FRACTURE ANALYSIS OF CRACKED SPHERICAL SHELLS

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ABSTRACT

Stress intensity factors (SIF) for cracked spherical shells have been determined by finite element analysis (FEA). The relationships between the SIF and the crack length for three crack configurations, defined as vertical, horizontal and 45°-angular, when the shell is compressed between two horizontal rigid plates, are all presented. It has been found that the SIF varies linearly through the shell thickness, which is consistent with the stress assumptions in classical shell theory. In addition, the applicability of the Green's functions to the crack problems in shells is discussed and theoretical and numerical results have been compared for the three crack geometries.

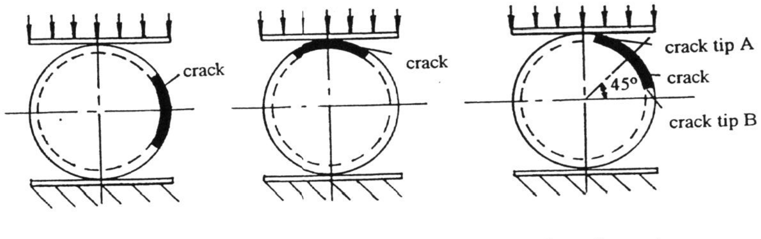
KEYWORDS

stress intensity factor, curvature effect, equatorial stress, meridional stress, membrane force, bending moment, crack initiation, crack propagation, shell theory.

INTRODUCTION

Methods to crack Macadamia nuts efficiently and to improve kernel recovery rates have been sought for many years and a number studies on shelling methods and machines for these nuts has been carried out. The fracture behaviour and mechanism of the nutshell, when compressed between two rigid plates, were studied, theoretically and numerically, by Wang *et al* [1, 2] recently. Macadamia nuts are commonly heated to reduce their moisture contents after being picked from the trees, which is beneficial for preservation. Furthermore the kernels become loose from their shells so that they are more easily cracked in subsequent processing. However, shrinkage and perhaps thermal stresses inevitably lead to the initiation of cracks in the nutshells after the drying process.

The work reported here was part of a larger research program concerned with Macadamia nuts. The aim of this paper is to present an analysis of the fracture of a Macadamia nutshell with an existing crack, being compressed between two rigid plates. Three configurations have been studied, where the crack orientations were defined as vertical, horizontal and 45°-angular respectively with reference to the loading planes, as shown in Fig. 1. The three crack geometries were all noted to be modeI due to the axisymmetrical type of loading.



(a) vertical crack (b) horizontal crack (c) 45°-angular crack

Fig. 1. The definition of the crack orientation in a nutshell

APPLICATION OF THE TWO BASIC THEORIES TO CRACKED SHELLS

Theoretical treatment of cracks, in initially curved plates or shells, has been limited because of the mathematical complexities. The presence of curvature in a shell makes its behaviour different to that of a flat plate, in that extensional loads will induce both membrane and bending stresses; while bending loads will also lead to both types of stress [3]. Based on different assumptions and boundary conditions, there are two basic theories applicable to cracked shells. One is called classical theory, the other is shear deformation theory.

In the classical theory, it is not possible to combine the membrane stress intensity factor, K_I^m, and the bending stress intensity factor, K_I^b, as a linear sum, with respect to shell thickness coordinate z, because of the difference in the θ-dependence of the membrane stresses and the bending stresses near the crack tip. However, the shear deformation theory has important improvements over the classical theory which permits the de-coupling of a tenth-order system of equations besides including the effect of transverse shear deformation [3]. It has been found that the θ-dependence of the membrane and bending stresses near the crack tip are identical. They differ only in the coefficients or amplitude of the singular stress field determined by K_I^m and K_I^b. This feature of the solution allows the membrane and bending stress fields to be combined in a superposition manner. The combined stress field for mode I is [4]: -

$$\sigma_x(r, \theta, z) = \frac{K_I(z)}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] \tag{1}$$

$$\sigma_y(r, \theta, z) = \frac{K_I(z)}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] \tag{2}$$

$$\tau_{xy}(r, \theta\, z) = \frac{K_I(z)}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)] \tag{3}$$

The stress intensity factor $K_I(z)$, which is consistent with the basic concept in fracture mechanics, depends on the thickness coordinate z. That is, it varies through the shell thickness. The shear deformation theory was adopted in the present work.

Theoretical determination of the SIF for a cracked shell by Green's functions, modified for curvature effects, was attempted. A schematic drawing of a shell is shown in Fig. 2 together with the spherical coordinates adopted. The loading points are called poles and the maximum horizontal circumference is called the equator. The assumptions, in classical shell theory, that both the meridional stress $\sigma_{\phi\phi}$ and the equatorial stress $\sigma_{\theta\theta}$ in the shell can be considered as the sum of the membrane stress, which is uniform through the shell wall, and the bending stress, which is linear across the wall thickness [5, 6], were adopted. Numerical solutions, by FEA, of the stress distribution in an uncracked nutshell, being compressed by two rigid plates, were made on a displacement of 0.5 mm between the poles, which corresponded to a contact load of 1 kN. The results are shown in Fig. 3. The angle χ was defined as the range from the equator; so that $\chi = 90° - \phi$ and $\chi = 0$ at the equator.

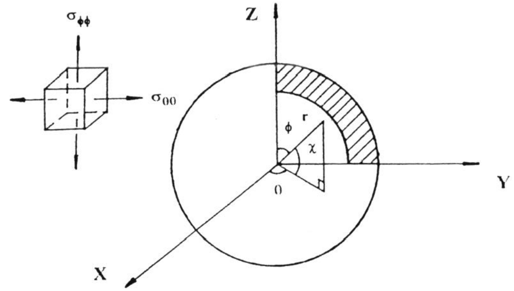
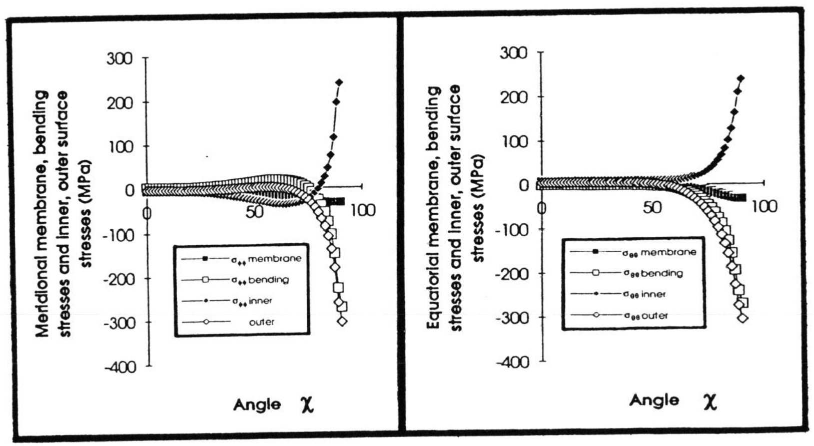


Fig. 2. Schematic drawing of a nutshell and notions of spherical coordinates



(a) meridional stresses (b) equatorial stresses

Fig. 3. The distribution of the meridional and equatorial stresses in an uncracked nutshell

It can be seen that the membrane or bending stresses in both the meridional and equatorial directions are non-uniform. Since only the equatorial stress was the driving force to open the crack, it was considered as a series of unevenly distributed wedge forces acting on the crack surface in the general Green's functions [7]

The Green's function method is strictly valid only for crack problems in flat plates and it can not be applied directly to curved shells. Results derived from both the classical and shear deformation shell theory indicated that, due to curvature effects, the stress intensity factors in shells might be considerably higher than in flat plates having the same crack length and the same thickness [4]. As mentioned before, the presence of curvature in a shell changes its behaviour from that of flat plates, in that stretching loads will induce both membrane and bending stresses, while bending loads will also lead to both types of stress. A treatment of curvature effects, by four interdependent stress intensity factor ratios which are all related to a geometrical parameter, has been given for the calculation of the SIF for cracked shells by modifying Green's functions [4].

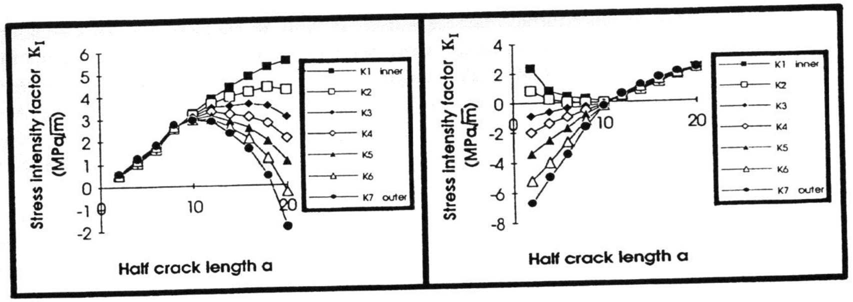
NUMERICAL RESULTS AND DISCUSSION

The stress field near a crack tip is quite complex and is characterized by the stress intensity factor. Since the stress distribution in the uncracked shell is rather complicated, it is difficult to determine the stress intensity factor for a given crack. No analytical solutions are presently available. To this end, the finite element method was used. The principle of the method is to derive the required stress intensity factor from the stress or displacement field [7].

To determination the stress and displacement in the cracked spherical shell, a finite element analysis was performed using LUSAS. A twenty-node solid continuum element was used throughout. A fine mesh was adopted in the crack tip region to simulate the singular behaviour. The contact behaviour between the shell and the rigid loading plates was modelled using a special interface treatment known as SLIDELINE. Each slideline comprised two surfaces; designated as the master surface and the slave surface. The slideline facility implied geometrical nonlinearity and required the use of NONLINEAR CONTROL operation in three datasets: incrementation, constants and output.

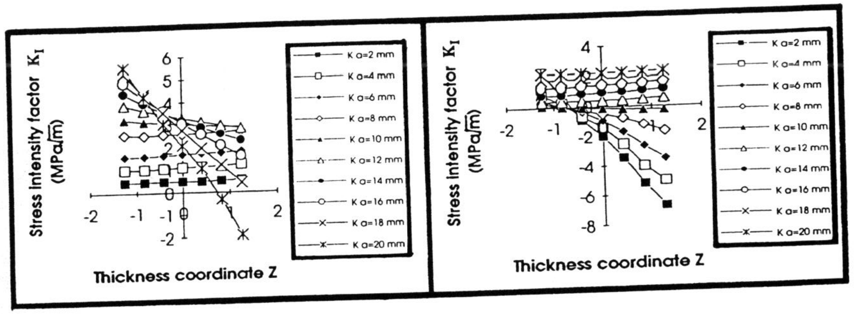
Studies have shown that the Macadamia nutshell is isotropic, with a Young's modulus of about 5.2 GPa and a Poisson's ratio of 0.3 [2]. The Macadamia nut was assumed to be an ideal sphere, the inner and outer radii of the nutshell were 10.5 and 13 mm, respectively, with the shell thickness being 2.5 mm. Based on the displacement or stress components derived from the FEA, the stress intensity factors for the vertical, horizontal and 45°-angular crack were calculated respectively by equation (2) in terms of the shear deformation theory. All the results discussed here were obtained for a displacement of 0.5 mm between the poles (corresponding to a contact load of 1 kN). The curves in Fig. 4 describe the variation of SIF for the vertical and horizontal cracks.

It is interesting to note that the curves in Fig. 4 are all a "broom" shape. For the vertical crack, the SIF on the inner surface increased linearly with the crack length; while that on the outer surface increased initially for short crack length and then decreased as crack length increased. However, exactly the opposite was observed for the horizontal crack. The values of the SIF around a half crack length a = 10 mm are the turning points of the "broom" from the "handle" to the "bristles".



(a) vertical crack (b) horizontal crack

Fig. 4. The SIF versus the half crack length



(a) vertical crack (b) horizontal crack

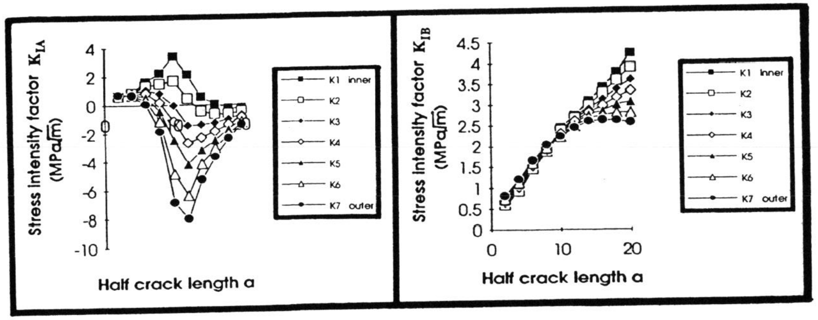
Fig. 5. The SIF versus the thickness of the shell wall

The stress intensity factor for a centre cracked plate subjected to uniform tension can be expressed as $K_I = \sigma\sqrt{\pi a}$. Clearly the longer the crack, the larger the SIF for a given σ. In a spherical shell, due to complex distribution of the equatorial stress, as shown in Fig. 3, the SIF for the cracked shell is dependent on not only the crack length but also the location of the crack in the shell. As shown in Fig. 3, the stress on the inner surface is always tensile and increases from the equator to the pole; and that on the outer surface the stress is tensile for χ < 50° and compressive otherwise, therefore the SIF on the inner surface for the vertical crack is positive for all crack lengths, but on the outer surface, decreases when the half crack length a = 10 mm, of which the crack tip is about at χ = 50°. On the contrary, for the horizontal crack, since the crack tip approaches the equator as the crack length increases, the SIF on the outer surface is below zero in the range of a < 10 mm and rises up to zero when the crack length reaches 10 mm; then it continuously increases with the crack length. Nevertheless, although the SIF for the horizontal crack on the inner surface decreases in the range of a < 10 mm and increases when a > 10 mm; it is still always above zero. It is notable that the equatorial stress on the inner surface decreases from the pole to equator, but the SIF for the horizontal crack exhibits a less consistent trend with the crack length. Similarly, for the vertical crack $\chi > 50^{o}$ when the half crack is longer than 10 mm; the SIF on the outer surface is always positive except at a = 20 mm even though the equatorial stress is compressive in the region of $\chi > 50^{o}$. Therefore, it is clear that the SIF for the cracked shell is not only closely related to the distribution of the equatorial stress, but also relies on the crack length.

Fig. 5 shows the variation of SIF through the shell wall, indicating that SIF is almost linear through the wall thickness, which is consistent with the stress assumptions in classical shell theory [5, 6]. As the crack tip approaches the pole (for long vertical crack and short horizontal crack), where there are high bending stresses (Fig. 3), the variation of the SIF is much greater than that near the equator. In other words, the SIF decreases rapidly from the inner to the outer surface because of high tensile stress on the inner surface and high compressive stress on the outer surface. This means that the crack would propagate first on the inner surface and further loading will be needed to extend the crack across the thickness. However, the SIF is nearly constant across the shell wall when the crack tip is around the equator (for short vertical crack and long horizontal crack), where there are low bending stresses, and so the propagation of a crack will probably start simultaneously at the inner and outer surfaces.

The fracture toughness of Macadamia nutshells was reported to be 0.78 MPa $\sqrt{m}$ [2]. Since the SIF for the vertical crack is greater than 0.78 MPa$\sqrt{m}$ when the half crack is longer than 2 mm, the vertical crack will propagate at the load level being discussed here, and when the crack is longer, it would start to propagate on the inner surface due to the higher SIF there; whereas when the crack is short, it may extend coincidently through the shell wall because of the almost constant SIF over the thickness. As to the horizontal crack, only when the half crack is longer than 14 mm does the SIF exceed the fracture toughness of the nutshell and then the crack can propagate simultaneously through the shell wall.

In order to analyse and understand the crack behaviour over the nutshell, the SIF for a 45°-angular crack has also been calculated by the FEA for a series of crack lengths. The variations of the SIF at the crack tip A and B, defined in Fig. 1, with the crack length are described in Fig. 6.



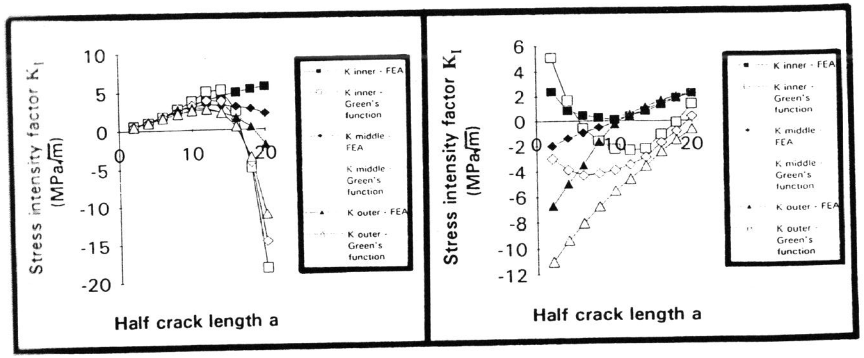
(a) at the crack tip A (b) at the crack tip B

Fig. 6. The SIF versus the half crack length for the 45°-angular crack

The crack will not extend at the crack tip A except when the half crack length is between 6 and 12 mm; the crack may propagate on the inner surface, but it will arrest in the mid layer because the SIF declines abruptly from the inner to outer surface. However the crack would extend at the tip B as the SIF is higher than the fracture toughness of the material. The same conclusion can be reached for the 45°-angular crack that, the closer to the pole the crack tip, the greater the difference of the SIF through the thickness; whereas the nearer to the equator the crack tip, the less the variation of the SIF through the thickness.

COMPARISON OF NUMERICAL AND THEORETICAL RESULTS

A comparative analysis of the SIF derived from the FEA method with the modified Green's function approach, taking into account the curvature effect, has been carried out for the vertical and horizontal crack over a range of crack lengths (from a = 2 mm to a = 20 mm), based on shear deformation theory. The results are shown in Fig. 7.



(a) vertical crack (b) horizontal crack

Fig. 7. Comparison between the numerical and theoretical SIF based on shear deformation theory

The theoretical solutions to the SIF depicted in Fig. 7 were obtained by the Green's function approach. The four stress intensity factor ratios, which characterise the curvature effect, were all established for uniform membrane force and bending moment acting on the crack surface [4]. For non-uniform stress states, although the general Green's function [7] could be substituted for uniform stress results, some deviation from the finite element results were observed in Fig. 7. The numerical and theoretical SIF for the vertical crack with $a \leq 14$ mm agreed well with each other because the crack was near the equator and the membrane and bending stresses acting on the crack surface were almost uniform (Fig. 3). However, as the crack length increased beyond a = 14 mm, the discrepancy between the numerical and theoretical results intensified due to the very non-uniform membrane and bending stresses around the poles. In the case of the horizontal crack, large difference between the finite element results and the Green's function prediction occurred for both short and long crack lengths, probably due to the steep gradient in the membrane and bending stress distributions. In the same way, comparison between the theoretical SIF with the FEA for the 45°-angular crack was also made. The results showed that the theoretical SIF at the crack tip A differed considerably from the FEA, whereas the two results at the crack tip B were very close.

Further verification of the applicability of the Green's functions to crack problems in shells has been made by considering FEA models with uniformly distributed pressure on the crack surface. The numerical results of the SIF appeared to be identical with the theoretical solutions for cracked shells. It shows the theoretical analysis of cracked problems in shells is confined to certain circumstances. The modified Green's functions can only be adopted in a limited range to give approximate solutions. Work is in progress to develop an analytical method for dealing with unevenly distributed stresses in cracked shells.

CONCLUSIONS

Due to the non-uniform distribution of the membrane and bending stresses in the shell, the SIF for three crack configurations showed quite different and complicated variations with respect to crack length. The SIF for a cracked shell, compressed by two rigid plates, depended not only on the crack length but also on the location of the crack in the shell. Under the displacement of 0.5 mm between the poles or the contact load of about 1 kN, giving the fracture toughness of the material of Macadamia nutshells $K_{IC} = 0.78$ MPa$\sqrt{m}$, the vertical crack would propagate, while the horizontal not. Moreover, like the vertical crack, the 45°-angular crack would also grow when it was longer than 4 mm.

The SIF varied linearly with the thickness of the shell wall, which was in accord with the stress assumptions in shear deformation theory of shells. For a crack tip located near the pole, the variation of the SIF through the wall thickness was much greater than that near the equator because of high bending stresses near the loading point. However, the SIF was nearly constant across the shell wall when the crack tip was around the equator due to low bending stresses there. Theoretical solutions established for uniform membrane force and bending moment were extended to non-uniform stress conditions, but limited to the cases where the stress gradient is only moderate.

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