ON THE ORIGIN OF JOINTS IN SEDIMENTARY ROCKS

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ABSTRACT

Joints, cracks, fissures, faults, and so on characterize the face of a geomaterial in many scales; they are responsible for inelastic and fracturing properties, permeability, viscous flow, and the creep of rocks. The problem of their origin is no less important for geophysics than the problem of the origin of species was for biology before Darwin. There are some speculations on the issues. This paper introduces some fresh ideas in this new direction of fracture science.

It is suggested that joints in a bed of a sedimentary rock appeared as a result of achieving a limiting equilibrium state of failure in the bed once or several times in its geological history. It is shown that a transition from an elastic state to a limiting equilibrium state in a bed may be accompanied by restructuring the uniform stress-strain field into a periodic one. It is hypothesized that the periodic stress distribution in a bed caused a periodic lattice of vertical joints or interfacial horizontal joints originating at the crests of waves. The role of the infilling processes is emphasized.

KEYWORDS

Joints, rocks, origin, fracturing, limiting equilibrium state, rock strata, periodic lattice of joints.

INTRODUCTION

Internal stresses are the cause of deformation and fractures. A joint in a rock is the track of a fracture or crack that appeared in the rock somewhere in its geological evolution. Vertical through-bed joints and horizontal interfacial joints in sedimentary rocks have been well known and described in literature. The geological data have documented a linear relationship between the joint spacing and bed thickness in sedimentary rocks (Price 1966; McQuillan 1973; Ladeira & Price 1981; Huang & Angelier 1989; Narr & Suppe 1991; Gross 1993; and Gross et al., 1995.) Many researchers hypothesize that the joints are caused by some fields of tensile horizontal stresses that somehow exist in a rock bed at distances even much greater than the bed thickness (Lachenbruch 1961; Hobbs 1967; Pollard & Segall 1987; and all of the authors cited above.) According to this viewpoint, the cracks and joints in sedimentary rocks originated in the past similarly to the cross cracks in fibers and laminae in aligned composite materials, which do arise from tensile stresses and are well studied both experimentally and theoretically (e.g., Cherepanov 1983.)

The present paper aims to treat an alternative point of view based on the following facts:

1. A sedimentary rock is or was a porous geomaterial that can or could be fractured by all-around compression stresses, with a short crack being initiated from a pore and developed along the direction of the maximum compressive stress (pressure), (Fairhurst and Cook 1966; Martin 1972; Cherepanov 1974).

2. For an opening mode crack to grow slowly in a brittle material, it may be sufficient that a small zone of tensile stresses exists in the neighborhood of the crack. Such a zone can be created by the wedging forces of tiny particles of ambient rocks penetrated into a free cavity of a fresh crack due to a process of sequential infilling (Cherepanov 1974, 1984, 1987.) Geological evidence supporting sequential infilling includes fractured piedmontite grains (Masuda & Kuriyama 1988) and curving cross joints (Engelder & Gross 1993.)

3. Slip shear interface cracks and joints can appear with no zones of tensile stresses (Jaeger & Cook 1976; Cherepanov 1974, 1983, 1987.)

4. Fracture initiation in a material can be described by a failure criterion that characterizes a limiting equilibrium state of the material (Jaeger & Cook 1976; Cherepanov 1974, 1987, 1988.)

Some researchers note a striking periodicity of the joint arrangement in certain beds (Segall & Delaney 1982; Pollard & Segall 1987; Olson & Pollard 1989; Gross 1993, Gross et al. 1995), although numerous statistical functions ranging from negative exponential to normal have been used to describe the distribution of joint spacing (Priest & Hudson 1976; Rouleau & Gale 1985; Rives et al. 1992.) The specific case of lithology-controlled joints leads to a typically skewed log normal or gamma distribution for joints belonging to an individual systematic set (Huang & Angelier 1989; Narr & Suppe 1991.)

It is relevant to notice that tensile horizontal stresses arise sometimes in a surface layer of the Earth. These stresses cause the formation of periodic hexagon-shaped vertical fractures observed in permafrost tundra (with the specific diameter from 10 to 100 meters in a plan view) and in deserts (with the specific diameter from 0.1 to 1 meter). Some geological evidence and the quantitative theory of these formations were provided (Cherepanov & Bykovtsev 1984a and b.) However, significant zones of tensile stresses deep inside the Earth's crust considered responsible for joints, are absolutely unrealistic and we reject this hypothesis generally accepted today.

In what follows, we consider the mathematical theory of limiting equilibrium states (plasticity) under compression stresses in a horizontal bed of a sedimentary rock, derive some mathematical solutions of the theory of plasticity, and try to understand the origin of joints based on the solutions.

BASIC EQUATIONS AND ASSUMPTIONS

The crust of the Earth has been stratified by layers of various sedimentary rocks, from particulated sediments near the Earth's surface to highly-compressed and strong rocks at the crust bottom. We consider the model of the stratified crust as a half-space with strata parallel to the Earth's surface, that is, the half-space boundary. To be specific, in what follows, we consider a certain jointing bed clamped between two adjacent non-jointing beds of a particulated or tough geomaterial. Designate t as the thickness, and d as the depth of the bed. Introduce the x and y Cartesian coordinates in the bed plane, and the z coordinate perpendicular to the bed plane. From the symmetry of this model formulation, it follows that the uniform stress field,

$$\sigma_z = \rho g d,\ \sigma_x = -\sigma,\ \sigma_y = -\sigma_1,\ \tau_{xy} = \tau_{xz} = \tau_{yz} = 0 \qquad (1)$$

holds in the bed under consideration (σ_x, σ_y, σ_z, τ_{xy}, τ_{xz}, and τ_{yz} are stresses). Here, ρ is the mean density of the rocks above this bed:

$$\rho = \frac{1}{d}\sum \rho_i t_i , \qquad (2)$$

g is the gravitation acceleration (ρ_i and t_i are the density and thickness of the ith bed), and σ and σ_1 are some constants called horizontal stresses. The stress field, Eq. (1), follows from the equilibrium equations for any constitutive equations of rocks if one assumes that: (i) all shear stresses equal zero; (ii) the normal stresses do not depend on x and y and (iii) the only external force in the system is gravitation. If one assumes additionally that the geomaterial of the bed under study is linearly elastic and experiences only vertical motion, one can derive:

$$\sigma_x = \sigma_y = \frac{\nu \sigma_z}{1-\nu} , \qquad (3)$$

where ν is Poisson's ratio of this bed.

Today, it is widely recognized that the uniform stress field in the bed, Eq. (1), oversimplifies the reality, namely: (i) considerable shear stresses can exist on interfaces between beds; (ii) the normal stresses can vary along x and y, and (iii) there are substantial residual stresses in the bed that can influence on horizontal stresses, σ_x and σ_y, first. These facts disagree with the assumption of an elastic state of a bed during its geological evolution from $d = 0$ to the current d.

Once or several times in the history the bed under study was probably fractured—that is, it was in a limiting equilibrium. The through vertical joints and horizontal interfacial joints are some tracks of these events in the past. The fracturing was caused, perhaps, by the fact that the strength of the bed rock was insufficient to endure the weight of the upper beds. The limiting equilibrium state is referred to as a stress state under which the material experiences fracturing and yielding.

We consider the stress distribution in the bed in the state of limiting equilibrium described by the following failure and yielding criterion (Cherepanov 1987):

$$I_D = -\sigma_s + \alpha I_1 . \qquad (4)$$

Here, α and σ_s are some positive empirical constants, I_D is the second invariant of the stress deviator, and I_1 is the first invariant of the stress tensor, namely:

$$I_D^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_x\sigma_z - \sigma_y\sigma_z + 3\tau_{xy}^2 + 3\tau_{zx}^2 + 3\tau_{yz}^2 , \qquad (5)$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z . \qquad (6)$$

Generally, the failure criterion of an isotropic material is

$$F_1(I_D\ ,\ I_1) = 0\ , \tag{7}$$

or, equivalently,

$$I_D = F_2\ (I_1)\ . \tag{8}$$

Here, F_1 and F_2 are some functions. The third invariant does not effect on failure. In sedimentary rocks subjected to all-around compression when all three principal stresses are negative, the $F_2\ (I_1)$ can be well approximated by a linear function, Eq. (4), with two empirical constants. For a comparison in metals, I_D does not depend on I_1 in a limiting equilibrium (when yielding occurs)—that is, $\alpha = 0$ for metals (Cherepanov and Annin 1988).

Assume that the rock in the bed can experience only the plane motion in the $x\ z$ plane when fracturing occurs—that is,

$$\tau_{xy} = \tau_{yz} = 0 \text{ and } \frac{\partial}{\partial y} = 0\ . \tag{9}$$

Let us integrate the equilibrium equation with respect to the x axis:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0\ . \tag{10}$$

over z from 0 to t. As a result we have:

$$t\,\frac{\partial \bar{\sigma}_x}{dx} = \tau^+_{xz} - \tau^-_{xz}\ . \tag{11}$$

Here, $\bar{\sigma}_x$ is the mean stress, σ_x, in the bed,

$$\bar{\sigma}_x = \frac{1}{t}\int_0^t \sigma_x\, dz\ , \tag{12}$$

and τ^+_{xz} and τ^-_{xz} are the shear stresses on the upper and lower interfaces of the bed.

If the stress state is symmetrical with respect to the middle of the bed, $z = t/2$, we have:

$$\tau^+_{xz} = -\ \tau^-_{xz}\ . \tag{13}$$

This symmetry is probably realized when the properties of adjacent layers and interfaces are identical. In the general case, we have:

$$\tau^-_{xz} = \eta_s\ \tau^+_{xz}\ , \tag{14}$$

where the coefficient, h_s, can vary from -1 to +1. Also, we assume that the principal stress, σ_y, is equal to:

$$\sigma_y = \eta_T\ (\sigma_x + \sigma_z)\ , \tag{15}$$

where η_T can vary probably from 0 to 1. In an elastic state, $\eta_T = \nu$; and in the general theory of plasticity, $\eta_T = 1/2$, because the inelastic deformation is accepted to be incompressible. In the theory treated below, we consider η_s and η_T as some given constants, while in the numerical calculations, we put $\eta_s = -1$ and $\eta_T = 1/2$.

Taking into account Eqs. (4), (5), (6), (9), (11), (14), and (15) yields the following governing equations of the problem:

$$t\,\frac{d\sigma}{dx} = (1-\eta_s)\,\tau\ , \tag{16}$$

$$\lambda^2(\sigma + \sigma_0)^2 + 3\tau^2 = \sigma_*^2\ , \tag{17}$$

where λ^2, σ_0 and σ_*^2 are the following positive constants:

$$\lambda^2 = 1-\eta_T + \eta_T^2 - \alpha^2 - 2\alpha^2\,\eta_T - \alpha^2\,\eta_T^2\ , \tag{18}$$

$$2\sigma_0 = \frac{-\sigma_z\left[-2\eta_T^2 + 2\eta_T + 1 + 2\alpha^2\,(1+\eta_T)^2\right] + 2\alpha\,\sigma_s(1+\eta_T)}{1-\eta_T + \eta_T^2 - \alpha^2 - 2\alpha^2\,\eta_T - \alpha^2\,\eta_T^2}\ , \tag{19}$$

$$\sigma_*^2 = \sigma_s^2 - 2\alpha(1+\eta_T)\,\sigma_s\sigma_z + \alpha^2\,(1+\eta_T)^2\,\sigma_z^2 - \eta_T^2\,\sigma_z^2 - \sigma_z^2 + \eta_T\,\sigma_z^2 + \lambda^2\sigma_0^2\ . \tag{20}$$

$$(\bar{\sigma}_x = \sigma,\ \tau_{xz}^+ = \tau,\ \sigma_z = -\rho g d)\ .$$

Here, the distribution of σ_x, σ_y, and σ_z in the bed along the z axis is ignored, so that $\bar{\sigma}_x = \sigma_x$, $\bar{\sigma}_y = \sigma_y$, and $\bar{\sigma}_z = \sigma_z$. However, τ_{xz} in Eqs. (5) and (17) is taken to be equal to τ_{xz}^+; this means that Eq. (17) describes the initial fracturing (a limiting equilibrium) of an upper portion of the bed under study, which is close to $z = t$. If $|\tau^-_{xz}|$ is greater than $|\tau^+_{xz}|$, the present analysis can be easily modified to the lower portion of the bed. The coefficients, η_T, and α, and the ratio, σ_z/σ_s, should meet the condition equations, $\lambda^2 > 0$, $\sigma_0 > 0$, and $\sigma_*^2 > 0$.

THE PERIODIC LAW OF HORIZONTAL STRESS DISTRIBUTION

Let us find the solution to the basic equations, Eqs. (16) - (20), in the bed under study. Introduce the new function, $f(x)$, as follows:

$$\sigma = -\sigma_0 + \frac{\sigma_*}{\lambda}\cos f(x)\ , \tag{21}$$

$$\tau = \frac{\sigma_*}{\sqrt{3}}\sin f(x) \tag{22}$$

Equation (17) is satisfied by Eqs. (21) and (22). Substitute σ and τ in Eq. (16) by Eqs. (21) and (22) to obtain:

$$\left(\frac{t}{\lambda} \frac{df}{dx} + \frac{1-\eta_s}{\sqrt{3}} \right) \sin f(x) = 0 . \tag{23}$$

Equation (23) has three solutions. The following solutions:

$$\sin f(x) = 0, \text{ hence: } \tau = 0 , \; \sigma = - \sigma_0 \pm \frac{\sigma_*}{\lambda} , \tag{24}$$

describe the uniform stress distributions in the bed under the limiting equilibrium. Designate as F_1 and F_3 the stress states, $\sigma = -\sigma_0 + \sigma_* \lambda^{-1}$ and $\sigma = -\sigma_0 - \sigma_* \lambda^{-1}$ correspondingly. The solution satisfying the equation:

$$\frac{t}{\lambda} \frac{df}{dx} + \frac{1-\eta_s}{\sqrt{3}} = 0 , \tag{25}$$

is equal to

$$f = - \frac{(1-\eta_s)\lambda x}{t\sqrt{3}} + C , \tag{26}$$

where C is a constant that is insignificant in the case under study when x varies from $-\infty$ to $+\infty$. It can be put zero; this means that the coordinate origin, $x = 0$, is chosen at a point where $\tau = 0$.

Substituting $f(x)$ in Eqs. (21) and (22) by Eq. (26) at $C = 0$, one finds

$$\sigma = -\sigma_0 + \frac{\sigma_*}{\lambda} \cos (kx) , \tag{27}$$

$$\tau = - \frac{\sigma_*}{\sqrt{3}} \sin (kx) , \tag{28}$$

where

$$k = \frac{(1-\eta_s)\lambda}{t\sqrt{3}} . \tag{29}$$

Designate as F_2 the stress state given by Eqs. (27) - (29). Equations (27) - (29) describe the space wave with the wave length, Λ:

$$\Lambda = \frac{2\pi t\sqrt{3}}{\lambda(1-\eta_s)} . \tag{30}$$

(k is the wave number, $k = 2\pi/\Lambda$). Particularly, from Eq. (27), it follows that $\sigma_x = \sigma$ may be tensile in a portion of a bed only if $\sigma_* > \lambda \sigma_0$.

So, we have obtained several different solutions: two of them are uniform, Eq. (24), and one is periodic, Eqs. (27) - (30). What solution is more appropriate to the geophysical problem under study?

To try to answer this question, consider the elastic energy, E, of the wave length bed just at the moment prior to fracturing when the bed is still elastic:

$$E = \frac{1}{2} \int_0^{\Lambda} \left(\frac{1}{\mu} I_D^2 + \frac{1}{k_e} I_1^2 \right) dx . \tag{31}$$

Here, μ is the shear modulus, and k_e is the volume compressibility coefficient of a rock. Substituting I_D in Eq. (31) by Eq. (4) yields the elastic energy of the bed in a limiting equilibrium state:

$$E_* = \frac{1}{2} \int_0^{\Lambda} \left[\left(\frac{1}{k_e} + \frac{\alpha^2}{\mu} \right) I_1^2 - \frac{2\alpha\sigma_s}{\mu} I_1 + \frac{1}{\mu} \sigma_s^2 \right] dx , \tag{32}$$

where

$$I_1 = (1 + \eta_T)(\sigma_x + \sigma_z) . \tag{33}$$

Here, E_* equals E_1 or E_2 or E_3, where E_1, E_2, and E_3 designate the respective values of E at the F_1, F_2, and F_3 limiting equilibrium states, and I_1 is negative because $\sigma_x + \sigma_z < 0$. Therefore, all summands in Eq. (32) are positive. The greater the absolute value of $\sigma_x + \sigma_z$, the greater is I_1, and hence E_*. Now, observe that the uniform solutions, Eq. (24), coincide with the maximum or minimum values of the periodic solution, Eqs. (27) and (28), at the crests of the waves when $x = 1/2\, \Lambda\, n$, where $n = 0, \pm 1, \pm 2, \ldots$.

From the above consideration, it follows that:

$$E_1 < E_2 < E_3 . \tag{34}$$

Suppose that the elastic energy of the bed under study achieves the value of E at a certain moment of the geological evolution. Four cases are possible:

I. $E < E_1$;
II. $E_1 < E < E_2$;
III. $E_2 < E < E_3$;
IV. $E_3 < E$.

The d, σ_z, E, E_1, E_2, and E_3 are some monotonically growing functions of time. Therefore, any of these functions can play the role of time. It is convenient to consider σ_z as time because σ, τ, E, E_1, E_2, and E_3 are simply expressed in terms of σ_z by Eqs. (17)-(20), (27)-(33). The η_T, η_s and α can generally depend on time, too. However, they are probably more conservative than σ_z and may be considered to be constant, at least, during the time span when the crack growth and infilling processes work.

If a system (the bed) experiences a transition from an elastic state with the energy, E, to a fractured state of limiting equilibrium with the energy, E_1 or E_2 or E_3, the difference,

$$\Delta E = E - E \quad (i = 1 \text{ or } 2 \text{ or } 3), \tag{35}$$

represents the elastic energy release spent for fracturing and the inelastic deformation of the system. This difference is positive, $\Delta E > 0$, and hence, we have:

If $E < E_1$, the transition to a fractured state of limiting equilibrium is impossible;
If $E_1 < E < E_2$, the transition to the F_1 state is possible;
If $E_2 < E < E_3$, the transition to the F_1 and F_2 states is possible;
If $E_3 < E$, the transition to the F_1, F_2, and F_3 state is possible.

Suppose that $\alpha = 0$, and $\eta_T = 0$. From Eqs. (17)-(20), it follows that $\sigma_0 = -\sigma_z$, $\sigma_* = \sigma_s$ and $\lambda^2 = 3/4$ in this case, so that $|\sigma_x| < |\sigma_z|$ at the F_1 state and $|\sigma_x| > |\sigma_z|$ at the F_3 state. It is clear that the F_3 state cannot be achieved by smooth geological developments of a bed in the chain of the events that follow: sedimentation, immersion, gravitation loading, sintering, deformation, fracturing, and infilling. Considerable lateral tectonic stresses of a larger scale due to plate tectonics are necessary to provide for $E > E_3$ and the F_3 states. Using Eqs. (27)-(29) and (31)-(33), it can be shown that the elastic energy corresponding to the hydrostatic state, $\sigma_x = \sigma_y = \sigma_z = -\rho g d$, of a bed is greater than E_2. The hydrostatic state can be achieved as a limit in a bed for a long time, at the cost of the infilling processes of mass redistribution and condensation due to the pressure effect of particles upon vertical fissures in the bed. Therefore, the F_2 state can be achieved in a bed even by a smooth geological development. Of course, the lateral tectonic pressure of a global origin can dramatically facilitate the transition to the F_2 state.

This analysis provides a substantiation of the F_2 state formation at certain moments of the geological evolution. The succeeding processes of infilling result in leveling the periodic stress distribution and relaxing the stress concentration. However, the scenarios may differ for different beds and zones. An interpretation of a mathematical solution may be not single and, sometimes, not simple. And, this is the case under study.

CONCLUSION

This analysis of the limiting equilibrium of a bed has discovered the possibility of the creation of a periodic law stress distribution in a bed at certain moments of its geological evolution that can cause the formation of some periodic lattices of joints in sedimentary rock.

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