NUMERICAL ANALYSIS OF A NONLOCAL DAMAGE MODEL FOR DUCTILE CRACK GROWTH

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ABSTRACT

A mesh independent finite element method is presented to describe strain softening with a nonlocal version of Gurson's damage model. The backward Euler scheme is used to integrate the constitutive equations. The nonlinear problem is solved by the Newton method which requires the derivation of the consistent tangent operator. A two step algorithm is presented which preserves the quadratic convergence of the global Newton iteration but involves the nonlocal description in an explicit manner. A numerical example involving mode II failure is presented to demonstrate the convergence and capacity of the method. Further, the method is applied to the ductile crack growth in a CT- specimen. Finally, the quality and the limits of the method are discussed.

KEYWORDS

Strain softening, Nonlocal damage, Ductile crack growth, Localization, Gurson Model

INTRODUCTION

The specimen size and geometry dependence of the crack resistance curves is a well known problem in the analysis of ductile crack growth. Several concepts have been developed to deal with this problem. Macromechanical approaches introduce additional fracture parameters while micromechanical concepts attempt to incorporate the failure process into the constitutive description of the material as e.g. the model presented by Gurson (1977) which describes the ductile failure process by nucleation, growth and coalescence of microvoids.

However, the application of the damage model involves strain-softening on the macroscopic level, while on the microscopic continuum scale this phenomenon may not exist. From the mathematical point of view, strain-softening leads to loss of ellipticity of the partial differential equations governing a given static boundary value problem. From the physical point of view, strain localization results in zero energy dissipation at failure. The consequence for finite element solutions is the observation of a strong mesh dependence. Therefore, a special treatment of the governing equations is necessary in order to limite the energy dissipation. In the past several regularization methods have been developed in order to solve this problem.

The introduction of the Cosserat continuum (e.g. Mühlhaus, 1989; de Borst, 1991) has the disadvantage that the regularization works only for mode II dominated failure because the additional rotational degrees of freedom must be activated in the problem. Material models taking rate effects into account (e.g. Needleman, 1988; Sluys and de Borst, 1992) work well only if very high, sometimes artificial viscosity parameters are chosen. In the recent years the exclusion of the localized zone from the field description was applied successfully by introduction of regularized discontinuities in the finite element formulation (e.g Larsson *et al.*, 1993; Oliver and Simo, 1994; Miehe and Schröder, 1994). Motivated from the physical drawback that the damage zone in a structure is always concentrated in a very small region before or around the actual crack tip, cohesive frictional models were developed (e.g. Tvergaard and Hutchinson, 1992; Hohe and Gross, 1996) which work well for problems in which the crack path is known a priori. A more general use is possible in the range of gradient plasticity (e.g. Aifantis, 1984; de Borst and Mühlhaus, 1992), which has been recently applied successfully. From the physical point of view, however, it is difficult to define the additional boundary conditions, a problem which arises also within the range of Cosserat continua.

Nonlocal approaches (e.g. Bažant *et al.*, (1984); Pijaudier-Cabot and Bažant, 1987) have the advantage of a wide range of applicability and a physical explanation can be given for the intervening characteristic internal length scale as e.g. the localization thickness in the real material. For these reasons, the nonlocal concept has been chosen in this work.

THE GURSON MODEL

Gurson's model in modified form as given by Needleman and Tvergaard (1994) uses the yield condition

$$\Phi = \frac{\sigma_e^2}{\sigma_M^2} + 2q_1 f^* \cosh\left(\frac{\sigma_{kk}}{2\sigma_M}\right) - \left(1 + (q_1 f^*)^2\right) = 0 \tag{1}$$

where

$$f^* = \begin{cases} f & \text{if: } f \leq f_c \\ f_c + \dfrac{1/q_1 - f_c}{f_f - f_c}(f - f_c) & \text{if: } f > f_c \end{cases} \tag{2}$$

Here, σ_{ij} denotes the macroscopic stress tensor, σ_e the macroscopic equivalent stress, σ_M the actual yield stress of the matrix material, f the real and f^* the effective void volume fraction respectively; q_1, f_c and f_f are material parameters. The void volume rate

$$\dot{f} = (1 - f)\,\dot{\varepsilon}_{kk}^{pl} + \mathcal{A}\,\dot{\varepsilon}_M^{pl} \tag{3}$$

is assumed to consist on the growth of existing microvoids and the nucleation of new voids, where a dot denotes the partial differentiation with respect to time. The first term in eq. (3) can be derived from the condition of plastic incompressibility of the matrix material, while the usual statistical approach is used for the nucleation as suggested by Tvergaard (1989):

$$\mathcal{A} = \frac{f_N}{s_N\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\varepsilon_M^{pl} - \varepsilon_N}{s_N}\right)^2} \dot{\varepsilon}_M^{pl}. \tag{4}$$

Here $\dot{\varepsilon}_M^{pl}$ denotes the equivalent plastic strain rate of the matrix material and f_N, ε_N, s_N are material parameters describing the nucleation of microvoids. The total plastic strain rate is split additive into an elastic part governed by Hooke's law and a plastic part for which an associated flow rule is assumed. An evolution equation for the equivalent plastic

strain rate of the matrix material is obtained by taking into account that the macroscopic and microscopic plastic work rate must be equal:

$$\dot{\varepsilon}_M^{pl} = \frac{\sigma_{ij}\dot{\varepsilon}_{ij}^{pl}}{(1 - f)\sigma_M} \tag{5}$$

Finally, a power-law hardening matrix material is adopted.

NUMERICAL INTEGRATION

Stress Integration

A key point for the implementation of the constitutive equations (1) - (5) is the choice of the integration scheme. Explicit schemes (cf. Ortiz, 1986) have the advantage of easy derivation and implementation. On the other hand they are instable and very small time steps are necessary to obtain an accurate solution. Therefore, implicit algorithms became very important in the last years because of their stability and good accuracy even for large time steps. Zhang and Niemi (1995) examined the generalized mid-point algorithm for the Gurson model in particular and found the Euler backward algorithm to be the best choice. For these reasons the implicit Euler backward scheme has been chosen in the present study, in which the problem to be addressed is that of updating the known state variables $\varepsilon_{ij}, \varepsilon_{ij}^{pl}, \sigma_{ij}, f$ and ε_M^{pl} on the actual time increment. For this purpose it is useful to introduce two scalar variables

$$p = -\frac{1}{3}\sigma_{kk} \qquad q = \sqrt{\frac{3}{2}s_{ij}s_{ij}}, \tag{6}$$

where s_{ij} denotes the stress deviator. The Lagrange multiplier $\dot{\gamma}$ is split into a pressure dependent and independent part

$$\Delta\varepsilon_p = -\dot{\gamma}\frac{\partial\Phi}{\partial p} \qquad \Delta\varepsilon_q = \dot{\gamma}\frac{\partial\Phi}{\partial q} \tag{7}$$

so that all variables of the equation system (1) - (5) can be expressed in terms of four unknown increments $\Delta\varepsilon_p, \Delta\varepsilon_q, \Delta f, \Delta\varepsilon_M^{pl}$. The following nonlinear system of equations, which has to be solved on each Gaussian point, is obtained:

$$\begin{aligned} 0 &= \Phi \\ 0 &= \Delta\varepsilon_p\frac{\partial\Phi}{\partial q} + \Delta\varepsilon_q\frac{\partial\Phi}{\partial p} \\ 0 &= \Delta f - (1 - f)\Delta\varepsilon_p - A\varepsilon_M^{pl} \\ 0 &= \Delta\varepsilon_M^{pl} - \frac{-p\Delta\varepsilon_p + q\Delta\varepsilon_q}{(1 - f)\sigma_M} \end{aligned} \tag{8}$$

The system of equations is solved by Newton's method, where the consistent tangent moduli are derived following the suggestions of Aravas (1987) and Zhang and Niemi (1995). During the constitutive calculations, where stress and state variables are updated, the total strain is known. The elasticity equation yields

$$\sigma_{ij}^{t+\Delta t} = C_{ijkl}(\varepsilon_{kl}^{t+\Delta t} - \varepsilon_{kl}^{t,p}) - K\Delta\varepsilon_p\delta_{ij} - 2G\Delta\varepsilon_q n_{ij} \tag{9}$$

where $n_{ij} = 3s_{ij}/2q$ and K, G being the elastic bulk and shear modulus, respectively. The derivation of the consistent elastoplastic tangent moduli requires the exact linearization of eq. (9) as:

$$d\sigma_{ij} = C_{ijkl}d\varepsilon_{kl} - Kd(\Delta\varepsilon_p)\delta_{ij} - 2Gd(\Delta\varepsilon_q)n_{ij} - 2G\Delta\varepsilon_q\frac{\partial n_{ij}}{\partial\varepsilon_{kl}}d\varepsilon_{kl} \tag{10}$$

where $\partial n_{ij}/\partial\varepsilon_{kl}$ is given by

$$\frac{\partial n_{ij}}{\partial\varepsilon_{kl}} = \frac{G}{q^{tr}}(3\delta_{ik}\delta_{jl} - \delta_{ij}\delta_{kl} - 2n_{ij}n_{kl}) \tag{11}$$

and $d(\Delta\varepsilon_p), d(\Delta\varepsilon_q)$ can be determined by the solution of a linear system of equations. After some algebra the following explicit expression for the consistent tangent moduli is obtained:

$$\frac{\partial\sigma_{ij}}{\partial\varepsilon_{kl}} = d_1\ \delta_{ij}\delta_{kl} + d_2\ \delta_{ik}\delta_{jl} + d_3\ n_{ij}n_{kl} + d_4\ \delta_{ij}n_{kl} + d_5\ n_{ij}\delta_{kl} \tag{12}$$

with the scalar values $d_1 - d_5$ as defined in Zhang and Niemi (1995).

Nonlocal Integration

The simplest form to introduce a localization limiter in a nonlocal sense is obtained by imposing a lower bound on the finite element size. This concept, however, has not only the disadvantage of limiting the mesh refinement, but also the worse deficiency of a dependence on the mesh alignment. A concept for strain softening solids in which all state variables were considered as nonlocal was successfully applied by Bažant *et al.* (1984). This rather complex method, however, posed many difficulties in real problems. Bažant and Pijaudier-Cabot (1988) found that the nonlocal concept can be simplified by considering only the damage variable as nonlocal. A weighting function is applied to the actual porosity increment as follows:

$$\Delta\bar{f}(\boldsymbol{x}_i) = \frac{\int_\Omega \Delta f(\boldsymbol{x})\varphi(\boldsymbol{x}-\boldsymbol{x}_i)d\Omega(\boldsymbol{x})}{\int_\Omega \varphi(\boldsymbol{x}-\boldsymbol{x}_i)d\Omega(\boldsymbol{x})} \tag{13}$$

where the bar on a variable denotes its nonlocal character and $\varphi(\boldsymbol{x})$ the weighting function being the normal distribution, normalized to unity:

$$\varphi(\boldsymbol{x}) = \frac{1}{\pi l_c^2}e^{-(\boldsymbol{x}/l_c)^2} \tag{14}$$

where $\boldsymbol{x}, \boldsymbol{x}_i$ denotes vectors to spatial points, Ω the volume studied and l_c the characteristic length. For finite element application the integral in eq. (13) is replaced by the sum over all elements:

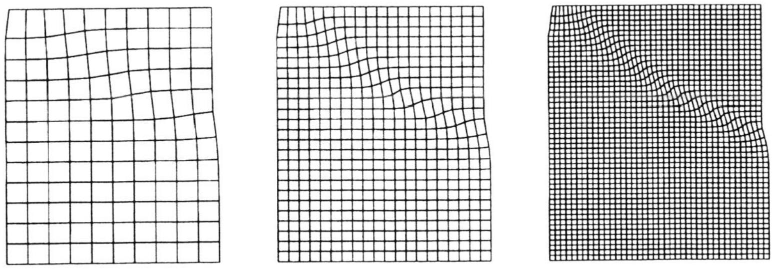
$$\Delta\bar{f}(\boldsymbol{x}_i) = \frac{\sum_{\Omega_j} \Delta f(\boldsymbol{x}_j)\varphi(\boldsymbol{x}_i-\boldsymbol{x}_j)\Delta\Omega_j}{\sum_{\Omega_j} \varphi(\boldsymbol{x}_i-\boldsymbol{x}_j)\Delta\Omega_j} \tag{15}$$

With eqns. (8) and (13), which form a system of integro-differential equations, the problem is now defined.
For the stress integration the classic return mapping algorithm is used. No spatial dependence is taken into account. By this method the consistent tangent operator, known from the 'local' theory, can be used without modification ensuring the quadratic convergence of the global iteration. The nonlocal porosity is computed after local and global convergence in a typical time step. Consequently, the algorithm is only explicit with respect to the nonlocal formulation and very small time steps are necessary to obtain a good accuracy of the global solution. However, some difficulties may arise in the presence of large porosity gradients as e.g. near the crack tip. They are caused by the explicit part of the algorithm which is the reason for jumps between elastic and plastic states with respect to time at some Gauss points. However, this effect does not seem to affect the accuracy, if small time steps are used.

RESULTS

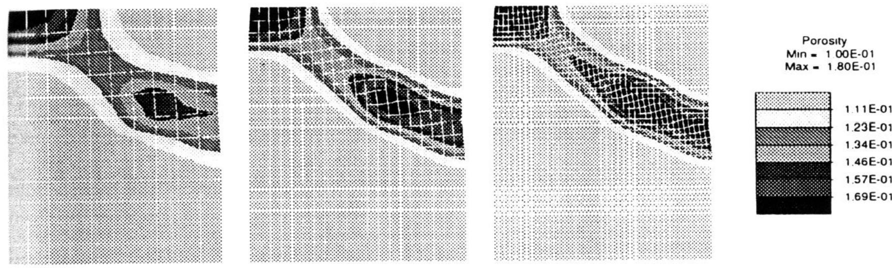
Shear band in a rectangular block

As a first example the presented method is applied to a rectangular block under tensile, strain controlled load (cf. Fig. 1). An imperfection in form of a higher initial porosity in one element was introduced in order to create a shear band localization. To simulate a real material behavior, the parameters for the Gurson model are taken from Klingbeil *et al.* (1993) who found a good agreement with experimental data. Three different meshes



a) b) c)

Figure 1: Localization thickness in three different meshes.



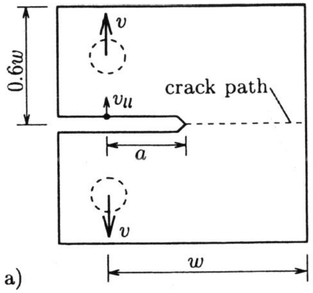
a) b) c)

Figure 2: Porosity contours in three different meshes.

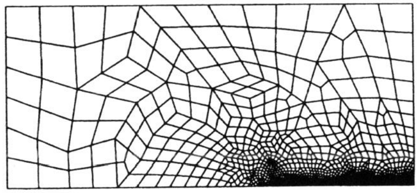
are used where the internal length was chosen equal to the element size in Fig. (1a). The comparison of the meshes in Fig. (1) shows that the shear band has a constant thickness as expected. In contrast to the nonlocal results, the same problem, solved with the local theory, shows that the localization thickness depends strongly on the element size in that sense, that the shear band localizes in one element. The contour plots of the porosity also show the good agreement of the three results (cf. Fig. 2) which proves the capacity of the presented method to describe mode II localizations without mesh dependence.

Ductile crack growth in a CT specimen

In this section, a CT-specimen is taken as example in order to verify the method for mode I simulations. The used geometry is given in Fig. (3a). The same material parameters as in the previous section are applied. The half-discretization as shown in Fig. (3b) is used



a)

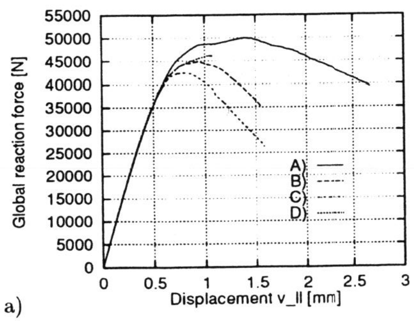


b)

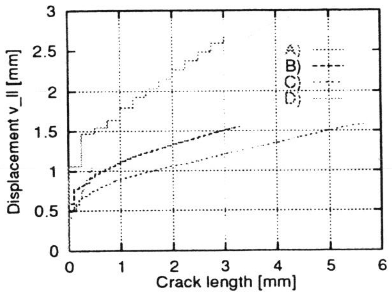
Figure 3: Geometry and discretization of a CT-specimen.

where the element size in the ligament and the characteristic length is varied in four cases as follows:

A) element size $0.25mm$ $l_c = 0$
B) element size $0.10mm$ $l_c = 0$
C) element size $0.05mm$ $l_c = 0$
D) element size $0.05mm$ $l_c = 0.10mm$



a)



b)

Figure 4: Nonlocal result of a CT-specimen.

In Fig. (4a) the load-deflection curves are plotted for all four cases. As expected, the comparison of the local results shows a remarkable mesh dependence. The load-deflection

curve of case B and D in which the mesh size corresponds to the nonlocal internal length, are in good agreement. This result shows that nonlocal solutions are convergent if the element size is chosen less or equal to the characteristic length. In Fig. (4b) the crack length is plotted over the displacement for the same cases. Whereas a qualitative agreement between the corresponding local and nonlocal cases B and D can be observed with respect to the load-deflection curves, this conclusion cannot be drawn for the crack length. This can be explained by the establishment of a diffuse crack in the nonlocal solution. However, this phenomenon seems to affect only the crack initiation phase.

SUMMARY

A finite element method to describe strain softening in a ductile material by means of Gurson's damage model was presented. The well-known mesh dependence encountered by the loss of ellipticity of the governing equations could be avoided by introduction of a nonlocal formulation with respect to the porosity only, which characterizes the damage state in the material. The constitutive relations, leading to a integro-differential system of equations, are solved by a time integration algorithm in two steps. In the first step the stresses are updated implicitly using the classical closest point projection in conjunction with the consistent algorithmic tangent moduli. In the second step the nonlocal damage variable is updated explicitly using the converged local solution.

Two numerical examples demonstrated the convergence and capacities of the presented method. However, some difficulties arising must be mentioned if the element size is chosen much smaller than the internal length. The reason for this behavior seem to be caused by the explicit part of the algorithm; numerical stability and sufficient accuracy of the solution can be obtained only for very small time steps resulting in a large amount of computing time. Investigations to improve the numerical scheme are in progress.

REFERENCES

Aifantis, E.C. (1984). On the microstructural origin of certain inelastic models. *J. Eng. Mat. Tech. ASME*, **106**, 326-334.

Aravas, N. (1987). On the numerical integration of a class of pressure-dependent plasticity models. *Int. J. Num. Meth. Eng.*, **24**, 1395-1416.

Bažant, Z., Belytschko, T.B. and Chang, T.P. (1984). Continuum theory for strain-softening. *J. Eng. Mech. Div. ASCE*, **110/12**, 1666-1692.

Bažant, Z.P. and Pijaudier-Cabot, G. (1988). Nonlocal continuum damage, localization instability and convergence. *J. Appl. Mech.*, **55**, 287-293.

de Borst, R. (1991). Simulation os strain localization: A reappraisal of the Cosserat continuum. *Eng. Comp.*, **8**, 317-323.

de Borst, R. and Mühlhaus, H.-B. (1992). Gradient-dependent plasticity: Formulation and algorithmic aspects. *Int. J. Num. Meth. Eng.*, **35**, 521-539.

Gurson, A.L. (1977). Continuum theory of ductile rupture by void nucleation and growth: part I - yield criteria and flow rules for porous media. *J. Eng. Mat. Tech.*, **99**, 2-15.

Hohe, J. and Gross, D. (1996, to appear). *Analysis of ductile crack growth by means of a damage yield strip model.* In: Localized Damage IV, (Ed. M.H. Aliabadi), Comp. Mech. Publication, Southampton.

Klingbeil, D.,Künecke G. and Schicker (1993). On the Application of Gurson's Model to Various Fracture Mechanics Specimens, *Report BAM-1.31 93/3, Laboratory 1.31, BAM Berlin.*

Miehe, C. and Schröder, J. (1994). Post-critical discontinuous localization analysis of small-strain softening elastoplastic solids, *Arch. Appl. Mech.*,**64**, 267-285.

Mühlhaus, H.B. (1989). Application of Cosserat theory in numerical solutions of limit load problems, *Ing. Arch.*, **59**, 124-137.

Needleman, A. (1988). Material rate dependence and mesh sensitivity in localization problems, *Comp. Meth. Appl. Mech. Eng.*, **67**, 69-86.

Larsson, R., Runesson, K. and Ottosen, N.S. (1993). Discontinous displacement approximation for capturing plastic localization, *Int. J. Num. Meth. Eng.*, **36**, 2087-2105.

Leblond, J.B., Perrin, G., Devaux, J. (1994). Bifurcation effects in ductile metals with nonlocal damage. *J. Appl. Mech.*, **61**, 236-242.

Oliver, J. and Simo, J.C. (1994). Modeling strong discontinuities by means of strain softening constitutive equations. *Proc. EURO-C 1994 Int. Conf. Computer Modelling of Concrete Structures* Eds H. Mang et al., Pineridge Press, Swansea, 363-372.

Ortiz, M. and Simo, J.C. (1986). An analysis of a new class of integration algorithms for elastoplastic constitutive relations. *Int. J. Num. Eng.*, **23**, 353-366.

Pietruszak, S. and Mroz, Z. (1981). Finite element analysis of deformation of strain softening materials,*Int. J. Num. Meth. Eng.*, **17**, 327-334.

Pijaudier-Cabot, G, Benallal, A. (1993). Strain localization and bifurcation in a nonlocal continuum. *Int. J. Solids Structures*, **30**, 1761-1775.

Simo, J.C. (to appear). Topics on the Numerical Analysis and Simulation of Plasticity. *E. Science Publishers B.V., North Holland.*

Sluys, L.J. and de Borst, R. (1992). Wave propagation and localisation in a rate-dependent cracked medium: model formulation and one-dimensional examples, *Int. J. Solids Struct.*, **29**, 2945-2958.

Tvergaard, V. (1989). Material Failure by Void Growth to Coalescence. *Advances in Appl. Mech.*, **27**, 83-151

Tvergaard, V. and Hutchinson, J.W. (1992). The relations between crack growth resistance and fracture process parameters in elastic-plastic solids, *J. Mech. Phys. Solids*, **40**, 1377-1397.

Needleman, A. and Tvergaard V. (1994). Mesh effects in the analysis of dynamic ductile crack growth. *Eng. Frac. Mech.*, **47**, 75-91.

Zhang, Ch. and Gross, D. (1993). Analysis of Ductile Crack Growth by a Simple Damage Model. *Transactions of the 12th International Conference on Structural Mechanics in Reactor Technology.* Edited by K. Kussmaul, Elsevier Science Publishers, Amsterdam.

Zhang, Ch. and Gross, D. (1994). Ductile crack analysis by a cohesive damage zone model. *Eng. Frac. Mech.*, **47**, 237-248.

Zhang, Z.L and Niemi (1995). A class of generalized mid-point algorithms for the Gurson-Tvergaard material model. *Int. J. Num. Meth. Eng.*, **38**, 2033-2053.