STRESS INTENSITY FACTOR HISTORIES FOR A 3D CRACK MOVING STEADILY AND UNSTEADILY

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ABSTRACT

This paper summarizes some results on stress intensity factor (SIF) histories for a 3D crack moving steadily and unsteadily.In the first part,elastodynamic SIF for a unbounded solid containing a planar crack that propagates at a constant velocity under 3D time-independent loading is considered .The solutions of a general loading are expressed out in term of superposition integral based on the fundamental solution procedure.In the last part,using the form of Willis et al (1995) of dynamic mode I weight function for a moving crack,the propagation of a planar crack with a perturbation from straightness of the edge and the associated perturbation of the SIF is investigated.The result shows that the oscillatory crack tip motion through brittle,locally heterogeneous materials could be the basis for careful recent measurements by Gross et al(1993).

KEYWORDS

Dynamic fracture,a half plane crack,combined mode,SIF,three-dimension,propagation,perturbation,interaction

INTRODUCTION

Some data from dynamic fracture measurements show that a number of three-dimensional effects are of importance to warrant further investigation.Achenbach and Gautesen(1977) solved the elastodynamic steady-state problem for a semi-infinite crack under 3D loading.Freund(1987) dealt with the case of incident mode I stress wave loading and some extensions have also been considered by Ramirez(1987) and Champion(1988).Most recently,Li and Liu(1994a-d,1995) published a series of papers for 3D elastodynamic crack problems and obtained the exact solution for the problem with a pair of opposed collinear concentrated loads acting on the crack faces at a fixed distance from the crack edge.
Experimental measurements by Fineberg et al(1992) indicate that the limiting fracture speed is significantly less than the Rayleigh velocity and the approach to this limiting speed is accompanied by the onset of a dynamic instability.Some insight regarding these issues can be obtained from the study of 3D crack advance through brittle,locally heterogeneous materials.Rice et al(1994) analysed the propagation of a planar crack with a nominally straight front in a model elastic solid with a single displacement component.They obtained then the solutions for some elementary cases where a crack front moves unsteadily through regions of locally variable fracture resistance.
This paper is divided into two parts.In the first part,the dynamic SIF histories for a half-plane crack extending uniformly in an otherwise unbounded elastic body under combined mode loading are considered.This problem is the three-dimensional analogue of the plane strain

problems solved by Fossum and Freund(1974).The SIF histories of a general loading are obtained in term of superposition integral based on the fundamental solution procedure.In the last part,by virtue of the form of Willis et al(1995),of dynamic mode I weight functions for a moving crack and its use in a first -order perturbation analysis of the deviation from straightness of the crack edge,we hope to learn how the crack front begins to surround and penetrate into various arrays of round obstacles.The results illustrate that the oscillatory crack tip motion through brittle,locally heterogeneous materials results from constructive-destructive interfences of stress intensity waves that can lead to continuing fluctuations of the crack front and propagation velocity.

SIF histories of a steadily moving crack

In this part,the fundamental solution,which is the response of point loading,is obtained firstly.Then SIF histories of a general loading system are obtained out in terms of superposition integrals.

Fundamental solution

Consider the elastic body containing a half plane crack depicted in Fig.1. The body is initially stress free and at rest. At time t=0, two pairs of point forces appear on the crack tip tending to slide open and to tear open the crack separately. Then while the crack tip moves in the x-direction, the forces remain acting at the origin.

Two coordinate systems are employed in this paper as shown in Fig.1.The relation between these two coordinate systems is:

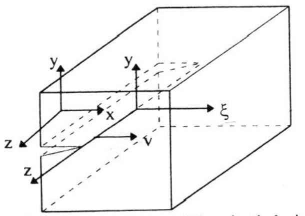

Fig.1. The geometry of the elastic body

$$\xi = x - vt, \quad y = y, \quad z = z$$

Because this problem is antisymmetric about the plane y=0, attention can be restricted to the upper half space y≥0. The boundary conditions in terms of displacements and stresses in the o-ξyz coordinate system are:

$$\begin{aligned}
&\sigma_{yy}(\xi,0,z,t) = 0 \\
&\sigma_{xy}(\xi,0,z,t) = P_1 H(t)\delta(\xi+vt)\delta(z) && \xi<0 \\
&\sigma_{yz}(\xi,0,z,t) = P_2 H(t)\delta(\xi+vt)\delta(z) && \xi<0 \\
&u_x(\xi,0,z,t) = 0 && \xi>0 \\
&u_z(\xi,0,z,t) = 0 && \xi>0
\end{aligned} \tag{1}$$

for z∈(−∞,∞) and t∈(0,∞).

This problem can be expressed in terms of the scalar dilatational wave potential φ and the vector shear wave potential $\vec{\psi} = (\psi_x, \psi_y, \psi_z)$.Transform techniques are used here to solve this problem. Similar to the solution procedure applied by Li et al(1994),the above boundary value problem can be transformed into the Wiener-Hopf equations

$$\mathbf{E}^+ + \mathbf{E}^- = -\frac{\mu}{\eta^2-\gamma^2}\mathbf{X}(\eta)\mathbf{D}(\eta)\mathbf{X}(\eta)\mathbf{U}^- \tag{2}$$

where

$$\mathbf{E}^+ = \begin{bmatrix}\Sigma^+_{xy}\\ \Sigma^+_{yz}\end{bmatrix} \quad \mathbf{E}^- = \begin{bmatrix}\Sigma^-_{xy}\\ \Sigma^-_{yz}\end{bmatrix} \quad \mathbf{D}(\eta) = \begin{bmatrix}\frac{R(\eta,i\gamma)}{\beta(\beta^2+\eta^2-\gamma^2)} & 0\\ 0 & \beta\end{bmatrix}$$

$$\mathbf{U}^- = \begin{bmatrix}U^-_x\\ U^-_z\end{bmatrix} \quad \mathbf{X}(\eta) = \begin{bmatrix}\eta & i\gamma\\ i\gamma & -\eta\end{bmatrix} \tag{3}$$

$$R(\eta,i\gamma) = 4\alpha(\eta,i\gamma)\beta(\eta,i\gamma)(\eta^2-\gamma^2)+[b^2(1-\eta/d)^2-2\eta^2+2\gamma^2]^2$$

where μ is the positive *Lame'* elastic constants,and a,b,c are the slownesses of dilatational , shear,and Rayleigh waves,respectively.

The Wiener-Hopf technique was employed to get the Wiener-Hopf factorization.After some analysis similar to those employed by Li et al,one can obtain the fundamental solutions.

$$K_{II} = \frac{\sqrt{2}}{(\pi|z|)^{3/2}}\frac{\partial}{\partial T}\int_1^T\left\{\mathrm{Re}\left[\frac{2d'\tau(\frac{1}{d'}+\frac{i}{b'}\sqrt{\tau^2-1}+i\tau)}{(d'^2+\tau^2)g_1(\tau)}f_1(\tau)\right]\right.$$
$$\left.+\mathrm{Im}\left[\frac{d'}{d'-i\tau}\frac{\sqrt{\frac{1}{d'}+\frac{i}{b'}\sqrt{\tau^2-1}+d'}\,(d'P_1-\tau P_2)}{(\frac{c_0^2}{db_0}+\frac{c_0}{c}\sqrt{c'^2-\tau^2}+d')S_+(d',-\tau)}\right]\right\}\frac{d\tau}{\sqrt{T-\tau}}H(T-1) \tag{4}$$

$$K_{III} = \frac{\sqrt{2}}{(\pi|z|)^{3/2}}\frac{\partial}{\partial T}\int_1^T\left\{\mathrm{Im}\left[\frac{2d'\tau(\frac{c_0^2}{db_0}+\frac{c_0}{c}\sqrt{c'^2-\tau^2}+i\tau)S_+(i\tau,-\tau)}{(d'^2+\tau^2)g_1(\tau)}f_1(\tau)\right]\right.$$
$$\left.+\mathrm{Im}\left[\frac{d'}{d'-i\tau}\frac{(\tau P_1+d'P_2)}{\sqrt{\frac{1}{d'}+\frac{i}{b'}\sqrt{\tau^2-1}+d'}}\right]\right\}\frac{d\tau}{\sqrt{T-\tau}}H(T-1) \tag{5}$$

where

$$g_1(\tau) = -b'^2\left(\frac{1}{d'^2}+(\frac{1}{b'}\sqrt{\tau^2-1}+\tau)^2\right)+$$
$$+\frac{d'^2\kappa}{b'^2}(-\frac{c_0^2}{db_0}+\frac{c_0}{c}\sqrt{c'^2-\tau^2}+i\tau)(\frac{c_0^2}{db_0}+\frac{c_0}{c}\sqrt{c'^2-\tau^2}+i\tau)S_+(i\tau,\tau)S_-(i\tau,\tau) \tag{6}$$

$$f_1(\tau) = -\frac{b'^2(-\frac{1}{d'}+\frac{i}{b'}\sqrt{\tau^2-1}+i\tau)\sqrt{\frac{1}{d'}+\frac{i}{b'}\sqrt{\tau^2-1}+d'}}{(\frac{c_0^2}{db_0}+\frac{c_0}{c}\sqrt{c'^2-\tau^2}+d')S_+(d',-\tau)}(d'P_1-\tau P_2)$$
$$-i\frac{d'^2\kappa}{b'^2}\frac{(-\frac{c_0^2}{db_0}+\frac{c_0}{c}\sqrt{c'^2-\tau^2}+i\tau)S_-(-i\tau,-\tau)(\tau P_1+d'P_2)}{\sqrt{\frac{1}{d'}+\frac{i}{b'}\sqrt{\tau^2-1}+d'}} \tag{7}$$

with

$$S_\pm(\eta,i\gamma) = \exp\left\{-\frac{1}{\pi}\int_{a'}^{b'}\tan^{-1}\left[\frac{4w^2(w^2-a^2)^{1/2}(b^2-w^2)^{1/2}}{(b^2-2w^2)^2}\right]f(w,i\gamma)dw\right\} \tag{8}$$

here

$$f(w,i\gamma) = \frac{dw(d^2\pm e(w,i\gamma))^2}{e(w,i\gamma)(d^2-w^2)(d(e(w,i\gamma)\pm w^2)\pm\eta(d^2-w^2))} \tag{9}$$

$$e(w,i\gamma)=\sqrt{w^2(d^2-\gamma^2)+d^2\gamma^2} \tag{10}$$

and $b_0 = db/\sqrt{d^2-b^2}$ $\quad c_0 = cd/\sqrt{d^2-c^2}$ $\quad d=1/v$

$$d'=d/b_0 \qquad b'=b/b_0 \qquad c'=c_0/b_0 \tag{11}$$

It is shown that those solutions will reduce to SIF histories for the two dimensional line load problem solved by Fossum and Freund [1975] when integrated over the range -∞<z<∞. Figs 2-3 shows that the normalized SIF versus normalized time $T=t/b_0|z|$ for various value of c/d,with Poisson's ratio v=0.3(b=1.87a,c=2.02a).From the graphs one can find that, for any value z>0, the stress intensity factors are zero up until the arrival of the first shear wave and there is a logarithmic singularity upon the first Rayleigh wave's arrival and the curves turn down sharply at the same time, then they go upward gradually. The factors will go to zero as the time goes to infinity, but this property varies largely with the ratio c/d. The velocity at which the crack travels and the property of the material have an important influence on the SIF histories in this problem.

As is known, the full dynamic stress intensity factors can be expressed as

$$KP_{II}(z,t)=\int_{-\infty}^{\infty}\int_0^{vt}K_{22}(z-z',t-x'/v)P_1(x',z')dx'dz'+\int_{-\infty}^{\infty}\int_0^{vt}K_{23}(z-z',t-x'/v)P_2(x',z')dx'dz' \tag{12}$$

$$KP_{III}(z,t)=\int_{-\infty}^{\infty}\int_0^{vt}K_{32}(z-z',t-x'/v)P_1(x',z')dx'dz'+\int_{-\infty}^{\infty}\int_0^{vt}K_{33}(z-z',t-x'/v)P_2(x',z')dx'dz'$$

Here $KP_{II}(z,t)$ *and* $KP_{III}(z,t)$ are the stress intensity factor histories for this general loading distribution. And $K_i(z,t)\ (i=22,23,32,33)$ provide weights for the general impact loads defined above.

SIF Histories of an unsteadily moving crack

Rice et al(1994) analysed a planar crack advancing through brittle,locally heterogeneous material in a model elastic solid with a single displacement component.Using the form of Willis et al (1995) ,of dynamic mode I weight functions for a moving crack,we study that problem solved by Rice et al in the 3D context of elastodynamic theory.The asperities are modeled as having the same elastic properties as the rest of the elastic medium,but with slightly higher fracture toughness.And the half-plane crack results model finite-sized cracks,assuming the lengths of the cracks are large compared to other parameters such as obstacles spacing along the crack.

Problem Statement

Consider a half-plane crack propagating in an unbounded solid,nominally in the x direction along the plane y=0. The crack front at time t lies along the arc x=a(z,t) while we assume to have the form $x=Vt+\varepsilon\,\phi(t,z)$,where the function $\phi(t,z)$ is assumed to be bounded,and ε is a small parameter. The crack front speed thus varies along the z axis and its shape deviates from straightness.

In the paper,we derive the 3D solution as a linearized perturbation about the 2D results for a crack moving at a steady speed V_0 under mode I situation.Hence for 2D situation

$$K=K_0=k(V_0)K^* \qquad G=G_0=g(V_0)G^* \tag{13}$$

where

$$k(V_0)=\frac{1-\dfrac{V_0}{c}}{\sqrt{1-\dfrac{V_0}{a}}}\frac{1}{S_+(1/V_0)} \qquad g(V_0)\approx 1-\frac{V_0}{c} \tag{14}$$

K^*, G^* *being* the static factors.

The energy vector factor g(V) has complex functional form but it can be taken to be in very simple form as eq.(15) for most practical purposes[6].

Following Willis JR et al and Fourier representation of the formula,SIF field associated with the perturbed crack is then

$$K(t,z)=k(V)K^*(1+I_a(z,t)-I_c(z,t)-I_f(z,t)) \tag{15}$$

where

$$I_{(*)}(z,t)=\sum_{n=-N}^{N}I_n^{(*)}(t)e^{i2\pi n\frac{z}{\lambda}} \quad *:=a,c,f \tag{16}$$

$$I_n^a(t)=-\frac{n\pi}{\alpha\lambda}\int_{-\infty}^{t}\dot{A}_n(t')F\left(2\pi n\frac{\alpha a(t-t')}{\lambda}\right)dt'$$

$$I_n^c(t)=-\frac{2n\pi}{\gamma\,\lambda}\int_{-\infty}^{t}\dot{A}_n(t')F\left(2\pi n\frac{\gamma\, c(t-t')}{\lambda}\right)dt'$$

$$I_n^f(t)=\frac{1}{\pi\alpha a}\int_{-\infty}^{t}\ddot{A}_n(t')dt'\int_0^1\frac{2}{u^2}F_r(\alpha\, au)\cos(2\pi\, n\alpha\, a(\frac{t-t'}{\lambda})udu \tag{17}$$

where

$$F(q)=\int_0^q\frac{J_1(p)}{p}dp=\int_0^qJ_0(p)dp-J_1(q) \tag{18}$$

and the details of $F_r(x)$ may be found in [14].

Using the relation between the energy release rate and the dynamic stress intensity factor,we obtain that

$$G=G^*g(V)(1+I_a(z,t)-I_c(z,t)-I_f(z,t))^2 \tag{19}$$

where $G^*=\dfrac{1-v^2}{2E}(K^*)^2$ is the rest energy release rate supplied to a straight crack front.

Following Rice et al,our simulations begin with a straight crack propagating with a uniform velocity $_0$ in the region $x<0$. The calculation of space and time varying dynamic crack propagation in the heterogeneous region $x>0$ (correct to first order) is done using the following procedure:

(1) Having crack front positions,velocities and accelerations at a general(discrete) time step $m\Delta t$

$$a(z,t)=\sum_{n=-N}^{N}A_n(t)\,e^{i2\pi n\frac{z}{\lambda}}$$

$$(z,t)=\sum_{n=-N}^{N}\dot{A}_n(t)e^{i2\pi n\frac{z}{\lambda}} \tag{20}$$

$$_{,}(z,t)=\sum_{n=-N}^{N}\ddot{A}_n(t)e^{i2\pi n\frac{z}{\lambda}}$$

Use the FFT procedure to calculate from current velocities $(z,m\Delta t)$,accelerations $_{,}(z,m\Delta t)$ the Fourier coefficients $\dot{A}_n(m\Delta t),\ddot{A}_n(m\Delta t)$;we first FFT the set $\{V(z_j,t)\}$ to get

$$\hat{A}_n(t)=\sum_{j=0}^{m-1}V(z_j,t)e^{-2\pi in\frac{j}{N}} \quad (m=2N) \tag{21}$$

The coefficient set $\{\hat{A}_n(t)\}$ is related to

$\dot{A}_n=\hat{A}_n/m$ for n=0,to m/2

$\dot{A}_{m/2}=\hat{A}_{m/2}/2m$ for n=m/2,-m/2

$\dot{A}_n = \hat{A}_{n+m/2} / m$ for n=-m/2+1 to -1

Verify that first order perturbation conditions $\frac{\dot{A}_n}{V_0} << 1, \frac{n\dot{A}_n \Delta t}{\lambda} <<$ are satisfied.

(2) Calculate local crack front velocities for the next time increment as follows:

(2.1)using the formala (30) and history of $\dot{A}_n, \ddot{A}_n$ to calculate the coefficient $I_n^a, I_n^c, I_n^f (m\Delta t)$; Next we rearrange from $\{\hat{I}_n^*(t)\}$ *to* $\{I_n^*(t)\}$, following the same rules as for rearranging from $\{\hat{A}_n(t)\}$ *to* $\{\dot{A}_n(t)\}$ above.

(2.2)use FFT to invert $I_n^\alpha(m\Delta t), (\alpha = a, c, F)$ *to* $I(z, m\Delta t)$

(2.3)use current crack front positions to calculate velocities $(z,(m+1)\Delta t)$during the next time step

$$(z,t) = \begin{pmatrix} c(1-\Lambda(z,t)), if\ \Lambda(z,t) < 1 \\ 0 \qquad\qquad if\ \Lambda(z,t) \geq 1 \end{pmatrix} \tag{22}$$

where

$$\Lambda(z,t) = \frac{G_{crit}(a,z)}{G^*(1 + I_a - I_c - I_f)^2} \tag{23}$$

(3)use difference formula to calculate accelerations $,(z,(m+1)\Delta t)$ during the next time step.

$$,(z,(m+1)\Delta t) = (V(z,(m+1)\Delta t) - V(z, m\Delta t)) / \Delta t \tag{24}$$

(4)Calculate the local crack front positions at the end of the next time step as

$$a(z,(m+1)\Delta t) = a(z, m\Delta t) + V(z,(m+1)\Delta t)\Delta t \tag{25}$$

(5)Write output,check exit criteria (location of crack front or violation of first order conditions);increase time index m by 1,go to step (1).

Fig 4 shows crack front profiles in the regions with a periodic array of circular asperities with radius R and center to center spacing L at successives times. Same as Rice et al (1994),we choose $\frac{R}{L} = 0.1$,and the speed of Rayleigh waves c=0.53851a=0.93273b. All calculations have been non-dimensionalized and $G_{crita}(left)/G_0 = 4.0$ and $G_{crita}(right)/G_0 = 2.0$,where G_{crita}(left) and G_{crita}(right) denote,respectively,the critical energy release rates of the left and right asperities in a fundamental wavelength λ=2L. G_0 denote non-asperity regions.The computations are done using m=2N=256 and Δt=λ/5Nc. At the initial instant,a straight crack was propagating with a uniform velocity $_0$ in the region x<0.The asperities block the crack advancement after it penetrates into the inter-asperity G_0 regions.As a result,the distribution of velocity initial uniform,turns wavy and shows instability feature in a successive instants.Then the weaker right asperity broke ,the left asperity also broke after some further crack front motion.

By virtue of the Mode I dynamic weight function and its numerical simulations in a first order perturbation analysis of the deviation from straightness of the crack edge,we observe that,oscillatory effects in crack motion,as denoted by Rice et al,are found to follow encounter of the crack front with regions of variable toughness and these may be also be interpreted in terms of constructive-destructive interference of stress intensity waves initialed by encounters of the crack front with asperities and then propagating along the front.These waves,including system of the dilatational ,shear and Rayleigh waves,interact on each other and with moving edge of crack,lead to oscillatory feature of crack front profiles. It seems that the type of oscillatory crack motion could be the basis for careful recent measurements by Gross et al(1993).

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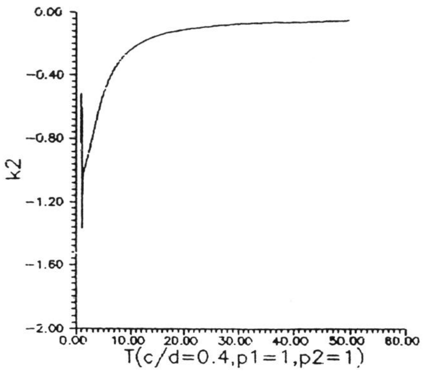

Fig 2 The normalized stress intensity factor histories $K_2 = (\pi|z|)^{3/2} K_{II} / \sqrt{2}$ vs $t / b_0|z|$.

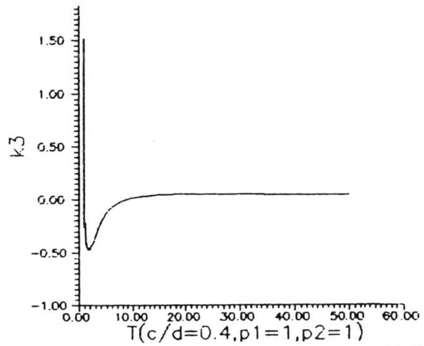

Fig 3 The normalized stress intensity factor histories $K_3 = (\pi|z|)^{3/2} K_{III} / \sqrt{2}$ vs $t / b_0|z|$.

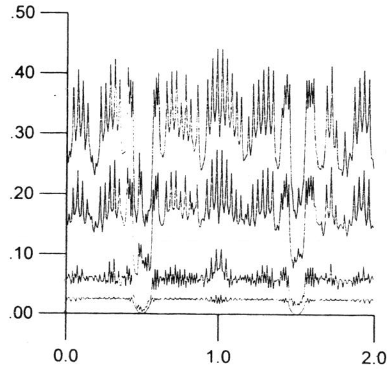

Fig.4 Positions a(z,t) versus z at successive times,for a crack at incoming speed $_0 = 0.3c$ hitting an infinite row of asperities with $G_{crit}(left) = 4.0G_0$ and $G_{crit}(right) = 2.0G_0$ (a) $t=50\Delta t$; (b) $t=100\Delta t$; (c) $t=200\Delta t$; (d) $t=300\Delta t$.Here $\Delta t=1/640$.

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