NON - CLASSICAL PROBLEMS OF COMPOSITE FAILURE

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ABSTRACT.

The report is devoted to presentation a quite brief description (as an annotation) of results of investigation of non-classical problems of fracture or failure mechanics of composites and corresponding mechanisms, which were obtained by author and his collaborators in the Institute of Mechanics (Kiev) starting with 1967.Three following problems and fracture or failure mechanisms relatively sequentially are considered: 1. fracture or failure in compression along reinforcing elements; 2. fracture or failure in the form of "bearing strain of ends" in compression; 3. fracture in the form of "separating into fibers" in compression or fension along reinforcing elements. The results on the three problems relate only to mechanics of composite materials, above mentioned mechanisms don't arise in the traditional mechanics of metals and alloys.

KEYWORDS.

Non-classical problems of fracture mechanics, micromechanics of composites, three-dimensional theory of deformable bodies stability.

INTRODUCTION.

Nowadays in fracture mechanics main concepts and approaches to formulation of fracture criteria were established: basic Griffith theory on the mechanics of brittle fracture, the concepts of quasibrittle fracture (Irwin, Orwan and others), Griffith energetic criterion of fracture and equivalent to it (but rather easy realized) Irwin force criterion, concept of non-depending on the contour of integrating (J - integral, Γ - integral) of an integral (Eshelby, Cherepanov and Rice) and the criterion of critical crack opening. The mentioned approaches and concepts are assumed and realized: 1 *when tension and shear (except compression) occur in the vicinity of a crack;* 2 *when in the process of deforming the sharp change of the body configuration does not occur (loss of stability does not precede to fracture);* 3 *when the sufficient change of deforming character before the fracture process (precritical growth of the crack is not present) does not occur.* In the defined sense the mentioned results and problems may be considered as classic problems of fracture mechanics, and the following investigations may be related to them nowadays.

1. Determination of stress intensity factors for bodies of a complex shape, containing cracks under various force, thermal and electromagnetic influences. For obtaining the mentioned results analytical, numerical (using a computer), experimental and experimental-theoretical methods are utilized. Results of these investigations (stress intensity factors) combined with the

mentioned fracture criteria give the necessary information of fracture of materials and structural elements in cases when these fracture criteria are applicable.

2. Experimental investigations of complex cases of fracture of materials and structural elements finalizing, as a rule, by results of a discriptive character without a proper analysis and attempts to formulate new criteria, corresponding to phenomena under consideration.

It should noted, that nowadays, the overwhelming number of publications relates to the classical problems of fracture mechanics in a previously mentioned sense. Apparently, due to the noted situation many scientists came to the conclusion of existance of the ideological crisis in fracture mechanics on the modern stage of its development.

The second trend, which we conditionally will call as non-classical problems of fracture mechanics, is connected with the following investigations.

1. The study of new mechanisms of fracture which are not described within the framework of the above mentioned concepts using appropriate analysis and the attempt to formulate new fracture criteria according to phenomena under consideration.

2. The investigation of separate problems for materials and structural elements as applied to new mechanisms of fracture under investigation with application of corresponding specially formulated fracture criteria.

It should be noted, that the elucidated classification (division on classical and nonclassical problems) is enough conditional and is not always strictly determined. Nevertheless, it relatively accurate determines the direction of investigations and the form of their novelty, that is presented as the quite important in the analysis of investigation results. We also note, that a number of fracture mechanisms is sufficiently expanded with allowance for microstructure of materials at various levels of its discription; this quantity, first of all, relates to fracture mechanics of composite materials, the account of microstructure of which is character.

It is typical the application of quite approximate calculating schemes for scientists, investigating non-classical problems and fracture mechanics mechanisms ; in case of composite materials such approximate schemes and models are used for an analysis of fracture in microstructure of composite. The application of approximate schemes and models leads to significant qualitative errors and, in many cases to quantitative distinctions; therefore, it is complecated to carry out the analysis of the mentioned problems and mechanisms of fracture mechanics in case of using approximate schemes and models. The noted position determines the significance of results on investigation of non-classical problems and mechanisms of fracture, which are obtained with sufficiently strict calculating schemes and models.

Two following features are typical for the mentioned investigation of the author and his co-author (in comparison with investigations of other authors).

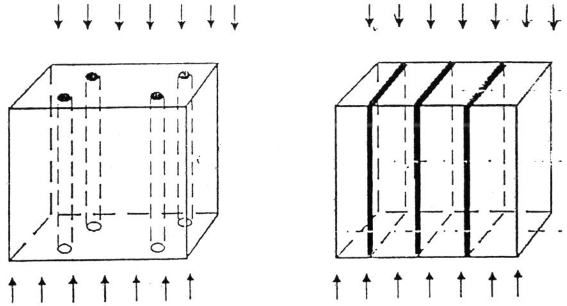
The *first* - are investigations have been carried out with application of the most strict and accurate formulations within the framework of deformable bodies mechanics; so, in case of investigation of the phenomenon of stability loss the three-dimensional linearized theory of stability of deformable bodies was applied and in case of investigation of the stress-strain state three-dimensional equations of statics of deformable bodies were applied. This comment completely relates to problems 1-3, associated with fracture mechanics of composites.

The *second* - is the analysis of problems 1-3 of fracture mechanics, considered in the present article and related to composite materials has been carried out within the framework of the continual approximation (a homogeneous orthotropic body model with averaged constants - three-dimensional formulation) and within the framework of a piece-wise homogeneous medium model (three-dimensional equations for the filler and the binder, conditions of continuousness at the boundary of interface).

We note, that the mentioned piecewise-homogeneous medium model (in three-dimensional formulation) is the most strict and accurate within the framework of deformable bodies mechanics as applied to composite materials; some refinements (specification of results) are possible at the expense of consideration of other boundary conditions on surfaces of interface and other determining equations for the filler and the binder.

1.FRACTURE OF COMPOSITES IN COMPRESSION ALONG REINFORSING ELEMENTS

Fracture mechanism under consideration is realized in "the most clean" form in unidirectional fibrous (Fig.1a) and laminated (Fig.1b) composites in compression along



a *b*

Fig.1

reinforcing elements; it is also realized in orthogonal-reinforced composites and in composites with slight bending in a structure. In these cases the beginning of fracture process is indentified with stability loss in a structure of a material, when the value of critical load (strength limit for this fracture mechanism) is determined not by dimensions and the shape of a specimen or a structural element, but by parameters characterizing the structure of composite (concentration of the filler and the binder etc.). A simple criterion of realisation of fracture mechanism under consideration exists - critical load, corresponding to stability loss in the structure of material, must be less then critical load, corresponding to stability loss of the whole specimen or a structural element.

In investigation (by virtue of internal character of instability) "an infinite" composite is considered and as a result of solving of a corresponding problem on eigenvalues the dependence of load on the parameter of wave formation is determined; typical dependences for four various laminated composites are presented in Fig.2. It is commonly assumed, that the phenomenon of stability loss in the structure of a material takes place, if the curve of dependence of the loading parameter on the wave-formation parameter for a given material has minimum with $\alpha \neq 0$. In the case P_{cr} and α_{cr} corresponds to the mentioned minimum, presented in Fig.2 these values for the material with the index"4"; with the assumed approach P_{cr} correspond to the theoretical strength limit in compression. For investigation of the mentioned fracture mechanism many authors used various aproximate models, based on applied two- and single-dimensional theories of plates and rods. By virtue of complexity of the

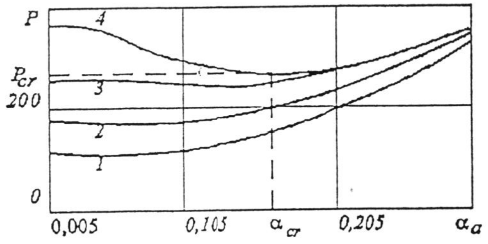


Fig.2

phenomenon under consideration, aproximate models and approaches lead to essential qualitative errors and, in many cases, to quantiative differences; in this connection it is complecated to carry out the analysis of the mentioned fracture mechanism using approximate models.

For the first time in author's studies the analysis of the mentioned fracture mechanism has been carried out with application of the three - dimensional linearized theory of stability of deformable bodies for elastic (brittle fracture) and plastic (fracture of composites with a metal matrix) models. Such an approach eliminates the occurance of errors, typical for various approximate models. Two approaches with application on the three-dimensional linearized theory of stability of deformable bodies are developed. The first approach is based on application of a homogeneous anisotropic body model with averaged constants; in this case, parameters, characterizing the structure of composite, are included in values of averaged constants. The second approach is based on application of a piecewise-homogeneous medium model, when, separately for the filler and the binder equations of the three-dimensional linearized theory of stability of deformable bodies are used and conditions of continuousness of stress vectors and displacements on the boundary of interface of the filler and the binder are satisfied. Within the framework of the first approach for determination of the theoretical strength limit in compression along the axis Ox_1 in case of internal fracture of a material the following expression is obtained.

$$(\Pi_1^-)_T = G_{12} \tag{1}$$

In case of near-the-surface fracture of a material in uniaxial compression along the axis Ox_1 for determination $(\Pi_1^-)_T^{\Pi}$ of the theoretical strength limit in near-the-surface fracture the following expression is obtained

$$(\Pi_1^-)_T^{\Pi} \approx G_{12}[1 - \frac{G_{12}^2}{E_1 E_2}(1 - \nu_{13}\nu_{31})(1 - \nu_{23}\nu_{32})] \tag{2}$$

In (1) and (2) via $E_1, E_2, \nu_{13}, \nu_{31}, \nu_{32}, \nu_{23}$ and G_{12} averaged elastic constants for an orthotropic body are denoted. Results (1) and (2) correspond to the case of a planar strain in a plane x_1Ox_2, when the reinforcing elements are directed along the axis Ox_1 (Fig.1a and 1b) in uniaxial compression along the axis Ox_1 as applied to brittle fracture. The results for plastic fracture of composites with a metal matrix and corresponding result in case of three-axial loading are also obtained. The continual theory under consideration, constructed within the framework of the mentioned first approach, along with determination of theoretical strength limits (results of the type (1) and (2)) also provides a way of explanation of the nature of fracture for the case under consideration. For determination of the nature of fracture (fracture mechanism) within the framework of the mentioned theory the character of propagation of small perturbations, caused by the initial stage of fracture is considered.

We present results of experimental investigations which testify on specific character of fracture in the case under consideration. As an example Fig.3 shows the character of fracture of a specimen from a glass-cloth-base laminate in compression perpendicular to reinforcing elements (perpendicular to a plane of reinforcement). Also as an example presented in Fig.4 is the character of fracture of specimens from a unidirectional glassplastic in uniaxial compression. There are other numerous publications, which also noted an analogous fracture mechanism of composites in compression. The situation, when fracture occurs over planes, which are almost perpendicular to direction of compressive load is common to mentioned investigations. Non-classical of fracture mechanism under consideration, lies in the fact that in case of metals and alloys in uniaxial compression fracture occurs under the angle about to direction of compressive load.

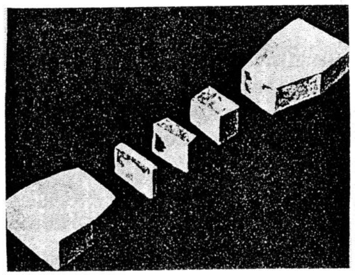


Fig.3

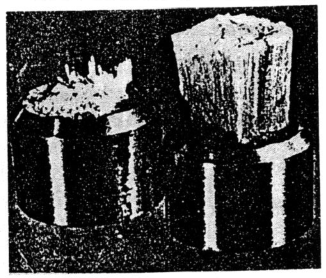


Fig.4

We note, that in these investigations (Fig.3, 4) phenomena, associated with fracture near ends were not considered; so, in case in Fig.3 in speciments near ends square of transverse section increased and in case in Fig.4 ends of specimens were fixed into rigid casings.

The continual theory of internal and surface fracture, under consideration, in compression, the fracture process (e.g. for brittle fracture) is represented in the following manner. *Initially near-the-surface fracture occurs with quantities of external load slightly less quantities of a minimal over value averaged shear modulus, which is determined as for a homogeneous anisotropic body. On further increasing of load (up to quantities slightly less over the value of a minimal averaged shear modulus) near defects internal fracture occurs. On reaching by external load the value of a minimal averaged shear modulus internal fracture occurs, which propagates avalanche-like (instantly) over planes, almost perpendicular to direction of external load.*

Analogous results are also obtained for plastic fracture; in this case a tangent averaged shear modulus, calculated in the moment of stability loss serves as a given shear modulus. On such approach (within the framework of the continual theory) long-term fracture of composites was also considered. Main results on the continual theory under consideration, corresponding to the mentioned at the beginning of this section the first approach are presented in cited monograph.

Within the framework of the second approach (a piecewise-homogeneous medium model) mechanisms of internal and near-the-surface fracture for fibrous (Fig.1a) and laminated (Fig.1b) composites with a polymer and metal matrix in unidirectional compression are investigated. For fibrous composites cases of one fiber (composite with a small concentration of the filler), for two fibers, infinite periodic series of fibers and a doubly-periodic system of fibers as applied to cases of internal and near-the-surface fracture are investigated. Main results are presented in cited monographs and are also partially incorporated into doctors thesis (the seconds doctorate degree) of I.Yu.Babich, Vic.N.Chekhov and Yu.N.Lapusta.

2. FRACTURE IN THE FORM OF BEARING STRAINS OF ENDS IN COMPRESSIONS.

Mechanisms with fracture in bearing strains of ends are sufficiently wide spread among various mechanisms, realized in fracture of composites. It lies in the fact, that, for instance, in uniaxial compression of speciments and structural elements from composites local fracture of a material near ends occurs, and the mentioned fracture, therewith, does not propagate far from ends and decreases in moving from them. As an example, presented in Fig.5 is the character of fracture in bearing strains of ends for a unidirectional boronaluminium composite (with 50% contens of boron fibers) in uniaxial compression; additional information are presented in cited monographs , where publications, containing results corresponding to those in Fig.5 are noted.

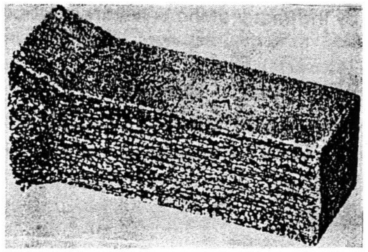


Fig.5

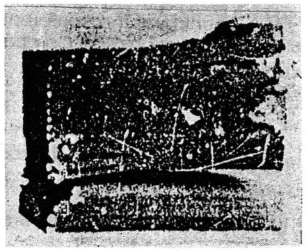


Fig.6

We note, that in experimental investigations of fracture mechanisms, considered in the previous section, design and technological techniques, excluding occurence of fracture mechanism bearing, strains of ends were utilized. As it was noted in the previous section, in particular, the development of bearing strains is eliminated by winding of specimens ends over lateral surfaces near ends and encapsulating them into casings from more rigid material (i.g. in Fig.4), via expansion of a square of transverse section near ends (for example, in Fig.3) ets. We also note, that experimental investigation of fracture phenomenon in bearing strains of ends is rather complex and not always leads to single valid results. It is associated with obtaining the result usually during experimental investigations (a photograph of a fractured near an end specimen or the quantity of strength limit), corresponding to already completed fracture process with bearing strains of ends. For investigation or description of the phenomenon under consideration it is necessary to obtain an information on initial stage of fracture (in an effort to find out reasons and a mechanism of a corresponding phenomenon); but a complex illustration of a destroyed near an end specimen (corresponding to the final stage of fracture) does not permit to indentify processes on the initial stage of fracture.

A mechanism of stability loss in the structure of composite, which has a character of near-the-surface stability loss near a loaded end with forms of stability loss, which attenuate in moving of the end is the most probable on the initial stage of fracture in bearing strains of ends (for unidirectional composites or orthogonal-reinforced composites in loading along axes of symmetry). Using such interpretation of the phenomenon a series of results with application of various approximate calculating models, based on single - and two-dimensional models of plates and rods are obtained.

In the studies of the author, for the first time, the analysis of the mentioned fracture mechanism was carried out with application of the three-dimensional linearized theory of stability of deformable bodies. Within the framework of such an approach results for the continual theory and a piecewise-homogeneous medium model are obtained. The continual theory for elastic (brittle fracture) and plastic (fracture of a composite with a metal matrix) models is developed; theoretical strength limits, corresponding to fracture in bearing strains of ends, therewith, are determined. As an example, we illustrate the quantity $(\Pi_3^-)_T^{SM}$ of the theoretical strength limit in compression in unixial compression along the axis Ox_3 of a unidirectional composite in case of brittle fracture

$$(\Pi_3^-)_T^{SM} \approx G'[1 - \frac{G'^2}{EE'}(1-\nu^2)(1-\nu'^2\frac{E}{E'})] \qquad (3)$$

In (3) as applied to a transverse-isotropic body model with an axis of isotropy, directed along the axis Ox_3 (for instance, fibrous unidirectional composite) the notation is introduced: E' - is Young modulus along the axis Ox_3 ; E- is Young modulus in a plane x_3 = const ; ν' and ν - are corresponding Poisson ratios; $G' = G_{23} = G_{13}$ - is a shear modulus. In conformity with (1) in case under consideration $(\Pi_3^-)_T$ - theoretical strength limit in compression along the axis Ox_3 in internal fracture has the following form

$$(\Pi_3^-)_T = G' \qquad (4)$$

In case of a fibrous unidirectional (along the axis Ox_3) composite inequality takes place

$$E' >> G' \qquad (5)$$

From (3)-(5) in case under consideration we obtain

$$(\Pi_3^-)_T > (\Pi_3^-)_T^{SM} \qquad (6)$$

Thus, in conformity with the continual theory the fracture process in bearing strains of ends begins slightly earlier, than internal fracture, corresponding to fracture of the whole specimen. This conclusion is in complete agreement with experimental results and explains the necessity to take additional design and technological techniques, partially described above, in order to obtain in experimental investigations results for a total (internal) fracture of the whole specimen. Analogous results are obtained for plastic fracture (composites with a metal matrix). Results on the continual theory are presented in cited monographs .

Using the mentioned approach a series of results for a piecewise-homogeneous medium models as applied to the simplest problems for laminated composites are obtained; these results were included into a doctoral thesis the second doctorate degree of Yu.V.Kohanenko.

3. FRACTURE IN THE FORM OF "SEPARATION OF FIBERS".

Fracture mechanism under consideration is not observed for homogeneous materials (metals and alloys) and is specific to composites with a distinct direction of primary reinforcement. This mechanism consists in separation of a material into separate parts along direction of action of compressive load under condition, that compressive load is directed along the direction of reinforcing in unidirectional composites and in one of directions of reinforcing in composites with obviously expressed direction of primary reinforcement.

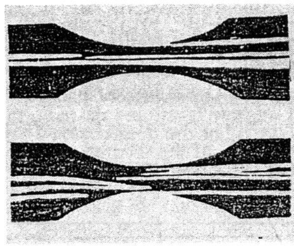


Fig.7

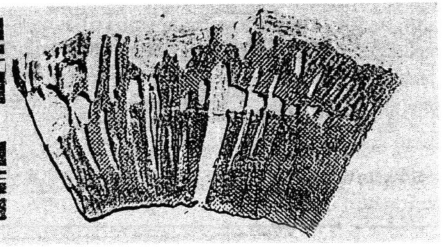


Fig.8

As an example Fig.7 shows the fracture character of a unidirectional glass-plastic in compression along the direction of reinforcement, Fig.8 shows the fracture character of a glass-plastic of longitudinal-transverses winding in compression along the direction of reinforcement and Fig.6 shows the fracture character of a unidirectional boron-aluminium composite in compression along the direction of reinforcement; additional information is presented in cited monographs, where publications, containing results Fig.6-8 type are also presented. It should be noted, that fracture Fig.6 type occured as a result af additional techniques, excluding occurance of fractures Fig.4 type. Fracture mechanism, considered in this section, following cited monographs, arbitrarily we will call "separation into the fibers". We note, that the mentioned fracture mechanism was repeatedly observed in fracture of wood. It is of interest to note, that similar phenomena are also observed in tension of a composite along the direction of primary reinforcement; references on sources, when cases of fracture "separation into fibers" type in tension, are presented in cited monographs.

This note has sufficient importance in connection with association of some authors the occurance of "separation into fibers" in compression with a transverse elongation (at the expense of Poisson ratio) reaching limited quantities. The mentioned approach can't explain the occurance of "separation into fibers" in tension, in the context of the transverse compression taking place in this case, and the material fractures along the direction of action of tensile load (perpendicular to transverse shortening). In this connection we come to conclusion, that in case of fracture ("separation into fibers") under consideration, a rather complex fracture mechanism in the structure takes plase. We also note, that even in case, of one would think, successful application of a concept of limited transverse elongation in case of compression of unidirectional composites it remains an unclarified a mechanism of fracture in the structure, since the concept of limited transverse elongation operates with integral characteristic and the force does not act in this direction.

Presented in Fig.6-8 inherent for fracture, is a total fracture over planes and surfaces, which are located along reinforcing elements. Therefore, it is quite logical to anticipate (apparently, it is beyond doubts) that the mentioned type of fracture occurs as a result of action of forces, directed perpendicular to reinforcing elements. Origin of this fracture mechanism as well as in compression along reinforcing elements, as in a series of cases in tension also along reinforcing elements allow to exclude from the fracture mechanism under consideration, of various type phenomena, associated with stability loss. Nevertheless, in uniaxial compression-tension along reinforcing elements external load is applied only along reinforcing elements; therefore, the fracture type under consideration may originate only at the expense of internal forces (stresses) *which act perpendicular to the direction of external load and originate as a result of the influence of the microstructure of composite*. This, it is necessary to determine a mechanism of origin of the mentioned stresses in the microstructure of a composite and find out the possibility reaching of these stresses (in microstructure) limited quantities, corresponding to the mentioned fracture type, on condition, that stresses along the direction of reinforcement (along the direction of applied external loads) do not reach limited quantities.

It is know from experimental investigations on micromechanics of composites, that in the structure of a composite owing to different reasons of technology, curvings of reinforcing elements originate; a series of photographs, illustrating the mentioned phenomenon are presented in cited monographs and other publications. On this basis in studies of the author for the first time it was proposed the explanation of fracture mechanism, by "separation into fibers" *at the expense of internal stress, which originate due to curvinds in microstructure and act on surface elements the normal to which coincides with the normal to curve interface of media*. Let us consider the proposed approach as applied to a microelement of a composite in Fig.9 including curved interface of a reinforcing element and a matrix; all values, related to the reinforcing element,are marked with the index "*a*" and all values, related to the matrix, are marked with the index "m". In Fig. 9 the notations are introduced: H - is a rise and Λ - is the length of half-wave of curvatures (it is considered for an example of periodic curvature); $\bar{n}$ and $\bar{\tau}$ -are unit vectors of the normal and the tangent to the curved interface; $\sigma_{11}^{(a)}$ and $\sigma_{11}^{(m)}$ - are stresses along the direction of external load, which

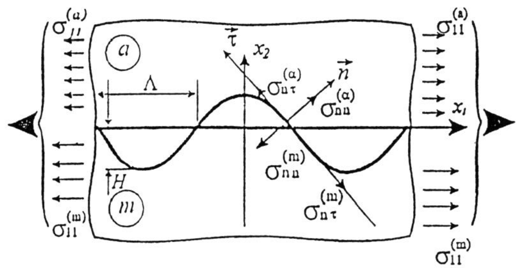


Fig.9

are balanced with external load; $\sigma_{nn}^{(a)}$, $\sigma_{n\tau}^{(a)}$, $\sigma_{nn}^{(m)}$ and $\sigma_{n\tau}^{(m)}$ - are stresses in the reinforcing element and the matrix, which are applied to the curved boundary and are self-balanced within each period of curvatures. It should be noted, that the last stresses change (sense of curvature) mathematical sign after a half-period of curving along the axis Ox_1; therefore, in tensile or compressive external load in separate parts of the curved boundary always originate tensile stresses, which may exceed adgesion strength.Analogous considerations take place relative to tangential stresses.

Let us consider certain qualititive estimations. For not very large curvatures conditions are satisfied

$$H << \Lambda \tag{7}$$

It is obvious, that for composites, reinforced in the direction of the axis Ox_1 (Fig. 9) in loading under consideration the unequality has place

$$|\sigma_{11}^{(m)}| << |\sigma_{11}^{(a)}| \tag{8}$$

by virtue of the natural unequality

$$E_m << E_a , \tag{9}$$

where E_m and E_a - are Young moduli of reinforcing elements and the matrix. In connection with the type of loading under investigation in brittle fracure relations are also valid

$$\sigma_{11}^{(a)} \approx E_a k_a ; \qquad \sigma_{11}^{(m)} \approx E_m k_m ; \tag{10}$$

where: k_a and k_m - are numerical coefficients.

By virtue of the condition of continuity of the stress vector on the curved boundary the relations take place

$$\sigma_{nn}^{(a)} = \sigma_{nn}^{(m)} ; \qquad \sigma_{n\tau}^{(a)} = \sigma_{n\tau}^{(m)} ; \tag{11}$$

By virtue of the condition (11) on the interface for stress $\sigma_{nn}^{(m)}$ and $\sigma_{n\tau}^{(m)}$ we can not obtain estimations of the second type of the expression (10); therefore for these stresses in spite of the presence of the unequality (9) we can not obtain now the estimation of the type (8). Thus, on the interface stresses $\sigma_{nn}^{(m)}$ and $\sigma_{n\tau}^{(m)}$ may be considerably larger in value, than one might obtain on the basis of estimation of the type of the second expression (10). In connection with the above elucidated it is necessary to clarify the following situation.

Let us consider, that stresses $\sigma_{11}^{(a)}$ and $\sigma_{11}^{(m)}$ (Fig. 9) are less of corresponding strength limits separately for reinforcing elements and the matrix

$$|\sigma_{11}^{(a)}| < \Pi_1^{\pm(a)} ; \qquad |\sigma_{11}^{(m)}| < \Pi_1^{\pm(m)} ; \tag{12}$$

The condition (12) eliminates from the origin of fracture of a composite owing to breakage of reinforcing elements and the matrix under the action of stress, directed along direction of external load; these conditions may be satisfied at the expense of limitations by the value of external load. It is necessary to prove, that rational limits of change of values H and Λ (Fig.9) are exist, satisfying the condition (7), where by conditions (12) are satisfied, and, for stresses $\sigma_{nn}^{(m)}$ and $\sigma_{n\tau}^{(m)}$ on the interface the following conditions are satisfied

$$|\sigma_{nn}^{(m)}| = A_p^+ \quad \text{or} \quad |\sigma_{n\tau}^{(m)}| = A_c^+ ; \tag{13}$$

here: A_p^+ and A_c^+ - are limits of adhesion strength in tension and shear as applied to the interface of reinforcing elements and the matrix. We note, that for majority of structural composites by virtue of presence of defects the condition is satisfied

$$A_p^+ < \Pi_1^{+(m)} \tag{14}$$

Thus, for the proposed explanation of "separation into fibers" fracture mechanism it is necessary to prove, that stresses $\sigma_{nn}^{(m)}$ *and* $\sigma_{n\tau}^{(m)}$ *(on the interface) no less of stresses* $\sigma_{11}^{(m)}$ *(Fig.9) may originate under conditions (7) and (12) are satisfied and the parameters* H *and* Λ *(Fig.9) change into real limits.*

We note, that in solving the mentioned problem in case of application of the continual theory it is necessary to construct such a theory, which would allow to determine selfbalanced (within curvatures) stresses; usually, continual theory provides determination of a stress on elements, dimensions of which essentially exceed dimensions of curvatures.

The above formulated problem has been positively solved within the framework of the three-dimensional continual theory.

Within the framework of the strict three-dimensional piese-homogeneous medium model for laminated and fibrous composites the above formulated problem has been also positively solved in studies of the author and S.D.Akbarov. Results on this trend are presented in cited monographs and in the Special Issue, and are also included into S.D.Akbarov doctoral thesis (the second doctorate degree).

It should be noted that the above considered problem was for a periodic curvature. In Fig.9 it corresponds to the periodic curvature along the axis. Analogous results are obtained as applied to local curvatures also.

It must be noted that author and his collaborators are investigating other non-classical problems of fracture or failure of composites, metals and alloys and corresponding mechanisms in the Institute of mechanics (Kiev). Additional information may be obtained from cited literature.

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