n=-\infty}^{\infty}\left\{E_1\left(\frac{r_{i,n}^2}{4\kappa t}\right) + \left(1-e^{-\frac{r_{i,n}^2}{4\kappa t}}\right)\cdot\left(1-2\frac{X_i^2}{r_{i,n}^2}\right)\cdot\frac{4\kappa t}{r_{i,n}^2}\right\}\right] \quad (6)$$

$$\tau_{xy}(x,y,t) = -\frac{\alpha EQ}{8\pi\lambda}\sum_{i=1}^{2}\left[\sum_{n=-\infty}^{\infty}\frac{8\kappa t X_i Y_n}{r_{i,n}^4}\left(1-e^{-\frac{r_{i,n}^2}{4\kappa t}}\right)\right] \quad (7)$$

Where, $X_1 = (x-L), X_2 = (x+L)$, $Y_n = y - 2nW$ and $r_{i,n}^2 = X_i^2 + Y_n^2$. $E_1(\bullet)$ is the integral exponential function defined as,

$$E_1(u) = \int_u^\infty \frac{e^{-x}}{x}dx \quad (8)$$

Note that the effect of heat dissipation from the plate surface was omitted simply because the present analysis concerns only the thermoelastic behavior in the beginning of heating, and the heating time t is restricted considerably short. For the large value of u, $E_1(u)$ behaves as e^{-u}/u (Abramowitz and Segun ed., 1968) and vanishes rapidly. Consequently, the sum up to $\pm\infty$ about n can be replaced by the sum up to some finite number N. The remaining term involved in Eqs.(5),(6) can be calculated with the help of the following formula (Moriguchi, *et.als.*, 1992).

$$\sum_{n=-\infty}^{\infty}\left(1-2\frac{X_i^2}{r_{i,n}^2}\right)\cdot\frac{4\kappa t}{r_{i,n}^2} = \frac{\pi^2}{2}\cdot\frac{4\kappa t}{W^2}\left[\frac{1-\cosh\pi\frac{X_i}{W}\cdot\cos\pi\frac{y}{W}}{(\cosh\pi\frac{X_i}{W}-\cos\pi\frac{y}{W})^2}\right] \quad (9)$$

It is assumed that the crack opening displacement induced by the infinite arrays of point heat sources is considerably small and hence, the temperature field is unchanged even if the crack opens.

SOLUTION OF ISOTHERMAL ELASTIC PROBLEM

The thermal stress field due to the infinite arrays of heat sources does not satisfy the stress boundary condition. In order to remove the stresses along the boundary, isothermal elastic field must be superposed as shown in Fig.3. Stress Intensity Factor (SIF) is computed in the isothermal field whose stress boundary value is obtained from the condition that the traction along the boundary after superposition should be zero.

In order to solve the isothermal problem, the Body Force Method (BFM) for two dimensional elastic problem was used. The BFM is one of the boundary type numerical technique for stress analysis and first proposed by H. Nisitani (1967). As far as author's knowledge, BFM is the most desired numerical method for crack analysis, and moreover, highly accurate solution can be easily obtained. In BFM, the boundary of the body is approximated by several boundary elements, as the same manner in usual Boundary Element Method. The boundary discretization for the present BFM is shown in Fig.4, where the linear element was used for the upper half region above the symmetric axis. The upper edge was represented by usual linear elements in the extent of $15W$ and one semi-infinite boundary element for the remnant part.

The body force doublet(Nisitani and Chen, 1987), which means discrepancy of displacement, is continuously distributed along the crack part as the form of product of the basic density function(Nisitani, 1978) and the weighting function. The basic density function for the crack problem expresses in itself the exact crack tip singularity, and therefore, determination of the weighting function through the discretization analysis becomes simple and easy. SIF can be obtained directly from the value of the weighting function at the crack tip.

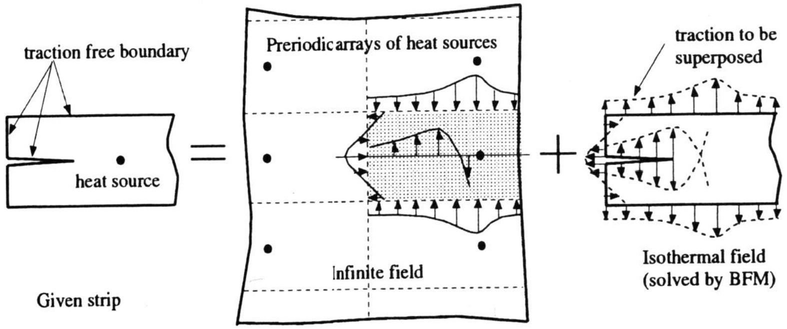


Fig.3 Superposition of stress fields

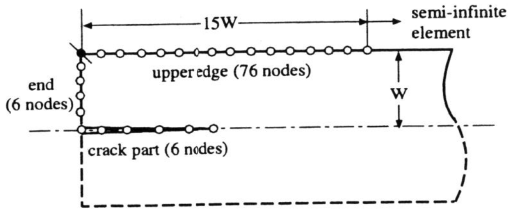
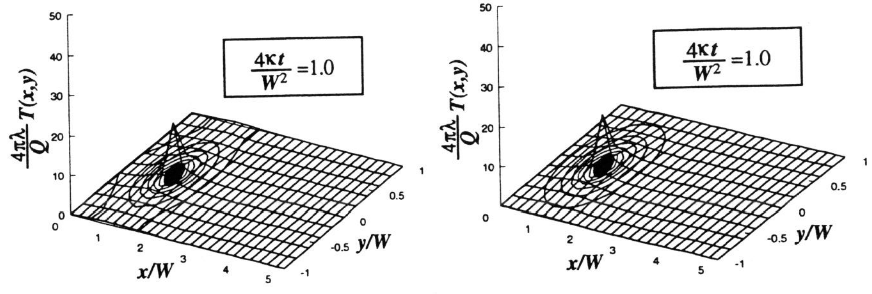


Fig.4 Discretization of the boundary for the present BFM analysis

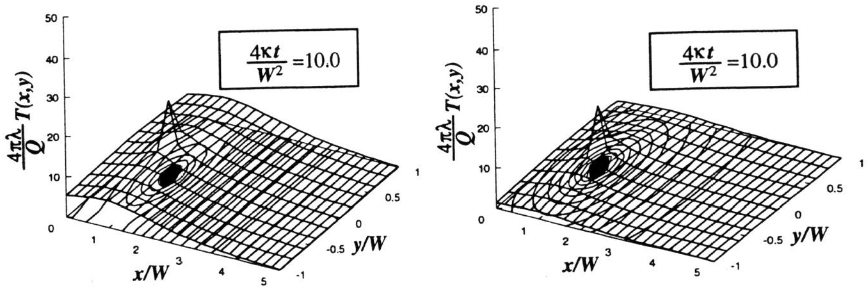
NUMERICAL RESULTS

Temperature analysis

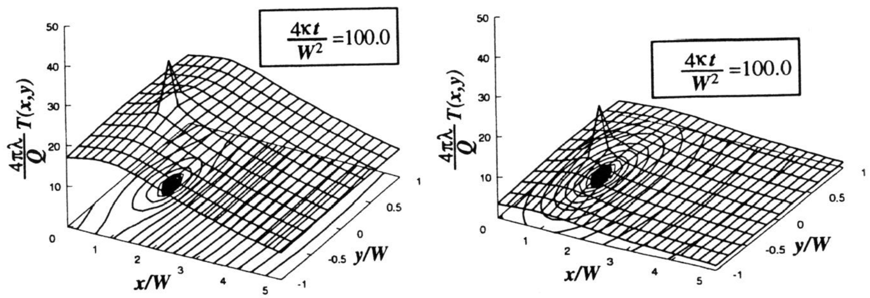
Fig.5 shows the difference between the temperature distribution in a semi-infinite strip and in the same area of an infinite plate due to an isolated point heat source applied at $x/W = 1.0, y/W = 0.0$. In the beginning of heating ($4\kappa t/W^2 = 1.0$), the temperature distributions are almost the same between two cases. The contour appears as the concentric circle with the origin at the heating point. As the heating time increases (Fig.5(b),(c)), the difference between the two becomes large. Although the axisymmetric distribution holds in an infinite plate independently from the heating time, this characteristic is no longer conserved in a semi-infinite strip and remarkable temperature rise near the end ($x/W = 0$) is observed.



(a)



(b)



(c)

Fig.5 Comparison of temperature distribution in the strip (left) and in an infinite plate (right) at $4\kappa t/W^2 = 1.0$ (a), 10.0 (b) and 100.0 (c)

Traction along the boundary

The traction distributions along the boundary to be superposed to the thermal stress field due to the point heat source are shown in Fig.6, where the normal stresses for the strip (solid line) is compared with one in an infinite plate (dot line) at the corresponding position. The difference of traction is small in the beginning of heating ($4\kappa t/W^2 = 1.0$), but this difference becomes large as heating time increases. It is noticed that the normal stress along the upper edge is tensile while the stress on the x axis is compressive with the singularity at the heating point. From Fig.8, time dependent behavior of the thermal SIF can be predicted at the tip of the edge crack in a strip. Moreover, once the resulting SIF reaches fracture toughness of the material, the crack would propagate toward the heat source until it reaches certain point leaving a small distance from the heat source.

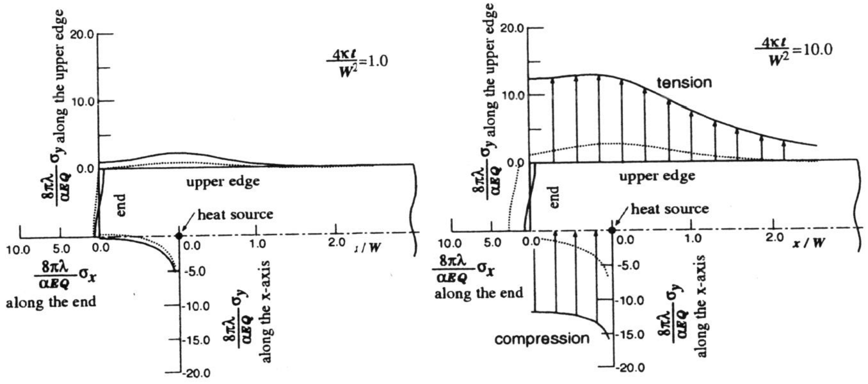


Fig.6 Traction along the boundaries and stress along the x axis

Thermal stress intensity factor

Fig.7 shows the time dependent behavior of the thermal stress intensity factor at the tip of the edge crack. While the non-dimensional heating time $4\kappa t/W^2$ is grater than 0.5, the most effective heating position is approximately $D/W = 0.4$. This is true even if the crack length changes widely within a range of $0.5 < c/W < 10.0$. It is also noticed that the crack length dependency of the relation between SIF and $4\kappa t/W^2$ diminishes for $c/W > 5.0$.

COMPARISON WITH EXPERIMENT

In order to verify the present results, the cleaving experiment of Silicon strips using a Nd:YAG laser as the heat source was carried out.

Table 1 shows the thermo-mechanical properties of the used material. The thickness of the plate was 0.36[mm] and the width was 10.0[mm]. The diameter of the laser spot was set to be 0.4 [mm], the heating time was fixed to be 0.3[s] and the minimum amount of laser

output required for crack extension was measured by increasing the output step by step up to 30[W]. The result of the cleaving test was arranged in Fig.8. The absorbed amount of laser output for the crack extension, Q were plotted against heating location D/W with error bars. The thick curve connects the mean values of the experimental data. It is seen that the most effective heating location for the thermal stress cleaving was $D/W \sim 0.4$ in both cases for the crack length ratio $c/W = 1.0$ and 5.0, and this fact agrees with the numerical results at $4\kappa t/W^2 = 4.0$ shown in Fig.7.

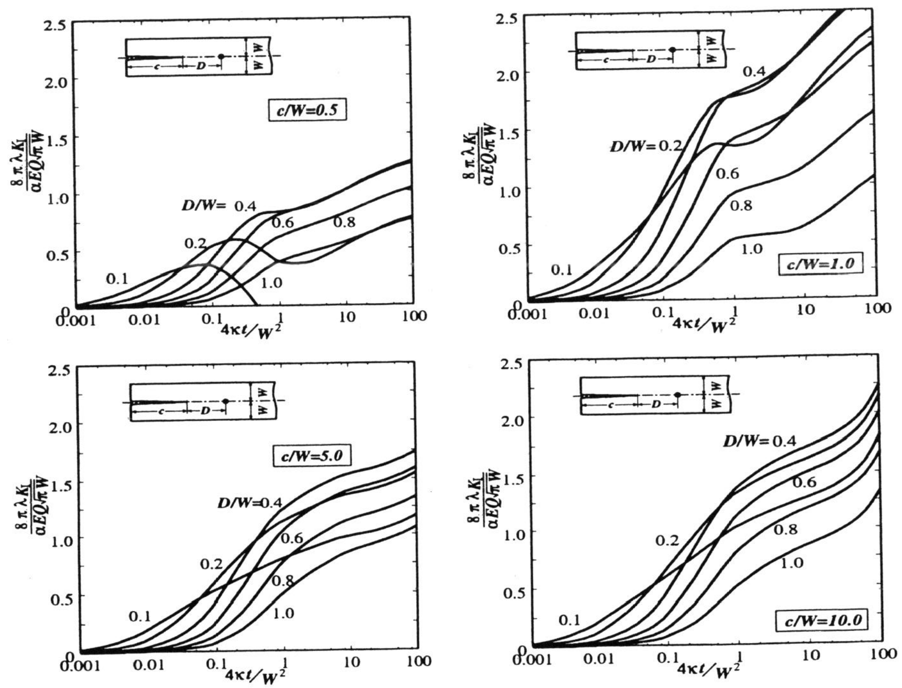
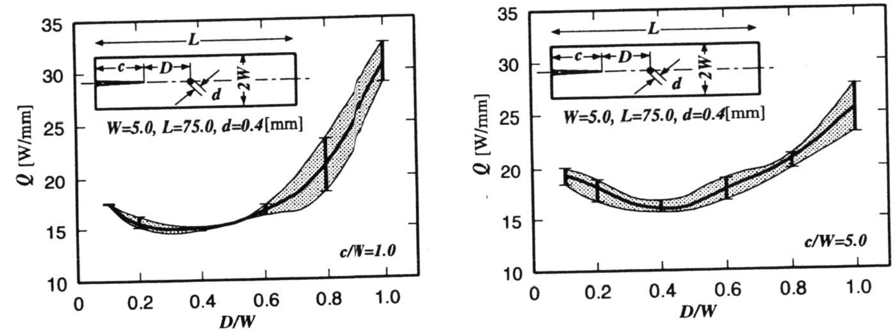


Fig.7 Thermal stress intensity factor versus heating time

Table 1 Thermo-mechanical properties of Silicon at room temperature

$\kappa[m^2/s]$	α $[1/K]$	λ $[W/mK]$	E [GPa]
83×10^{-6}	2.62×10^{-6}	156	117



(a) $c/W = 1.0$ (b) $c/W = 5.0$

Fig.8 Experimental result of Silicon plate cleaving

CONCLUSION

Calculation of SIF for the edge crack in a semi–infinite strip due to the thermal stress of a point heat source was shown and the computer work was demonstrated under various geometric and heating time conditions. It was found that the most effective heating location is the point ahead of the crack tip for approximately 0.4 times the half strip width regardless of the crack length. In order to verify present analysis, cleaving test of Silicon strip using a Nd:YAG laser was carried out and good agreement with the numerical results was found.

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