

EVOLUTION EQUATIONS OF STABLE AND UNSTABLE CRACK PROPAGATION

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ABSTRACT

A crack propagation equations have been reconsidered from the point of view of thermodynamics of irreversible processes. The stable crack growth was assumed as a semi-equilibrium, linear dissipative process. The general structure of the crack growth evolution equations was proposed. For stable crack growth the R-curves determination has been proposed taking into account two dominant dissipative processes and using mixed numerical – experimental method. The fast, unstable crack growth was assumed to be governed by crack growth equation of the same general structure as for the stable crack extension. However, it was also postulated that dissipation function contains term with crack tip acceleration.

KEYWORDS

Cracks propagation, stable crack growth, fast crack growth, R-curves.

INTRODUCTION

Fracture mechanics is an interdisciplinary field of science that aims at prediction of strength of structural or machine element at the presence of cracks. The essence of fracture mechanics is to define the proper fracture criteria to predict the onset of crack propagation as well as the stable and unstable crack extension. It is expected that fracture criterion introduces certain parameters which would be able to characterise **material** from the point of view of its resistance to the crack initiation and growth. While the problem of the stationary crack growth initiation is more or less well understood and various material constants are established (K_{IC} , J_{IC} , G_{IC} , K_{Id} , ...) the process of stable and unstable crack growth still requires additional research and theoretical basis to define the proper crack growth equations and material characteristics.

So far the existing theories of the stable crack propagation fail to provide the unique, geometry independent material characteristics of the crack growth resistance. It was reported by many authors (e.g. Turner, 1990). In the case of unstable crack growth the existing theories are not

able to predict the basic features of the fast crack growth process observed experimentally (Neimitz, 1995). In the present paper the general structure of the stable crack growth equations will be discussed along with a certain suggestions aiming at the numerical – experimental approach to establish material parameters that may characterise the dissipation processes accompanying the fracture of specimens. It will also be proposed that this structure is generalised to unstable, fast crack growth although it is not strictly justified.

STABLE CRACK GROWTH

The stable crack growth (sometimes called subcritical) is an example of stable processes when small changes in external excitations (external load or load point displacement) are not followed by drastic changes along the crack tips trajectories. Moreover the stable crack growth process is semi – equilibrium one in this sense that crack growth is immediately stopped when external loading (load controlled process) or external displacement (displacement controlled process) is slightly reduced. We will not discuss here the creep crack growth, stress corrosion cracking and HAC, although these processes can also be discussed in the general frame of proposed equation. The stable crack growth can be considered as an example of the linear dissipative processes being very close to the state of equilibrium. For this situation both the free energy and entropy are potentials and extremes of these functions determine the attractor state. The theorem of the minimum entropy production tells us that in domain where the Onsager's relations are valid, it means in the linear domain, the system evolves into steady state (under given boundary conditions). The steady state which is an attractor state must be a nonequilibrium state in which dissipative processes take place with the non – zero speeds. For this state also entropy of the system becomes independent of time but it steadily increases. In this particular state the entropy production is as small as possible under the given boundary conditions. In this situation the equilibrium state becomes possible as soon as boundary conditions allow the entropy production to vanish. In other words the system becomes passive. These properties reflect the properties of the subcritical crack growth which can be considered as a sequence of the equilibrium states, when the time is not an independent variable and enters equations only as a parameter.

The classical resistance curves concept to characterise the stable crack growth lies within the framework of the above described semi-equilibrium processes. This concept is based on an assumption of a local (in time) equilibrium of the crack driving force and material resistance force. It can be written in the general form:

$$X(\sigma_a, a, geometry) = X_R(\Delta a) \quad (1)$$

where X is driving force and X_R is resistance force, σ_a is applied stress and a is crack length. In practical situation X is usually replaced by one of known fracture parameter: J , G , K , $CTOD$, $CTOA$. The question arises: can these parameters describe uniquely the crack growth process? Leaving the answer for this question open at the moment one should say that resistance curves (J - R , G - R , ...) obtained experimentally are not the one that characterise tested material in most cases (Turner et al, 1990, Neimitz et al, 1996). It may stem from the wrong definition or interpretation of the driving force or from the improper determination of the R -curves. Both problems will be discussed in the next paragraphs.

THERMODYNAMICAL ANALYSIS

The fracture process is an irreversible one. One of the first who considered the crack growth process within the framework of thermodynamics of irreversible processes was J.Rice (Rice, 1978). He arrived at the formula for entropy production rate in the form:

$$\dot{S}_i = (G - 2\dot{\gamma})\dot{a} \quad (2)$$

where \dot{S}_i is an entropy production rate, G is classical energy release rate, $\dot{\gamma}$ is specific fracture energy, \dot{a} is a crack tip velocity. However, this relation was derived under strong assumptions that process is isothermal, quasi static and reversible. Under these assumptions one may not see the reasons for entropy production. Similar equation but for more general case, including irreversibility of the fracture process was derived (Neimitz, 1995) in the form:

$$T\dot{S}_i = \left[(F - F_q|_{a,\tau}) \frac{d\Delta}{da} - L_q|_{\Delta,\tau} \right] \dot{a} \quad (3)$$

where F is an external force, F_q is quasi conservative force, L_q is internal reversible force due to the internal parameter (the crack length a). Δ is load point displacement, T is temperature. For $\dot{\Delta} = 0$ one has:

$$L_q \dot{a} = -T\dot{S}_i \quad (4)$$

Thus internal force can be a source of dissipation provided that

$$L_q \dot{a} = -L_d \dot{a} \quad (5)$$

where L_d is internal dissipative force. Relation (5) must hold because internal power due to internal forces must be equal to zero since it does not enter the first fundamental law of thermodynamics (Ziegler, 1977). It may be shown (Neimitz, 1995) that $L_d = -L_q = G$ (=energy release rate). For plastic dissipation $F - F_q = F_d$ where F_d is called external dissipative force and entropy production rate can be defined as:

$$T\dot{S}_i = L_d \dot{a} + F_d \dot{\Delta} \quad (6)$$

Equation (3) can be rewritten for growing cracks in the form:

$$C = (F - F_q) \frac{d\Delta}{da} - L_q = \frac{T\dot{S}_i}{\dot{a}} = D \quad (7)$$

adopting Turner's model (Turner, Kolednik, 1994)

In this case C is the driving energy rate due to external and internal forces. $D = L_d + F_d d\Delta/da$ is dissipation rate. Eq. (7) strongly supports Turner's model of the „two step elementary crack increment“. Thus, the product $C\dot{a}$ should be understood as a sum of a part of work rate due to

external forces that is not reversibly stored within the material and release of the internal power associated with the material separation. C should be computed for a given boundary value problem. D , the dissipation function should be computed independently of C from the micro – macro mechanics approach taking into account evolution of a dominant dissipation processes Equation (7) can be easily reanalysed and compared with the concept of introducing the T^* integral (Atluri et al, 1984) to the fracture mechanics analysis.

$$T^* = G = J_{ff} + I \quad (8)$$

where J_{ff} is J far field contour integral, T^* is J contour integral computed over the very small contour ϵ surrounding the crack tip. I is the area integral over the area between far field and ϵ contours. The I integral contains terms characterising the reversible terms of the stored energy and irreversible ones $I = I_{ir} + I_{re}$.

Thus

$$G = J_{ff} + I_{ir} + I_{re} \quad (9)$$

Now if stable crack evolution equation is written in the form (the R curve concept)

$$G = 2\dot{\gamma}$$

where 2γ is here a material property characterising the energy rate associated with a new crack surface formation and Eq. (9) is used one obtains:

$$(J_{ff} - I_{re}) - 2\dot{\gamma} = I_{ir} \quad (10)$$

which if multiplied by \dot{a} is similar to Eq. (3) with the similar physical interpretation of particular terms within it.

RESISTANCE CURVES DETERMINATION – SUGGESTIONS.

The analysis within this paragraph concentrates on the most popular J - R curve concept. The „classical” equation is as follows.

$$J = J_R \quad (11)$$

In this case the J integral can not be understood as in the case of stationary cracks. For growing crack the J integral is not path independent and it is not a measure of a stress field in front of the crack for arbitrary contour. It should be computed here along the far field contour surrounding domains of plastic dissipation. J_R measured directly from the test using P - Δ diagram for various geometries must provide different R curves. Measuring „global” parameters (force P and load point displacement Δ) and the area under the P - Δ curve all dissipative processes are taken into account. In many cases they are geometry dependent. The domain of plastic deformation, for example, strongly depends on the specimen thickness and the initial length of the crack. Thus the J integral being a „global” parameter for growing cracks must be sensitive to the crack geometry. However, it can be utilised to the analysis of the crack growth provided dissipation function is defined correctly.

The dissipation function D (now $D \equiv J_R$) should be computed from the analysis of dissipative processes and should contain computable (or measurable) parameters depending on geometry and material parameters independent of size and geometry of specimen. As a first approximation it has been assumed here that two dissipative processes dominate during the crack propagation history: plastic dissipation and process zone creation (voids, microcracks, ...). Both processes can be analysed from the point of view of micromechanics including dislocations motion, creation and interaction as well as voids nucleation and growth. However, this approach might be time consuming with uncertain result that could involve many new parameters. Certainly, results could not be generalised. Therefore a simpler approach is proposed. It was assumed that the energy dissipated by plastic deformation is proportional to a product of ξV where ξ is specific energy rate of plastic deformation, V is the volume of material where the plastic deformation takes place. Similarly the amount of energy dissipated within the process zone is equal to a product of $\eta \Delta A$ where η is specific energy rate of process zone formation and ΔA is extension of the crack surface. ΔA can be taken from experiment, V can be computed with the help of finite element method. This simple assumption followed from the experimental – numerical analysis performed on 40HMNA steel (according to Polish Standards – which is equivalent to the 4340 steel) heat treated to 1100 MPa and 1287 MPa yield points. Three point bend specimens with various sizes and relative initial crack lengths have been machined and precracked. This steel was selected because of a very weak work – hardening. The results reporting classical J_R and $CTOAR$ curves along with numerical results were published elsewhere (Neimitz et al 1996). Numerical analysis of a stable crack growth was performed with the help at ADINA 6.1.4. The smallest element size was 0.05 mm. The material was assumed to follow HMH yield criterion and associated flow rule with multi linear isotropic hardening. Full Newton iteration was employed. The crack growth was modelled by the node release technique with node shifting. Experimentally registered load point displacements $\Delta(t)$ and crack lengths $a(t)$ were used as an input for computations. It turned out from the analysis that the total dissipation due to plastic deformation was proportional to the volume of the plastic zone which was growing from the size of almost small scale yielding to the whole unbroken specimen ligaments. Similar situation was observed concerning dissipation within process zone and its size. The width of the process zone was assumed to be constant with crack growth and equal to 0,3 mm. The assumed material parameters ξ and η can be computed from a selected arbitrary J - R curve for a tested material solving two equations:

$$J_{p(1)} = J_{(1)} - J_{el(1)} = V_{(1)} \xi + \Delta A_{(1)} \eta \quad (12)$$

$$J_{p(2)} = J_{(2)} - J_{el(2)} = V_{(2)} \xi + \Delta A_{(2)} \eta$$

Subscripts 1 and 2 refer to two arbitrary points along the selected J - R curve. The linear form of Eq. (12) was assumed for a tested material because of its weak strain hardening. If the quantities ξ and η computed from (12) are material parameters one should „reconstruct” the J - R curves for other geometries and sizes provided $V_{(i)}$ and $\Delta A_{(i)}$ are known. It was done for tested material and several specimen sizes where the pairs of $\xi = 0,81 \text{ N/m}^4$, $\eta = 14,679$ for plane strain and of $\xi = 1,279 \text{ N/m}^3$, $\eta = 24,074 \text{ N/m}^3$ for plane stress provided a tool to obtain the J - R curves for different specimens in a good agreement with experiment. However, it should be mentioned that the linear form of Eqs (12) although very simple makes parameters ξ and η very sensitive to changes of other quantities (J_{pl} , V , ΔA). Thus, they have to be evaluated very carefully and certain unified method of measurement and computations must be proposed preceded by additional research with other materials. The proposed approach is certainly

oversimplified but it aimed at the simplest method that being in agreement with general theory provides relatively simple tool for practical applications. In more extended version it should include at least two different modes of plastic deformation.

FAST, UNSTABLE CRACK GROWTH

Many of the experiments performed indicate that the fast crack growth is strongly non-linear dissipative process, which takes place far from the equilibrium state. Its features including oscillations with increasing amplitudes and chaotic behaviour locate this process within so called deterministic chaos. The thermodynamics of the non-linear, nonequilibrium, dynamic systems does not provide us with precise tools to solve particular problem. As far there is no proof available concerning the theorem of the minimum entropy production for cases where the fluxes are not linear functions of thermodynamical forces. It is also known that far away from equilibrium, the system must evolve towards certain steady state, but this state can not be characterised by properly selected potential. If we do not know the potential we can not say anything about stability of states to which the system evolves. If we can not define the minimum of potential the oscillations may not be controlled by the second law of thermodynamics. For such a systems, according to Prigogine: „ Stability does not follow from the general laws of physics”. Thus, fluctuations that arise may not vanish but they can considerably increase leading to a chaotic behaviour. These features are clearly observed during the fast crack growth (Fineberg et al, 1992) and are visualised by mirror, mist and huckle surfaces as well as branching process.

The classical fast crack growth equations are of semi-equilibrium type and can be written in the form of Eq (1) where X is usually replaced by dynamic stress intensity factor or dynamic energy release rate. These functions depend on crack tip speed but are independent of the crack tip acceleration. This follows from the one term approximation of the crack tip stresses. Recent solution (Freund, Rosakis, 1992) introduces more terms containing crack tip acceleration. Theoretical analysis based on semi-equilibrium crack growth equations is not able to predict the basic features of the fast crack growth (Neimitz, 1995). It is so for linear elastic materials. For nonelastic materials the situation is even worse since there are only a few closed form solutions for these materials but for simply mode III of loading and steady state or for simplified Dugdale-Panasyuk (D-P) model of crack (Neimitz, 1995, 1996). The nonlinear zone of dissipation is observed even for relatively brittle materials. Thus, this zone should be taken into account in the fast crack growth analysis. It is possible if the Eqs(3) or (7) are adopted as a crack growth equation. In this case the left hand side of the Eq(3) represents release of the energy into the zone of dissipation. It can be easily computed for D-P model. The right hand side of this equation represents dissipation within the plastic and/or process zone. This term contains crack tip acceleration for transient crack growth since the zone of dissipation changes its size in time. In general the left hand side of the crack growth equation should contain acceleration also but it is not so because of the crack kinetic model adopted (Neimitz, 1995). Results of computer simulations of the fast crack growth based on the proposed crack growth equation have already been published (Neimitz, 1996). They reflect all features of the fast crack growth observed experimentally. However, it should be written that the equation derived for stable crack growth may not be generalizable to the unstable crack growth which is strongly nonlinear, irreversible process being far away from the state of equilibrium. For this case not all quantities can be properly defined. Thus, the extension of the Eq (7) to the fast crack growth should be considered as a possible generalisation only.

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