LIMIT EQUILIBRIUM OF ELASTO-PLASTIC PIECEWISE-HOMOGENEOUS CYLINDRICAL SHELLS WITH NON-THROUGH CRACKS

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ABSTRACT

A procedure for approximate solution of the problems of stressed state and limit equilibrium of piecewise-homogeneous shells with non-through (surface or inner) cracks is proposed. It takes into account plastic strain zones in the vicinity of cracks. The essence of this procedure lies in the fact that, using an analogue of the Leonov-Panasyuk-Dugdale model, the three-dimensional problem is reduced to a two-dimensional one. The latter problem is reduced to a system of singular integral equations with unknown limits of integration and discontinuous right-hand terms. An algorithm for the numerical solution of such systems (in conjunction with conditions of plasticity and finiteness of forces and momemts in the vicinity of a crack) is presented. The dependence of the crack opening and the size of plastic zone on loading, geometric and physico-mechanical parameters is investigated for closed piecewise-homogeneous cylindrical shells built up from two dissimilar semi-infinite shells with a surface crack in one of them.

KEYWORDS

Piecewise-homogeneous structure, elasto-plastic shell, non-through crack, singular integral equations.

INTRODUCTION

The importance of investigating the stress distribution in the vicinity of cracks in solids of various configurations and under various load conditions increases with the extension in the field of practical uses of fracture criteria. At present the stress distribution in the vicinity of through cracks in homogeneous shell structures is adequately studied (Panasyuk et al., 1976; Osadchuk, 1985). There are considerably fewer investigations devoted to anisotropic and nonhomogeneous shells with through cracks (Nykolyshyn, 1993a; Kushnir and Olijnyk, 1993). In the case of non-through crack the problem becomes three-dimensional and, if the development of plastic strains is taken into account, difficult to solve. Therefore, simplified models which are consistent with experimental data, deserve attention. For instance, the δ_c-model of Leonov-Panasyuk (1959) and the model of Dugdale (1960) work well for plane thin-walled structural elements, where fracture is preceded by the development of large zones of plastic strains. This was shown by Trufyakov et al. (1975) in experiments on model specimens of welded joints of structural steel with low and medium strength. It should be noted that a satisfactory agreement of theoretical and experimental results was also observed in the case, when the configuration of plastic zones in the test specimens differed from that assumed in the δ_c-model. An analogue of this model was used for investigation of stressed state and limit equilibrium of shell structures, in which the membrane stresses far exceed the bending ones. Shallow

(Erdogan and Ratwani, 1972), non-shallow isotropic (Osadchuk et al. 1984) and transversly isotropic shells (Nykolyshyn, 1993a) weakened by through and non-through cracks have been studied.

However, a major part of real elasto-plastic shell structures is nonhomogeneous. Investigations of stressed state and limit equilibrium of such shells are unknown to the authors. Therefore, an analogue of the δ_c-model is proposed below for the study of the opening of non-through cracks in a piecewise-homogeneous cylindrical shell.

GOVERNING RELATIONS AND EQUATIONS

Consider a piecewise-homogeneous closed cylindrical shell built up from two dissimilar semi-infinite shells. One of its parts is weakened by a longitudinal inner crack with a length $2l_0$. The shell is referred to the three orthogonal coordinates (α, β, γ) being the principal curvature lines of the median surface and the outer unit normal to it, respectively. The origin is placed in the middle of the crack. The known conditions of ideal mechanical contact are satisfied at the interface $\alpha = \alpha^*$ ($\alpha^* = l^*/R$, R is the radius of the median surface). The crack is located in the section $\beta = 0$ and is bounded by lines parallel to the coordinate ones. $2d_1$ and $2d_2$ are the distances between the crack and the inner and outer surfaces of the shell, respectively (see Fig. 1). It is assumed that the shell is under external loading and the faces of crack may be subjected to self-equilibrated forces and moments. The study is restricted to the case of external loading symmetric about the crack, as well as to symmetric forces and moments acting on the crack faces. During the deformation process the faces of crack do not come into contact. Deep cracks only are considered ($d_3 = d_1 + d_2 \leq 0.6h$, where $2h$ is the shell thickness). The size of the crack, the level of external load and the properties of material are assumed to be such that plastic strain develops through the whole shell thickness in a narrow strip in the vicinity of the crack.

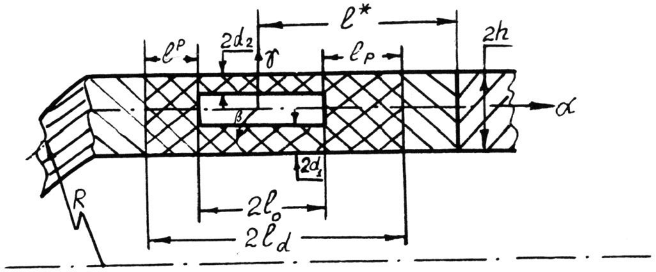

Fig. 1. Schematic illustration of a model.

According to the δ_c-model, the zones of plastic strains are substituted by surfaces of discontinuity of elastic displacements and rotation angles, while the action of material in the plastic zone on the material in the elastic one is replaced by appropriate forces and moments. It is assumed that the prolongation of the crack to the inner and outer shell surfaces, i.e. the region $\beta = 0$, $\alpha \in]-\alpha_0; \alpha_0[$, $\gamma \in [-h; -h+2d_1] \cup [h-2d_2; h]$ is subjected to constant stresses $\sigma^0 = (\sigma_B + \sigma_T)/2$. Here $\alpha_0 = l_0/R$, σ_B and σ_T are the strength and yield limits of the material of shell containing the crack. l_p is the length of the plastic zone on the crack line extension at the crack tip located nearer to the interface, $N^{(1)}$ and $M^{(1)}$ are the normal force and bending moment describing the action of the plastic zone; l^p, $N^{(2)}$ and $M^{(2)}$ are the corresponding quantities for the plastic zone at the crack tip located farther from the interface. It should be noted that appropriate plastic conditions

for thin shells must be satisfied by every pair of unknown forces and moments $N^{(1)}$, $M^{(1)}$ and $N^{(2)}$, $M^{(2)}$.

Thus, in the frame-work of the accepted model the three-dimensional elasto-plastic problem for a non-through crack of length $2l_0$ is replaced by the two-dimensional elastic problem for a shell with a through crack of length $2l_d$ under the following conditions

$$N_2(\alpha) = \begin{cases} N_2^{(2)} - N_2^0, & -(\alpha_0 + \alpha^p) \leq \alpha \leq -\alpha_0 \\ N^l - N_2^0 + N^{(3)}, & |\alpha| < \alpha_0 \\ N_2^{(1)} - N_2^0, & \alpha_0 \leq \alpha \leq \alpha_0 + \alpha_p \end{cases} \tag{1}$$

$$M_2(\alpha) = \begin{cases} M_2^{(2)} - M_2^0, & -(\alpha_0 + \alpha^p) \leq \alpha \leq -\alpha_0 \\ M^l - M_2^0 + M^{(3)}, & |\alpha| < \alpha_0 \\ M_2^{(1)} - M_2^0, & \alpha_0 \leq \alpha \leq \alpha_0 + \alpha_p \end{cases}$$

$$\alpha^p = l^p/R, \quad \alpha_p = l_p/R, \quad 2l_d = 2l_0 + l_p + l^p, \quad \alpha_0 + \alpha_p < \alpha^*.$$

Here N^l and M^l are the normal force and the bending moment being the reaction of the material on the break of internal bonds above and below the crack. According to the adopted assumptions on stresses in these zones they are determined by

$$N^l = 2d_3\sigma^0, \quad M^l = 2\sigma^0(h - d_3)(d_2 - d_1). \tag{2}$$

$N^{(3)}$, $M^{(3)}$ are the forces and moments applied to the crack faces, N_2^0, M_2^0 are the forces and moments acting on the crack line in a shell without a crack. The plasticity condition for $N^{(i)}$, $M^{(i)}$ (i=1,2) is chosen as the Tresca yield condition for a surface layer

$$N^{(i)}/(2h\sigma_T) + 3|M^{(i)}|/(2h^2\sigma_T) = 1 \tag{3}$$

or the condition of a plastic hinge

$$\left[N^{(i)}/(2h\sigma_T)\right]^2 + |M^{(i)}|/(h^2\sigma_T) = 1. \tag{4}$$

For solving the above problem a correlation is established between the considered shell with the crack and the intact shell with sources of internal stresses of unknown density located on the crack line. The density of sources is chosen such that the stressed-strained states of the shell with the crack and the model shell are identical. Using the key system of differential equations in displacements for homogeneous shells with cracks (Osadchuk, 1985) and the conditions of ideal mechanical contact and extending all desired quantities and mechanical characteristics on the whole region occupied by the piecewise-homogeneous shell considered, a key system of partly degenerated differential equations with step-function type coefficients (Kushnir et al., 1991)

$$L_{i1}u + L_{i2}v + L_{i3}w = g_i'(\alpha, \beta, \epsilon_{kl}^0, \kappa_{kl}^0) + g_i''(\alpha, \beta) \qquad i = \overline{1,3}, \quad k, l = 1, 2. \tag{5}$$

is obtained. Here L_{ij} $(i, j = \overline{1,3})$ are the differential operators no higher than fourth order with discontinuous coefficients; $g_i'(\alpha, \beta, \epsilon_{kl}^0, \kappa_{kl}^0)$ are functions obtained by extension on the whole region and further action of differential operators no higher than second order on the strain components ϵ_{kl}^0 and κ_{kl}^0; $g_i''(\alpha, \beta)$ are the functions allowing for the conditions of ideal mechanical contact on the interface $\alpha = \alpha^*$ and involving coefficients of the delta-function type and its derivatives;

$$\kappa_{11}^{0} = \kappa_{12}^{0} = 0, \qquad \kappa_{22}^{0} = R^{-1}\left\{[\theta_2(\alpha)]\,\delta(\beta) + R^{-1}\,[w(\alpha)]\,\frac{\partial}{\partial\beta}\delta(\beta)\right\},$$

$$\epsilon_{11}^{0} = \epsilon_{12}^{0} = 0, \qquad \epsilon_{22}^{0} = R^{-1}\,[v(\alpha)]\,\delta(\beta),\ \ \forall\alpha : -(\alpha+\alpha^{p}) \le \alpha \le \alpha_0 + \alpha_p; \tag{6}$$

$$\epsilon_{kl}^{0} = \kappa_{kl}^{0} = 0,\ \ \forall\alpha : \alpha \le -(\alpha_0 + \alpha^{p}) \cup \alpha \ge \alpha_0 + \alpha_p;\ \ \theta_2(\alpha) = R^{-1}\left(\frac{\partial w}{\partial\beta} - v\right);$$

$\delta(\beta)$ is the Dirac function; square brackets indicate a jump of the quantity in the brackets.

THE PROCEDURE FOR OBTAINING SINGULAR INTEGRAL EQUATIONS

The solution of the system of differential equations (5) is constructed using the Green tensor matrix $\|G^{jl}(\alpha,\beta,\xi,\varphi)\|$, whose components are determined as particular solutions of the following system of equations

$$\sum_{j=1}^{3} L_{ij}G^{jl}(\alpha,\beta,\xi,\varphi) = \delta_{il}\delta(\alpha-\xi)\delta(\beta-\varphi), \quad (i,l = \overline{1,3}). \tag{7}$$

Here $L = \|L_{ij}\|$ is a matrix of operators of the system (5), δ_{il} is the Kronecker symbol.

Representing the function $\delta(\beta-\varphi)$ and the solution of the system (5) as trigonometric series, we obtain a system of ordinary differential equations with piecewise-constant coefficients for determining the unknown coefficients, i.e. functions of coordinate α . Its solution is constructed using a method of finding the fundamental system of solutions (Kushnir, 1980). Now using the 2π-periodical Green tensor, it is possible to write the integral representation of displacements as

$$u_i(\alpha,\beta) = \iint\limits_{D}\left\{\sum_{j=1}^{3} G^{ij}(\alpha-\xi,\beta-\varphi)\,g_j(\xi,\varphi)\right\}d\xi d\varphi, \quad i = \overline{1,3} \tag{8}$$

where $u_1 = u$, $u_2 = v$, $u_3 = w$, $g_j(\xi,\varphi) = g_j'(\xi,\varphi) + g_j''(\xi,\varphi)$, D is the whole region of the median surface of the shell.

Using representations (8) and the corresponding formulae, the forces and moments may be found at any point of the shell. Satisfying now the boundary conditions (1) on the crack faces and the force conditions of ideal mechanical contact on the interface gives a system of six singular integral equations in the derivatives of the generalized displacement jumps across the crack faces and the jumps of displacement derivatives across the interface

$$\begin{aligned}
&\int_{-\alpha_1}^{\alpha_2}\left\{\Psi_1'(\xi)K_{i1}(\xi,\alpha) + \Psi_3'(\xi)K_{i3}(\xi,\alpha)\right\}d\xi \\
&\qquad + \int_0^{2\pi}\sum_{j=5}^{8}\Psi_j(\theta)K_{ij}(\theta,\alpha)\,d\theta = 2\pi(E_1h)^{-1}f_i(\alpha), \quad i = 1,3 \\
&\int_{-\alpha_1}^{\alpha_2}\left\{\Psi_1'(\xi)K_{i1}(\beta,\xi) + \Psi_3'(\xi)K_{i3}(\beta,\xi)\right\}d\xi \\
&\qquad + \int_0^{2\pi}\sum_{j=5}^{8}\Psi_j(\theta)K_{ij}^{s}(\theta,\beta)\,d\theta = \delta_{i8}A_1, \quad i = \overline{5,8}
\end{aligned} \tag{9}$$

where

$$\alpha_1 = \alpha_0 + \alpha^{p}, \quad \alpha_2 = \alpha_0 + \alpha_p,$$

$$\Psi_1(\xi) = [v(\xi)], \quad \Psi_3(\xi) = -R\,[\theta_2(\xi)], \quad \Psi_5(\beta) = \left[\frac{\partial}{\partial\alpha}u(\alpha,\beta)\right]_{\alpha=\alpha^*},$$

$$\Psi_6(\beta) = \left[\frac{\partial}{\partial\alpha}v(\alpha,\beta)\right]_{\alpha=\alpha^*}, \quad \Psi_7(\beta) = \left[\frac{\partial^2}{\partial\alpha^2}w(\alpha,\beta)\right]_{\alpha=\alpha^*}$$

$$\Psi_8(\beta) = \int\left[\frac{\partial^3}{\partial\alpha^3}w(\alpha,\beta)\right]_{\alpha=\alpha^*}d\beta,$$

$$a_{11} = 1, \quad a_{13} = a_{31} = c^2(1-\nu_1^2), \quad a_{33} = (3+\nu_1)/(1+\nu_1),$$

$$a_{65}^{s} = 2\left[E_2/(1+\nu_2) - E_1/(1+\nu_1)\right]/(1-\nu_1) - (3-\nu_2)\,(E_2 - E_1)\,/(1-\nu_1^2),$$

$$a_{66}^{s} = a_{68}^{s} = a_{86}^{s} = a_{88}^{s} = 0, \quad a_{5i}^{s} = a_{7i}^{s} = 0, \ (i = \overline{5,8}),$$

$$a_{67}^{s} = 2c_1^2\left[E_2/(1+\nu_2) - E_1/(1+\nu_1)\right],$$

$$a_{85}^{s} = c_1^2\{2\left[E_2(2-\nu_2)/(1-\nu_2^2) - E_1(2-\nu_1)/(1-\nu_1^2)\right]/(1-\nu_1)$$
$$-\frac{1}{3}(3-\nu_1)\left[E_2(2-\nu_2)/(1-\nu_2) - E_1(2-\nu_1)/(1-\nu_1)\right]/(1-\nu_1^2)\},$$

$$a_{87}^{s} = \frac{1}{2}\left[E_1/(1-\nu_1) - E_2/(1-\nu_2)\right],$$

$$c^2 = h^2/(3R^2(1-\nu_1^2)), \quad c_1^2 = c^2(1-\nu_1^2), \quad f_2 = N_2(\alpha), \quad f_3 = M_2(\alpha),$$

$$K_{ij}(\xi,\alpha) = \frac{1}{2}a_{ij}\,\mathrm{cth}\left(\frac{\xi-\alpha}{2}\right) + K_{ij}^{0}(\xi-\alpha) + K_{ij}^{*}(\xi,\alpha), \quad (i,j = 1,3)$$

$$K_{ij}(\theta,\alpha) = K_{ij}^{0}(\theta,\alpha) + K_{ij}^{*}(\theta,\alpha), \quad (i = 1,3, \ \ j = \overline{5,8}),$$

$$K_{ij}(\beta,\xi) = K_{ij}^{0}(\beta,\xi) + K_{ij}^{*}(\beta,\xi), \quad (i = \overline{5,8}, \ \ j = 1,3),$$

$$K_{ij}^{s}(\theta,\beta) = \frac{1}{2}a_{ij}^{s}\,\mathrm{ctg}\left(\frac{\theta-\beta}{2}\right) + K_{ij}^{0s}(\theta,\beta) \quad (i,j = \overline{5,8}).$$

The regular parts of kernels K_{ij}^{0}, K_{ij}^{*}, K_{ij}^{0s} are continuous for the whole set of real values of arguments and are not presented here in view of their cumbersome form; A_1 is an integration constant.

The solutions of the system of singular integral equations must satisfy the conditions

$$\int_{-\alpha_1}^{\alpha_2}\Psi_m(\xi)\,d\xi = 0 \quad (m = 1,3), \qquad \int_0^{2\pi}\Psi_8(\theta)\,d\theta = 0 \tag{10}$$

ensuring the continuity of displacements and rotation angles at the crack tip and, besides, the 2π-periodicity of the function $\Psi_8(\theta)$.

The integration limits (the plastic zone lengths l_p and l^p) in the system of integral equations (9) are unknown. Furthermore, four unknown forces and moments $N^{(i)}$, $M^{(i)}$ (i=1,2) are contained in the functions $f_1(\alpha)$, $f_3(\alpha)$. Hence, these equations must be solved in conjunction with additional conditions. The plasticity condition (3) or (4) is chosen for both values of i and the finiteness of forces and moments in the vicinity of the crack is enforced, i.e. the stress intensity factors at both crack tips must be equal to zero:

$$K_N^i = 0, \quad K_M^i = 0, \quad (i = 1, 2). \tag{11}$$

Thus, the complete system of equations is obtained for solving the problem at hand. But the right-hand sides of the first and second equations, i.e. the functions $f_1(\alpha)$, $f_2(\alpha)$ are discontinuous. Numerical experiments have shown that direct methods of solving such systems lead to a considerable error near the point of discontinuity. Therefore, the solution of the system of integral equations (9) is constructed as

$$\Psi_k(\alpha) = h_k(\alpha) + \Phi_k(\alpha), \quad k = 1, 3 \tag{12}$$

where $h_k(\alpha)$ are solutions of the canonical singular integral equations with the discontinuous right-hand sides

$$\int_{-\alpha_1}^{\alpha_2} \frac{h_i(\xi)}{\alpha - \xi} d\xi = f_k(\alpha), \quad k = 1, 3, \quad -\alpha_1 \leq \alpha \leq \alpha_2. \tag{13}$$

The solution of these equations is found using the inversion formulae for integrals of the Cauchy type. Inserting the transform (12) into system (9) we obtain a system of singular integral equations in $\Phi_k(\alpha)$ similar to (9), but with the continuous right-hand sides containing integrals of the products $h_k(\alpha)K_{ik}(\xi, \alpha)$ $(i, k$=1,3) and unknown quantities $N^{(1)}$, $N^{(2)}$, $M^{(1)}$, $M^{(2)}$. Hence, $\Phi_k(\alpha)$ is represented as

$$\Phi_k(\alpha) = \Phi_k^0(\alpha) + \sum_{i=1}^{2} \left(N^{(i)} \Phi_k^i(\alpha) + M^{(i)} \Phi_k^{i+2}(\alpha) \right), \quad k = 1, 2 \tag{14}$$

and every $\Phi_k^m(\alpha)$, $(m = \overline{0, 4})$ is found using the mechanical quadrature method which allows us to reduce the calculation of every function $\Phi_k^m(\alpha)$ to the solution of a system of linear algebraic equations.

NUMERICAL SOLUTION ALGORITHM

The following algorithm is used for the construction of numerical solution of the problem. The initial values of parameteres α_1 and α_2 are first chosen (in particular, the solution of the corresponding problem for homogeneous shell (Nykolyshyn, 1993b) may be taken as an optimal choice). Then the right-hand sides of the system of integral equations for determining $\Phi_k(\alpha)$ are found and the solution of corresponding system of linear algebraic equations is constructed. The forces $N^{(i)}$ and moments $M^{(i)}$ $(i = 1, 2)$ are obtained next, and the plasticity condition (3) or (4) is verified at both zones of plastic strains. If the plasticity condition in both zones is satisfied with a given error, the problem is solved; otherwise l_p (and analogously l^p) are changed. Integrating the solutions of system of integral equations (9) and inserting them into the following expression

$$\Delta(\alpha, \gamma) = [v(\alpha)] + \gamma [\theta_2(\alpha)] \tag{15}$$

gives the relation for determining the opening of the crack faces. According to the δ_c-model, the fracture of the shell is assumed to begin when the maximum opening of the crack reaches the critical value δ_c, i.e. after changing $\Delta(\alpha, \gamma)$ by δ_c the expression (15) becomes the criterion for establishing the relation between the applied load, the crack size, and the physical and geometrical parameters of the shell under the conditions of limit equilibrium state.

The numerical analysis is carried out for the piecewise-homogeneous shell made from aluminium ($E_1 = 0.65 \times 10^{11}$ Pa, $\nu_1 = 0.3$, $\sigma^0 = \sigma_T = 1.1 \times 10^8$ Pa) and epoxy resin ($E_2 = 2.6 \times 10^9$ Pa, $\nu_2 = 0.35$, $\sigma^0 = \sigma_T = 4.3 \times 10^6$ Pa) with a surface crack. The shell is subject to internal pressure. The basic stressed state of such a shell was investigated by Kushnir et al. (1995). The calculations were performed using the following values of parameters: $h/R = 0.01$; $l_0/h = 5$; $d/h = 0.6$ (here $2d$ is the crack depth). The maximum opening of the crack is reached at a point $\hat{\alpha} = 0$, $\gamma = h - 2d$. Figure 2 represents the dependence of the maximum opening on the distance between the crack centre and the interface ($\Delta^* = \Delta/\Delta_0$, Δ_0 is the opening of the crack in the homogeneous shell, $\eta = (2l_0 + l^p + l_p)/(2\alpha^* R)$). The curves 1 and 2 correspond to the crack located in aluminium and in epoxy resin, respectively. From Fig. 2 we notice, that the presence of similar cracks in different materials of the shell has different influence on its strength.

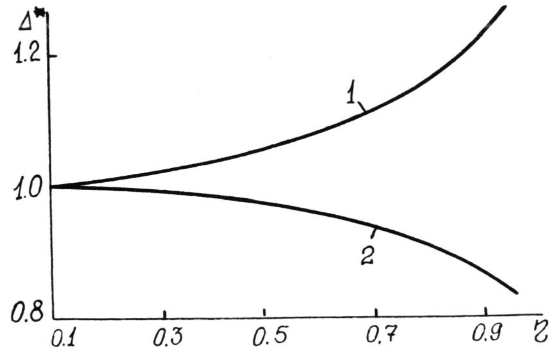

Fig. 2. The behaviour of maximum opening *vs.* the distance between the crack centre and the interface.

The calculations were also done for other values of parameters. The analysis of these results shows, that the maximum opening of the crack is reached at the crack tip on the shell surface $\gamma = h$ or at $\alpha = 0$, $\gamma = h - 2d$. In the first case ($\gamma = h$) the crack will propagate along the shell, in the second case ($\gamma = h - 2d$) the non-through crack propagates along the depth and may become a through one. If the opening of the through crack is less than the critical one, the fracture process stops, otherwise the through crack propagates along the shell.

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