THE CONSTRAINT OF ELASTIC-PLASTIC CRACK TIP FIELDS

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Abstract.

Crack tip constraint has been analysed for a wide range of common plane strain crack geometries, including single edge cracked bars in tension and bending and centre cracked panels. Constraint estimation is based on a non-linear finite element analysis, using a two parameter characterisation. The first parameter J, describes the deformation level. The second parameter, which describes crack tip constraint, is a non-singular term, based on either an elastic T-stress or a Q field. For the single edge cracked bending and tension geometries Q is divided into two components a distance independent term related to the T-stress and a distance dependent term which arises from global bending on the ligament.

Key words: Constraint, T-Stress, Q fields, Global Bending

1. Introduction to Constraint Based Fracture Mechanics

The J-integral provides a single parameter characterisation of the crack tip stress field for a limited range of highly constrained loading configurations and deformation levels. Proposals to extend the characterisation of the stress field beyond single parameter characterisation are known as two parameter fracture mechanics. A two parameter approach developed by Bilby *et al.* (1986) and more recently by Betegón and Hancock (1991), Al-Ani and Hancock (1991), and Du and Hancock (1991) is based on the elastic T-stress. T is the second term in the Williams expansion whose significance was first discussed by Rice (1974). Neglecting higher order terms, the elastic stress field can be expressed in the form:

$$\sigma_{ij}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{1j} \tag{1}$$

Crack geometries which develop constrained flow fields feature positive values of T stress, while geometries which exhibit unconstrained flow field feature negative values of T. In single edge cracked bend bars geometries with a/W$\geq$ 0.3 exhibit positive values of T and fully constrained fields, as do single edge cracked bars in tension with $a/W \geq 0.5$. For the centre cracked panels the full range of geometries ($0.1 \leq$ a/W ≤ 0.9) exhibit unconstraint flow fields and negative values of T.

O'Dowd and Shih (1991b) have used both the small scale yielding field (SSY) and the HRR field as reference fields to which constraint loss is indexed. The small scale yielding field can be expressed as the HRR field plus a collection of minor higher order terms. In the present work the small scale yielding field is used as a reference field so that the stress fields are expressed in the form :

$$\frac{\sigma_{ij}}{\sigma_0} = \frac{\sigma_{SSY}}{\sigma_0} + Q \tag{2}$$

Q is usually obtained from small strain solutions as the difference between full field solutions and the small scale yielding field. For moderate levels of deformation the difference field (Q) corresponds approximately to a uniform hydrostatic term in front of the crack tip.

2. Numerical Analysis

Finite element analysis using ABAQUS V.5.3 (1992) was used to analyse plane strain single edge cracked bars in bending (SECB) and tension (SECT), centre cracked panels (CCP). The crack to width ratios were a/W=0.1 to a/W=0.9 with and interval of 0.1. For the single edge geometries the width to height ratio was 1/3 and for the CCPs the width to height was 2/3.

In uniaxial tension the material response can be described by Hooke's law at stresses less than the yield stress σ_0 and the plastic response was approximated to a Ramberg-Osgood stress-strain relation with a strain hardening n=13. Poisson's ratio, ν, was 0.3. The analysis of single edge cracked bars in tension and bending are extensively described in Karstensen *et al.* (1995) for hardening rates n=3,6,13 and ∞.

3. Constraint Estimation Expressed by T

When the plastic zone around the crack tip is very small compare to dimensions such as the crack length or the ligament, small scale yielding conditions are present.

With increasing levels of deformation this field changes for geometries which develop a negative T-stress. As an illustration Fig. 1 shows the development of the crack tip fields for two different geometries: a shallow cracked bend bar and a centre cracked panel. The figure shows the hoop stress directly ahead of the crack normalised by the yield stress as a function of the non-dimensional distance $\frac{r\sigma_0}{J}$. The small scale yielding field is shown as a solid bold line in both figures.

For the single edge cracked bend bar the stress profile is initially close to the small scale yielding value and as the level of deformation increases the stress profile remains parallel to the small scale yielding field. For the two lowest deformation levels the difference between the small scale yielding field and the full field solution is independent of distance until $\frac{r\sigma_0}{J} \geq 10$. At higher deformation levels the loss of constraint becomes distance dependent as the global bending field is encountered.

For centre cracked panels the loss of constraint is significant even at low levels of deformation such as contained yielding. However even for high level of deformation $\frac{c\sigma_0}{J} \approx 10$ the stress profiles remain broadly parallel to the small scale yielding field.

Modified boundary layer formulations (Karstensen *et al.* (1995)) have been used to estimate the loss of constraint exhibited by geometries with negative values of T. The fields σ_{MBLF} are expressed by two-term polynomials of the form:

$$\frac{\sigma_{MBLF}}{\sigma_0} = \frac{\sigma_{SSY}}{\sigma_0} + a_1\left(\frac{T}{\sigma_0}\right) + a_2\left(\frac{T}{\sigma_0}\right)^2 \tag{3}$$

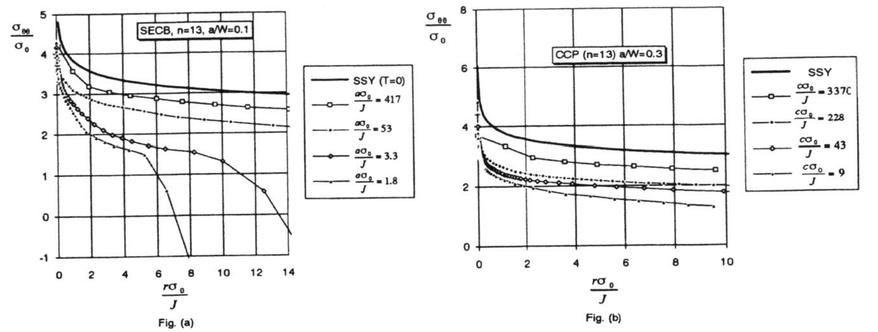

Fig. 1. The hoop stress directly ahead of a crack as a function of $\frac{r\sigma_0}{J}$ for two different geometries at several levels of deformation. (a): SECB (a/W=0.2, n=13) and (b): CCP (a/W=0.3, n=13).

In the full field solutions the stress at a distance $\frac{r\sigma_0}{J}$ = 2 were examined and compared with those in modified boundary layer formulations, using the scheme suggested by Betegón and Hancock (1991). In this case a value of T is calculated from K or the elastic component of J using the biaxiality data given by Sham (1991). Figures 2 and 3 show the stresses in the full field solutions normalised by those in the modified boundary layer formulation, σ_{MBLF}, at the same value of T for two different cases. Figure 2 is a single edge tension bar and Fig. 3 is a centre cracked panel where the full range of a/W is shown. The level of deformation is assessed in terms of the applied load normalised by the limit load. The limit loads were determined numerically from a non-hardening analyses.

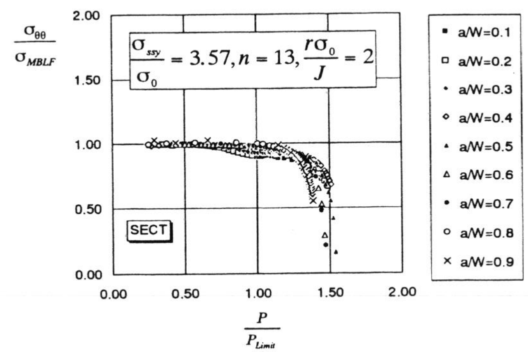

Fig. 2. The hoop stress normalised by the stress from a modified boundary layer formulation for a distance $\frac{r\sigma_0}{J}$=2 at the same value of T, as a function of load normalised by the limit load, n=13.

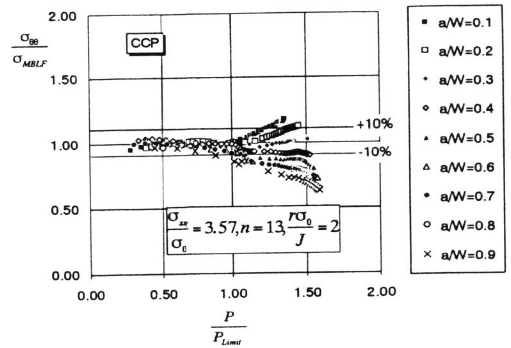

Fig. 3. The stress directly ahead of a crack in a CCP, normalised by the MBLF stress field at a distance $\frac{r\sigma_0}{J}$=2, as function of the load normalised by the limit load, n=13.

4. Constraint Loss due to Global Bending

Figure 1 shows that for geometries in which the ligament is subjected to an opening moment the stress profiles become non-parallel with the small scale yielding field at high levels of deformation. This is due to global bending on the ligament which is a feature of both for single edge cracked bars in bending and tension. For these geometries it is convenient to decompose Q into components Q_T and Q_P as proposed by Karstensen *et al.* (1995)

$$Q = Q_T + Q_P \tag{4}$$

Q_T is related to T and can be understood as

$$Q_T = \frac{\sigma_{MBLF} - \sigma_{SSY}}{\sigma_0} = +a_1(\frac{T}{\sigma_0}) + a_2(\frac{T}{\sigma_0})^2 \tag{5}$$

and Q_P can be calculated as a difference between the full field solutions and the small scale yielding field plus the constraint lost by an eventually negative value of T.

$$Q_P = \frac{\sigma_{\theta\theta}}{\sigma_0} - \frac{\sigma_{SSY}}{\sigma_0} - Q_T \tag{6}$$

In Fig. 4 Q_P is plotted as a function of level of deformation as measured by $\frac{P}{P_{Limit}}$. It is significant to note that the results for all the a/W ratios fall on a common curve which depends on the distance from the crack tip, as described by Karstensen *et al.* (1995).

The shape of the curve only depends on the strain hardening rate n, and the distance $\frac{r\sigma_0}{J}$ at which Q is measured. The relation between Q_P and the applied load can be described by a relation of the form:

$$Q_P = k_2(n)\left(\frac{r\sigma_0}{J}\right)\left(\frac{P}{P_{Limit}}\right)^{n+1} \tag{7}$$

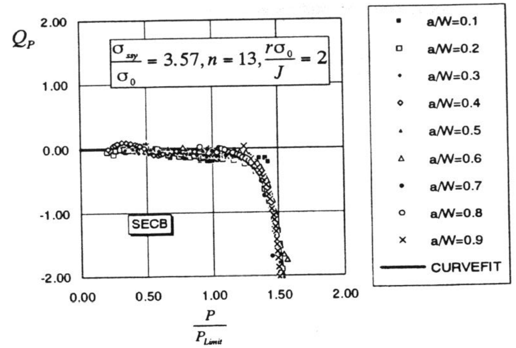

Fig. 4. Q_P as a function of $\frac{P}{P_{Limit}}$ for single edge bend bars at a distance $\frac{r\sigma_0}{J}$ = 2 from the crack tip, for all a/W ratios and n=13.

where $k_2(n)$ is a proportionality constant which depends on the strain hardening rate but is independent of the geometry (a/W ratio or the ligament size c). For n=13 k_2=-0.0038 for single edge cracked bars in bending and k_2=-0.0041 for single edge cracked bars in tension.

Kumar *et al.* (1981) expressed the relation between the plastic component of the J integral, J_P, and the load as

$$J_P = \alpha\sigma_0\varepsilon_0 c h_1(\frac{a}{W}, n)(\frac{P}{P_{Limit}})^{n+1} \tag{8}$$

where h_1 is a function of the $\frac{a}{W}$ ratio and the strain hardening exponent, tabulated by Kumar *et al.* (1981).

Figure 5 demonstrates that Q_P is linearly dependent on J_P, and increases with the distance $\frac{r\sigma_0}{J}$ from the crack For low and moderate hardening rates the distance dependence of Q_P can be approximated by the relation:

$$Q_{P(\frac{r\sigma_0}{J})} = k_1(n)\left(\frac{r\sigma_0}{J}\right)\left(\frac{J_P}{c\sigma_0}\right) = k_1(n)(\frac{r}{c})\left(\frac{J_P}{J}\right) \tag{9}$$

However in this case k_1 is dependent on the loading mode (bending or tension), for the single edge bars in bending k_1 is largely independent of a/W, while for the single edge tension bars k_1 is strongly dependent on a/W.

5. Discussion

The stresses in a series of full field solutions of edge cracked bars in bending and tension and for centre cracked panels have been compared with the stresses predicted from a modified boundary layer formulation at the same value of T. For the single edge cracked bars the difference between the prediction based on T and the crack tip field in full field solutions has been denoted Q_P through Equation (4). In loading modes which involve opening bending moments on the ligament both J-T and J-Q characterisation break down. Shih and O'Dowd (1992) have discussed criteria for the limits of J-Q characterisation. At low levels of deformation Q is independent of distance and is identical to the constraint characterisation based on T. However at high deformation levels

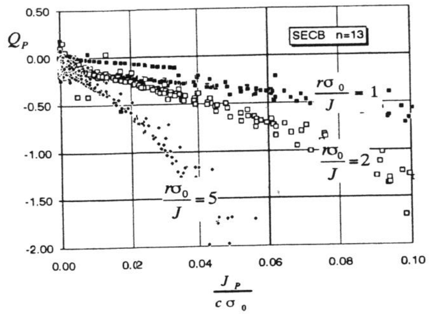

Fig. 5. Q_P as a function of $\frac{J_P}{c\sigma_0}$ for single edge cracked bars in bending for all a/W at distances $\frac{r\sigma_0}{J}$=1,2,5 from the crack tip.

Q varies with distance for the edge cracked bars and the criterion suggested by Shih and O'Dowd (1992) is a limit on the Q gradient term Q':

$$Q' = \frac{dQ}{d\frac{r\sigma_0}{J}} \tag{10}$$

Q' may be compared by the distance derivative of Q_P. The criterion for break down of two parameter characterisation expressed in terms of the distance dependency gradient can only be caused by change of Q_P as Q_T is independent of distance. From equation (9)

$$Q' = \frac{dQ_P}{d\frac{r\sigma_0}{J}} = k_1(n)(\frac{J}{c\sigma_0}) \tag{11}$$

With this relationship the ASTM requirement for J dominance of deeply cracked bars corresponds to $Q' \leq -0.20$. With this approach the limits of two parameter characterisation of shallow cracked bend bars and the limits of single parameter characterisation of deeply cracked bars becomes identical at:

$$\frac{c\sigma_0}{J} \geq 25 \quad \text{and} \quad Q' \geq -0.2 \tag{12}$$

The existence of a valid Q field beyond the predictions based on T requires the existence of a distance independent term. Figure 1 shows the stress profiles do not remain parallel as the loss of constraint increases with distance from the tip. The necessary conclusion is that in the case of edge cracked bars, the distance independent Q term is entirely accounted for by T, and that the deviation from both J-T and J-Q characterisation arises from the global bending field.

In Fig. 6 the stress field ahead of a shallow cracked bar (a/W=0.2) from the numerical analysis is compared with the prediction from Equation (9) and (7). Two different levels of deformation are shown and there is demonstrated good agreement between the prediction and the field through conditions in which constraint loss occurs both by the formation of genuine Q/T fields and through conditions in which constraint loss occurs by global bending.

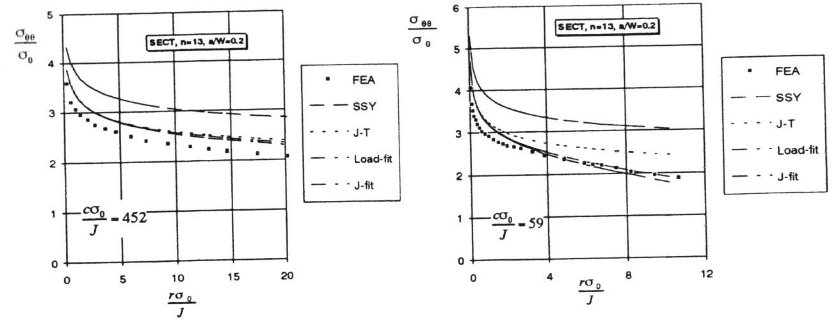

Fig. 6. The hoop stress at a distance $\frac{r\sigma_0}{J}$=2 directly ahead of a SECT, a/W=0.2, n=13 at several levels of deformation.

Centre cracked panels are loaded with displacement boundary conditions and with displacement controlled symmetry conditions. A crack closing moment develops on the ligament as a reaction of the boundary condition along the symmetry line normal to the crack. The ligament is thus loaded with a combination of a crack opening force and a crack closing moment.

At low levels of deformation the stress field for centre cracked panels is well defined by the modified boundary layer formulation. As the deformation level exceeds linear elastic characterisation, the force to moment ratios on the ligament changes and the associated bi-axiality obtained from initial a linear elastic characterisation becomes less appropriate at high deformation levels as discussed more detailed by Karstensen (1996).

6. Conclusions

The development of crack tip constraint has been systematically examined for edge cracked bars subject to tension and bending and for centre cracked panels. The initial loss of crack tip constraint is controlled by the sign of the non-singular T stress which is associated with fields which can be described by the small scale yielding field plus a distance independent term (Q).

Within contained yielding crack tip characterisation can rigorously be achieved by T for both the single edge geometries and the centre cracked panel. J-T characterisation does however extend in practice well beyond the formal limits of contained yielding if a value of T is calculated from the elastic component of J. At high levels of deformation this characterisation breaks down. For the single edge cracked bars the break down is due to the global bending field impinging on the crack tip. This results in a distance dependent Q term. For the centre cracked panels J-T characterisation works well for low to moderate level of deformation, and the stress field under those conditions are characterised by the stresses obtained from a modified boundary layer formulation.

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