THE EQUILIBRIUM OF DILATING SHEAR CRACK WITH ROUGH FACES

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ABSTRACT

A general 2-D solution has been obtained analytically for the crack with interacting rough faces. The interaction has been modelled at a large scale as (1) Mohr-Coulomb friction and (2) the crack opening being a known function of sliding. The detailed consideration of a crack with saw-like faces has shown that the energy release rate is less than for a conventional shear crack. Tensile stress concentration caused by the crack opening reduces the angle of kinking though not sufficiently to make the crack propagate in its own plane. At a certain magnitude of shear loading the opening reaches its maximum value determined by the height of asperities. Starting from this point the zone where the maximum opening is reached rapidly increases and the dependencies of crack face displacements and the area of crack opening on the load become non-linear. This however does not affect values of the stress intensity factors.

KEYWORDS

Shear Crack, Saw-Like Faces, Friction, Dilation, Opening, Stress Intensity Factors.

1. INTRODUCTION

Shear cracks are usually considered as smooth cuts which faces slide due to the action of shear tractions. The real roughness of the crack faces is modelled by friction usually depending on the normal compressive component of the external load (eg, Cherepanov, 1979). In this case there is no normal opening of the crack. It is however known that roughness of the crack faces contributes to frictional resistance (eg, Patton, 1966; Barton, 1973) and also causes crack opening (dilation) due to interaction between the asperities of the opposite faces (eg, Barton, et al., 1985; Barton, 1986; Cherepanov, 1987; see also Goodman, 1989). The simplest model illustrating this influence is saw-like crack faces (eg, Goodman, 1989). The horizontal displacement of the crack faces causes a corresponding vertical displacement (see also Bazant, 1980). If, in addition, there is normal stress applied to the faces then its action will manifest itself as friction resisting the shearing with the resisting stress proportional to the magnitude of the compression.

The interaction between the rough crack surfaces under shear changes the distribution of compressive stresses acting on the crack plane and thus affects the sliding and eventually the stress concentration at the crack tip. Tong et al. (1995) computed the stress intensity factors for an edge crack with the rough tip in which both compressive and shear stresses associated with the face interaction are linear functions

In the present paper the 2-D problem for a crack with interacting rough faces is solved in closed form for an arbitrary dependence between normal and shear displacement and a particular case of saw-like crack is considered in detail.

2. CRACK WITH ROUGH FACES. GENERAL EQUATIONS

Consider a straight crack with rough faces in a plane. This crack will be treated at a scale level at which particular elements of the roughness are not seen. It is assumed that there exists an intermediate size H such that $d<<H<<l$, where d is the characteristic size of roughness, $2l$ is the crack length, Fig. 1. Then the overall effect of the interaction between the faces can be accounted for by introducing volume elements of the size H and considering the stress and strains averaged over such volume elements.

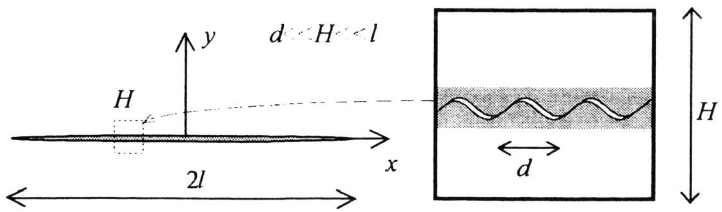

Fig. 1. Homogenisation of the crack face roughness.

When the large-scale stress and strain are introduced, interaction of the asperities will manifest itself in (a) *friction* and (b) *dilation* ie the crack opening even if the crack itself is under compressive load.

Let L be a contour of a crack of unit half-length, $L=\{ z : -1<\mathrm{Re}z<1,\ \mathrm{Im}z=0 \}$. The *friction* will be characterised by the Mohr-Coulomb criterion:

$$z \in L: \quad |\tau_{xy}(x)| + \sigma_y(x)\tan\phi(x) = c(x) \quad \sigma_y(x) < 0, \tag{1}$$

where σ_y and τ_{xy} are normal and shear stresses, $\phi(x)$ is friction angle, $c(x)$ is cohesion.

Dilation is represented as a dependence, f, between the tangential and normal components of relative displacements of the crack faces. This dependence is defined by the type of roughness and the way they interact with each other.

If one uses [.] to denote the discontinuity (jump) of a field through the crack contour, the boundary conditions for the elastic problem for the crack with interacting faces will include the friction condition (1), the following conditions

$$z \in L: \quad [\sigma_y(x)] = [\tau_{xy}(x)] = 0, \quad [u(x)] = f([v(x)]) \tag{2}$$

and the conditions determining the behaviour of the stresses at infinity. In this problem the normal and shear stresses are continuos functions, while the displacement components u, v experience jumps interrelated through the given function f.

The complex potentials for the additional stress field produced by such a crack can be expressed through the (unknown) distribution of the displacement discontinuity (eg, Savruk, 1981) under plane-strain conditions as follows

$$\Phi(z) = \frac{1}{2\pi}\int_{-1}^{1}\frac{Q(\xi)}{\xi - z}d\xi, \quad \Psi(z) = \frac{1}{2\pi}\int_{-1}^{1}\left(\frac{\overline{Q(\xi)}}{\xi - z}\overline{d\xi} - \frac{\bar{\xi}Q(\xi)}{(\xi - z)^2}d\xi\right) \tag{3}$$

$$Q(\xi) = \frac{E}{4(1-\nu^2)}\left(f'([u(\xi)]) - i\right)\frac{d}{d\xi}[u(\xi)] \tag{4}$$

Here z is a point in the plane, $Q(\xi)$ is the unknown function; E, ν are Young's modulus and Poisson's ratio respectively.

Let us assume that the contact between the crack faces is always kept. The criterion of sliding (1) can be now written as follows

$$|\tau_0(x) + \Delta\tau(x)| + [\sigma_0(x) + \Delta\sigma(x)]\tan\phi(x) = c(x) \tag{5}$$

where τ_0 and σ_0 are the applied shear and normal stresses, $\Delta\tau$ and $\Delta\sigma$ are the additional shear and normal stresses created by the displacement discontinuity (sliding and opening) to compensate the excess of τ_0 over the frictional strength. This means that $\mathrm{sgn}(\Delta\tau)=-\mathrm{sgn}(\tau_0)$, but $\mathrm{sgn}(\tau_0+\Delta\tau)=\mathrm{sgn}(\tau_0)$, since the compensating stress cannot change the direction of sliding.

Now (5) can be rewritten by multiplying both parts with $\chi=\mathrm{sgn}(\tau_0)$, which after simple algebra gives:

$$\Delta\tau(x) + \Delta\sigma(x)\tan(\chi\phi(x)) = \left(c(x) - |\tau_0(x)| - \sigma_0(x)\tan\phi(x)\right)\chi \tag{6}$$

Then equation (6) after substituting complex potentials (3) and using the Kolosov formulae (eg, Muskhelishvili, 1953) leads to the following singular integral equation

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\mu(\xi)d\xi}{\xi - x} = \left(c(x) - |\tau_0(x)| - \sigma_0(x)\tan\phi(x)\right)\chi, \quad |x| < 1 \tag{7}$$

where $\mu(\xi)=\mathrm{Re}[(\tan(\chi\phi(\xi))+i)Q(\xi)]$ is a new unknown function.

The solution of equation (7) has to satisfy the condition of uniqueness of displacement. In the case when $\phi(x)=\phi=\mathrm{const}$ (this will be the only case considered further) this condition leads to

$$\int_{-1}^{1}\mu(\xi)d\xi = 0 \tag{8}$$

The appropriate solution has the form (eg, Muskhelishvili, 1953)

$$\mu(\xi) = \frac{-\chi}{\pi\sqrt{1-\xi^2}}\int_{-1}^{1}\frac{\sqrt{1-x^2}}{x - \xi}\left(c(x) - |\tau_0(x)| - \sigma_0(x)\tan\phi\right)dx \tag{9}$$

3. EQUILIBRIUM OF A CRACK WITH SAW-LIKE FACES

Consider now a special case of a crack with saw-like faces (eg., Patton, 1966; Cherepanov, 1987; Goodman, 1989), Fig. 2a. For the sake of simplicity all asperities will be assumed to possess the same size, d, and the same angle of inclination, ϕ. The friction between the contacts will be characterised by a constant cohesion, c, while the genuine friction angle at the contacts will be neglected in comparison with the inclination angle. The external load will be assumed uniform with the shear and normal components τ_0 and σ_0 respectively. It will also be assumed that when the crack opening reaches its maximum, v_{max} (Fig. 2b), the overall friction angle remains the same, ϕ, ie there is still a certain penetration of the saw-like faces into each other sufficient to ensure the friction.

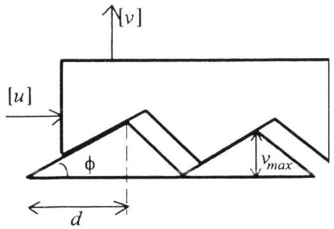

Fig. 2a. Saw-like crack faces.

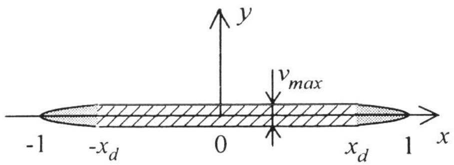

Fig. 2b. Maximum crack opening.

In this case

$$\tau_e = |\tau_0| - c + \sigma_0 \tan\phi = const, \quad [v] = \begin{cases} [u]\tan\phi & \text{if } [u] < d \\ d\tan\phi & \text{if } [u] > d \end{cases}$$
$$Q(x) = \frac{i\tau_e x}{\sqrt{1-x^2}}\left\{1 + [\cos\phi \exp(i\chi\phi) - 1] H(x^2 - x_d^2)\right\} \tag{10}$$

where $H(x)$ is the step function, $\pm x_d$ are coordinates of the points where the vertical displacement, v, first reaches its maximum, v_{max}=$d$tanϕ, Fig. 2b, ie the points with horizontal displacement, $[u]$=d. In this consideration τ_e, plays the role of the only loading parameter. It will further be called the effective shear stress.

After substituting (10) into (3) and finding the corresponding integrals, the potentials take the form

$$\Phi(z) = G_1(z, x_d) + (G_1(z,1) - G_1(z, x_d))\cos\phi \exp(i\chi\phi)$$
$$\Psi(z) = -\Phi(z) - G_2(z, x_d) - (G_2(z,1) - G_2(z, x_d))\cos\phi \exp(i\chi\phi) \tag{11}$$

where

$$G_1(z,x) = \arcsin x - \frac{z}{\sqrt{z^2-1}}\arcsin\left(\frac{x\sqrt{z^2-1}}{\sqrt{z^2-x^2}}\right), \quad G_2(z,x) = \frac{\partial}{\partial z}(zG_1(z,x))$$

Now the stress intensity factors, K_I, K_{II} can be obtained

$$K_I = \tau_e\sqrt{\pi}\cos\phi\sin\phi, \quad K_{II} = \tau_e\sqrt{\pi}\cos^2\phi \tag{12}$$

4. CRITERIA OF CRACK GROWTH

Let us now consider classical criteria of crack growth. Two criteria will be discussed: the energy criterion and the criterion of maximum tensile stresses.

4.1. *Energy criterion*

The energy criterion has the form (eg, Rice, 1968)

$$\delta U = 2\gamma \tag{13}$$

where γ is the specific surface energy of fracture, δU is the energy release rate which for the crack under consideration in plain strain is

$$\delta U = \frac{1-\nu^2}{E}\left(K_I^2 + K_{II}^2\right) = \frac{1-\nu^2}{E}\pi\tau_e^2\cos^2\phi \tag{14}$$

When ϕ=0 this expression is reduced to the conventional one for a shear crack. It should be emphasised that for the crack with rough surfaces the energy release rate is *less*, by the factor of $\cos^2\phi$, than for the conventional shear crack. This is also seen from (12): the Mode II stress intensity factor is less than for the conventional crack by the same factor. Even the appearance of K_I in the expression for the energy release rate does not compensate for the reduction of K_{II}.

Both these reductions should be attributed to the presence of the normal displacement. Indeed, the crack opening displacement creates *compressive* stresses in the material at the place of the crack (see Section 5). These additional compressive stresses increase friction and thus decrease the effective stress shearing the crack. It can therefore be concluded that the crack dilation increases the stress required to initiate the crack growth.

4.2. *Criterion of maximum singular tensile stresses*

It is known that shear cracks usually do not grow in their own plane but rather kink. In order to determine the influence of the dilation of the direction of kinking the criterion of maximum singular tensile stresses (eg, Cherepanov, 1979) will be used.

According to Cherepanov (1979) the angle of kinking, θ, is

$$\theta = 2\arctan\left(\frac{1-\sqrt{1+8\lambda^2}}{4\lambda}\right), \quad \lambda = \frac{K_{II}}{K_I} \tag{15}$$

After substituting (12) into (15) one has λ=cotϕ. Figure 3 shows the resulting dependence. As expected, the dilation reduces the kinking angle, although this mechanism is not sufficient to make the crack grow in its own plane.

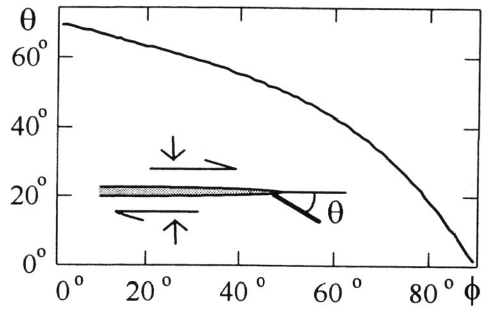

Fig. 3. The angle of crack kinking, θ, vs. the dilation angle.

5. DISPLACEMENTS OF CRACKS FACES

By integrating Q in (10) one has the following expressions for relative displacements of the crack faces and the coordinate of the zone of maximum opening, x_d (see also Fig. 2b)

$$[u(x)]=\begin{cases}\tau_e\dfrac{4(1-\nu^2)}{E}\sqrt{1-x^2}-d\tan^2\phi & \text{if } |x|<x_d\\ \tau_e\dfrac{4(1-\nu^2)}{E}\sqrt{1-x^2}\cos^2\phi & \text{if } x_d\le|x|\le 1\end{cases}$$

$$[v(x)]=\begin{cases}d\tan\phi & \text{if } |x|<x_d\\ \tau_e\dfrac{4(1-\nu^2)}{E}\sqrt{1-x^2}\cos\phi\sin\phi & \text{if } x_d\le|x|\le 1\end{cases} \qquad (16)$$

$$x_d=\sqrt{1-\frac{1}{t^2}},\quad t=\frac{4(1-\nu^2)\tau_e\cos^2\phi}{Ed} \qquad (17)$$

Figure 4 shows the dependence of the half-length, x_d, of the maximum opening zone on the dimensionless loading parameter t which is the normalised effective stress.

When $t<1$, ie $\tau_e<1/4Ed(1-\nu^2)^{-1}\cos^{-2}\phi$, (note, the crack half length is 1, so $d<<1$) the distribution of the horizontal component of displacement discontinuity is the same as for conventional shear crack save for the corresponding factor, while the vertical component is directly proportional to the horizontal one. After t reaches 1 the zone of maximum opening appears instantaneously. At this point the rate of its propagation is infinite. Then the zone enlarges with a reduced rate eventually covering the whole crack as t approaches infinity.

This instantaneous appearance of the zone of maximum opening has however no implication on the deformation energy, U, associated with the crack. Indeed, by transferring the expression for energy release rate (14) to an arbitrary crack length, l, integrating the result over l from 0 to 1 and taking into account that a crack of zero length does not contribute to the energy, one has

$$U=\frac{1-\nu^2}{2E}\pi\tau_e^{\,2}\cos^2\phi \qquad (18)$$

It is seen now that the energy smoothly depends on the applied load and does not depend on the length of the zone of maximum opening.

As the crack opens it affects overall dilatancy and permeability of the cracked material (see also Barton, et al., 1985). The contribution of the crack into both dilatancy and permeability is proportional to the total area of the gaps between the asperities which in its own turn is proportional to the area of crack opening. From (16), (17) one has

$$S(t)=\int_{-1}^{1}[v(x)]dx=\begin{cases}\dfrac{\pi}{2}td\tan\phi & \text{if } t\le 1\\ d\tan\phi\left[t\arcsin(t^{-1})+\sqrt{1-t^{-2}}\right] & \text{if } t>1\end{cases} \qquad (19)$$

where t is the normalised effective shear stress (17).

The area of the crack opening reaches its maximum, $S_{max}=2d\tan\phi$, when $t\to\infty$. The relative area of crack opening, $S(t)/S_{max}$ is plotted on Fig. 5. As expected, it first linearly increases with t and then, after $t=1$, sharply slows its growth.

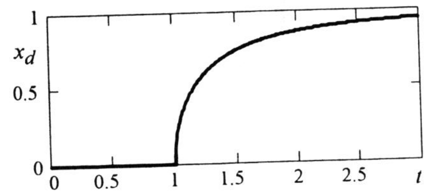

Fig. 4. The coordinate of the zone of maximum opening vs. the normalised effective stress, t.

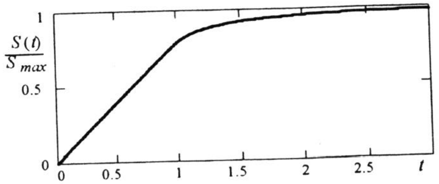

Fig. 5. Relative area of the crack opening vs. the normalised effective shear stress.

The length of this zone however affects the distribution of compressive stresses acting on the crack plane due to the interaction between the rough faces. Indeed, it is followed from (11) that

$$\Delta\sigma(x)=-\frac{\tau_e}{\pi}\sin 2\phi\left[\arccos(x_d)+\frac{x}{\sqrt{1-x^2}}\ln\frac{x_d\sqrt{1-x^2}+x\sqrt{1-x_d^{\,2}}}{\sqrt{|x^2-x_d^{\,2}|}}\right] \qquad (20)$$

Figure 6 shows this distribution for different values of x_d. It is seen that the stress distribution is highly non-linear, opposite to what was assumed by Tong et al. (1995). Moreover, it has logarithmic singularities at the ends of the zone of maximum opening.

6. CONCLUSION

A general 2-D solution is obtained for the crack with interacting rough faces. The interaction is modelled at a large scale as (1) Mohr-Coulomb friction and (2) the crack opening being a known function of sliding. A particular case of a crack with saw-like faces is considered in detail. The energy release rate for such a crack is less than for a conventional shear crack even despite the presence of the Mode I stress intensity factor. However the tensile stress concentration does reduce the angle of kinking (calculated from the criterion of maximum tensile stresses) though not sufficiently to make the crack propagate in its own plane.

At a certain magnitude of shear loading the opening reaches its maximum value determined by the hight of asperities. Starting from this point the zone where the maximum opening is reached rapidly enlarges and the dependencies of crack face displacements and the area of crack opening on the load become non-linear. This however does not affect the stress intensity factors and thus the conditions of crack propagation.

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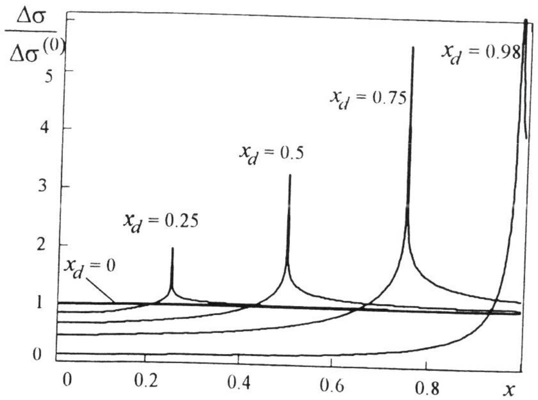

Fig. 5. Distribution of additional normal stress due to the crack opening; $\Delta\sigma^{(0)}=-\tau_e/2\sin 2\phi$ is the stress distribution for $x_d=0$.

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