APPLICATION OF THE GRIFFITH ENERGY APPROACH TO NON-CLASSICAL PROBLEMS OF FRACTURE

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ABSTRACT

The Griffith energy approach is applied to study two types of dispersed fracture: conventional, gradual damage and abrupt damage. Gradual damage under uniaxial straining and abrupt damage of a stretched rod are considered to illustrate the effects described by the models. Besides the model of abrupt damage is applied to study energetics of a fast moving longitudinal shear crack.

KEYWORDS

Energy approach, dispersed fracture, damage, abrupt damage.

INTRODUCTION

Griffith in his classical paper (1920) firstly applied the energy approach to investigation of strength of materials. Instead of the conventional theories of strength connecting exhausting of material resistance to failure with reaching a threshold stress or strain state the concept of fracture as the energy consuming process was developed. The energy equation allowing for the energy required for fracture was explicitly used. The specific fracture energy or the surface energy was introduced as a material property which determines crack resistance. Besides in opposite to the theories of strength which consider intact materials, the defects were explicitly introduced whose evolution leads to failure. In the Griffith theory defects are implemented as cracks. As a result the theory predicts a critical load for a given size of the crack or *vice versa* a critical size of the defect which can be sustained under the given load. The theory of fracture essentially enriched the material sciences. The clear, evident from the physical point of view Griffith's idea that fracture is the energy consuming process appeared to be fruitful in many other problems where fracture is induced by defects different from a single crack. In the present paper the energy approach will be applied to study two types of dispersed fracture under action of stress and/or temperature, namely, gradual damage and abrupt damage. The former is conventional damage characterised by progressive fracture of the internal structure with growth of stress. The latter is similar to a phase transformation and is characterised by step-like change of mechanical properties of a solid upon reaching the threshold stress state.

ENERGY THEORY OF DAMAGE

The theories of damage or dispersed fracture describe processes of conception and evolution of material microdefects such as microcracks, pores, voids, inclusions and so on. The traditional theories of damage taking their origin from the pioneering papers (Kachanov, 1958; Rabotnov, 1959; Il'ushin, 1967) give continuum description of dispersed fracture in solids. The state of a material particle is characterised by the strain tensor. The response of a material particle to history of its state is determined by the stress tensor and some measure of damage which is either a scalar (in the simplest case) or a tensor. For these measures of damage the kinetic equations were postulated. Thus the damage measure is an internal variable and the theory does not differ formally from the theories of plasticity or visco-plasticity. But damage and plasticity or viscosity are of different physical nature. The conventional damage theories take into account only mechanical and thermal forms of energy. However damage first of all is related to the transfer of the thermomechanical energy into the surface one. Besides external sources of damage such as microcrack growth due to external fields or some tools not depending on strain and temperature may exist. The theory taking into account these phenomena following the Griffith energy approach must be based on the energy equation (Kondaurov *et al.*, 1989).

Consider a body whose particles in the reference configuration κ is identified by the position vector $\mathbf{X}$ and in the actual configuration χ at the time instant t by the position vector $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$. Let us denote by $\mathbf{F}(\mathbf{X}, t)$ the deformation gradient, by $\mathbf{v} = \partial\mathbf{x}/\partial t$ the particle velocity, by ρ_κ and ρ densities of mass in κ or χ, respectively, by $\mathbf{b}$ the vector of body forces, by $\mathbf{T}_\kappa$ the Piola stress tensor, by $\theta(\mathbf{X},t)$ the absolute temperature, by U the density of the internal energy without the density $U_\bullet$ of the intrinsic energy of damage, by $\mathbf{q}_\kappa$ the vector of heat flux, by r the density of distributed heat sources and by $r_\bullet$ the density of the external sources of damage. The energy equation takes the form

$$\frac{d}{dt}\int_\kappa \rho_\kappa\left(U + U_\bullet + \frac{1}{2}\mathbf{v}\mathbf{v}\right)d\kappa =$$
$$\int_{\partial\kappa}\left(\mathbf{T}_\kappa^{T}\cdot\mathbf{v} + \mathbf{q}_\kappa\right)\mathbf{n}_\kappa d(\partial\kappa) + \int_\kappa \rho_\kappa\left(r + r_\bullet + \mathbf{b}\mathbf{v}\right)d\kappa \tag{1}$$

where $\mathbf{n}_\kappa$ is the unit normal to the boundary $\partial\kappa$ in the reference configuration. The law of mass conservation and equation of motion are

$$\rho \det \mathbf{F} = \rho_\kappa, \qquad \rho_\kappa \frac{\partial\mathbf{v}}{\partial t} = \nabla_\kappa\cdot\mathbf{T}_\kappa^{T} + \rho_\kappa\mathbf{b} \tag{2}$$

where ∇_κ is the gradient with respect to $\mathbf{X}$ and $\mathbf{I}$ is the unit tensor. With the help of (2) Eq(1) may be written in the form which constitutes the Griffith energy approach

$$\rho_\kappa \dot{U} = \mathbf{T}_\kappa : \dot{\mathbf{F}} + Q + Q_\bullet \tag{3}$$

where $Q = \nabla_\kappa \mathbf{q}_\kappa + \rho_\kappa r$ is the conventional rate of heating while $Q_\bullet = \rho_\kappa\left(r_\bullet - \dot{U}_\bullet\right)$ is the energy rate due to damage. Note that the external sources $r_\bullet$ as well as r are under control and in particular they may be put equal to zero. However in any process of damage $\dot{U}_\bullet$ is different from zero. In the limiting case of a single crack $\dot{U}_\bullet$ and $r_\bullet$ are described by delta functions with their bearers at the crack tip.

The measure of damage will be defined by a second rank tensor π. Similarly to the thermal couple (θ, η) where η is the density of the entropy, let us introduce the couple (π, Π) where Π is the quantity energetically conjugated to π, i.e. a second rank tensor which will be called the entropy of damage. With the help of these couples the second law of thermodynamics may be formulated as the modified Clausius-Duhem inequality (Kondaurov *et al.*, 1989) according to

$$\delta_M + \delta_T + \delta_\bullet \ge 0 \tag{4}$$

where $\delta_M = \theta\dot{\eta} - Q/\rho_\kappa$ is the internal (mechanical) dissipation, $\delta_T = \mathbf{q}_\kappa \nabla_\kappa\theta/(\rho_\kappa\theta)$ is the thermal dissipation and

$$\delta_\bullet = \pi : \dot{\Pi} - \frac{Q_\bullet}{\rho_\kappa} = \pi : \dot{\Pi} + \dot{U}_\bullet - r_\bullet \tag{5}$$

is the dissipation due to damage. Using (1) and (3) reduces the dissipation inequality (4) to the form

$$-\dot{U} + \frac{1}{\rho_\kappa}\mathbf{T}_\kappa : \dot{\mathbf{F}} + \theta\dot{\eta} + \pi : \dot{\Pi} + \delta_T \ge 0 \tag{6}$$

We confine our consideration to the simplest case of damage of a thermo-elastic solid. The response of such a material

$$\{U(\mathbf{X},t), \mathbf{T}_\kappa(\mathbf{X},t), \theta(\mathbf{X},t), \mathbf{q}_\kappa(\mathbf{X},t), U_\bullet(\mathbf{X},t)\}$$

is determined by the current values of the independent state parameters $\{\mathbf{F}(\mathbf{X},t), \eta(\mathbf{X},t), \Pi(\mathbf{X},t), \nabla_\kappa\theta(\mathbf{X},t)\}$. Then the standard procedure of local continuation of the process and the postulate of the thermodynamical compatibility permit to obtain necessary and sufficient conditions of validity of (6)

$$\mathbf{T}_\kappa = \rho_\kappa\, \partial U/\partial\mathbf{F}, \quad \theta = \partial U/\partial\eta, \quad \partial U/\partial(\nabla_\kappa\theta) = 0, \quad \pi = \partial U/\partial\Pi \tag{7}$$

Let us additionally suggest the possibility of the passive continuation ($\dot{\Pi} = 0$) of the process from any state. In this continuation the intrinsic energy and distributed sources vanish. Then from (7) it follows that

$$\delta_M = 0, \qquad \delta_\bullet = 0 \tag{8}$$

These equations lead to the equations for the thermal and damage entropies

$$\rho_\kappa \dot{\eta} - \nabla_\kappa\left(\theta^{-1}\mathbf{q}_\kappa\right) = \rho_\kappa\left(\theta^{-1}r + \delta_T\right)$$

$$[\partial U(\mathbf{F}, \eta, \mathbf{\Pi})/\partial \Pi + \mathbf{R}(\Pi)]:\dot{\Pi} = r_\bullet H(\dot{U}_\bullet) \tag{9}$$

where $\mathbf{R}(\Pi) = \partial U_\bullet(\Pi)/\partial \Pi$ is the material tensor of resistance to damage and $H(z)$ is the unit step function. From (9) it follows that both passive ($\dot{\Pi} = 0$) and active $(\dot{U}_\bullet \neq 0)$ processes are possible. If the process is active from (9) it follows that

$$(\partial U/\partial \Pi + \mathbf{R}):\dot{\Pi} = r_\bullet(t), \qquad \dot{\Pi} \neq 0 \tag{10}$$

The damage process may be either reversible or irreversible. An example of the former type is the process of partial melting, an example of the latter type is the process of micro-fracturing without healing. In the case of an irreversible damage process the following inequality must hold

$$\dot{U}(\Pi) = \mathbf{R}(\Pi):\dot{\Pi} \geq 0 \tag{11}$$

This is also a condition for the change of a passive process into an active one and *vice versa*.

To illustrate the presented model let us consider the simplest isothermal case. Strains ε are assumed to be infinitesimal so that the stress state may be described by the symmetric tensor σ. Damage will be described by a scalar measure ω. The initial configuration is assumed to be stress-and damage-free $(\omega = 0,\ U_\bullet = 0)$. External sources of damage are absent $(r_\bullet = 0)$. The density of the internal energy is taken in the simplest form, i.e.

$$\rho U(\varepsilon, \omega) = K I_1^2/2 + GJ^2 - \alpha_p I_1 \omega - \alpha_s J\omega$$

$$I_1 = \varepsilon{:}\mathbf{I}, \quad J^2 = (\varepsilon - I_1\mathbf{I}/3):(\varepsilon - I_1\mathbf{I}/3) \tag{12}$$

The density of the intrinsic energy of damage is taken in the form

$$\rho U_\bullet(\omega) = \gamma_1\omega + \gamma_2\omega^2/2 \tag{13}$$

Here $K, G, \alpha_p, \alpha_s, \gamma_1, \gamma_2$ are the material constants. From (10) it follows that in the active process $(\omega \geq 0,\ \dot{\omega} > 0)$

$$\omega = \left(\alpha_p I_1 + \alpha_s J - \gamma_1\right)/\gamma_2 \tag{14}$$

The stresses are obtained from (7) and (12) according to

$$\sigma = \left(KI_1 - \alpha_p\omega\right)\mathbf{I} + \left(2G - \alpha_s\omega/J\right)(\varepsilon - I_1\mathbf{I}/3) \tag{15}$$

Note that for $\omega \neq 0$ the expression (15) is non-linear. From (14) it can be seen that in order to start damage the following equation must hold

$$\alpha_p I_1 + \alpha_s J - \gamma_1 = 0 \tag{16}$$

As an example let us consider the case of the monotone uniaxial straining $\varepsilon_{11} = \varepsilon,\ \varepsilon_{ij} = 0$ for $i \neq 1,\ \ j \neq 1$. From (14)-(16) it follows that

$$\sigma_{11} = \begin{cases} \Lambda\varepsilon \\ \left(\Lambda - \gamma_2\alpha_+^2\right)\varepsilon + \gamma_2\alpha_+^2\varepsilon_0^+, \\ \left(\Lambda - \gamma_2\alpha_-^2\right)\varepsilon + \gamma_2\alpha_-^2\varepsilon_0^- \end{cases} \qquad \omega = \begin{cases} 0 \\ \alpha_+\left(\varepsilon - \varepsilon_0^+\right) \\ \alpha_-\left(\varepsilon - \varepsilon_0^-\right) \end{cases} \qquad \text{for} \quad \begin{cases} \varepsilon_0^- \leq \varepsilon \leq \varepsilon_0^+ \\ \varepsilon > \varepsilon_0^+ > 0, \quad \dot{\varepsilon} > 0 \\ \varepsilon < \varepsilon_0^- < 0, \quad \varepsilon < 0 \end{cases}$$

where

$$\alpha_\pm = \left(\alpha_p \pm \alpha_s\sqrt{2/3}\right)\gamma_2, \qquad \varepsilon_0^\pm = \gamma_1/\alpha_\pm, \qquad \Lambda = K + 4G/3$$

Figure 1 shows dependence of the stress σ_{11} on the strain $\varepsilon_{11} = \varepsilon$.

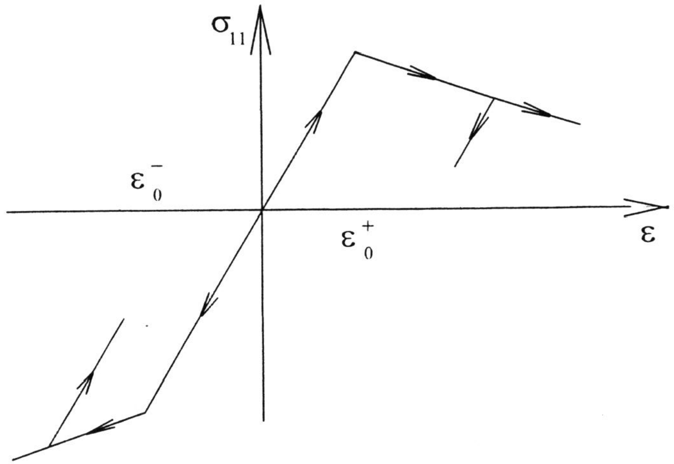

Fig. 1. Stress-strain diagram for uniaxial straining with damage

This simplest application of the developed model shows already the characteristic features of the damage process: the existence of a threshold stress for the onset of damage, the elastic unloading, the dilatancy, the occurrence of damage under the action of both tension and shear, different ultimate strength under tension and compression. The possibility of a falling portion on the stress-strain diagram should especially be emphasised.

ABRUPT DAMAGE

Mesostrucrure of some solids such as polycrystalline brittle metals or rocks under action of high stress undergoes substantial step-like changes. In some cases when defects of the damaged body are numerous it can be again considered as a continuum. In these cases actually a phase transformation takes place caused by the stress rather than the temperature. Hence thermal effects may be neglected while the energy consumption must be taken into account.

Solids undergoing substantial changes of the internal mesostructure are usually described by the models of the strain softening material. However existence of strain softening elasto-plastic materials is questionable (Nikitin, 1996). A model considered in this paper is alternative to that with strain softening. We consider a solid transforming under action of stress into another solid with different mechanical properties and different stress-free state. Thermal effects are neglected while energy consumption will be allowed for.

Assume for simplicity that the parental and damaged solids are both elastic and undergo infinitesimal strain ε. The elastic potential W_p and stress σ for the parental material may be written in the form

$$W_p = \frac{1}{2}\varepsilon:\mathbf{C}:\varepsilon, \qquad \sigma = \mathbf{C}:\varepsilon \qquad (17)$$

Here **C** is the elasticity tensor of the parental solid. The elastic potential of the damaged body in the same as in (17) reference configuration must involve zero and first order terms in strain

$$W_d = W_* - \varepsilon:\mathbf{C}^d:\varepsilon_* + \frac{1}{2}\varepsilon:\mathbf{C}^d:\varepsilon, \qquad \sigma = \mathbf{C}^d:(\varepsilon - \varepsilon_*) \qquad (18)$$

where $\mathbf{C}^d$ is the elasticity tensor of the damaged solid, ε_* is the kinematic tensor of damage and W_* is the energy of the structural transformation due to damage.

Abrupt damage in general takes place when a function of the invariants of the stress tensor reaches a threshold value. In the simplest case one of the next equations may be taken as a criterion of abrupt damage

$$\sigma_1 = \sigma_d \qquad or \qquad J_2 = \tau_d \qquad (19)$$

where σ_1 is the maximal principal stress, J_2 is the second invariant of the stress deviator and σ_d, τ_d are material constants. Abrupt damage is analogous to a thermal phase transformation. The counterparts of the temperature and the latent heat of a transformation are σ_d or τ_d and W_*, respectively.

Nikitin (1995) has shown that a homogeneous stress state in the strain softening material is unstable: strains are localised and dynamic process starts even in the case of quasi-static, displacement controlled loading. The same reasonings are applied to the solid under consideration. Thus even in the case of the quasi-static loading a static equilibrium becomes unstable upon reaching of the threshold stress and damage may occur dynamically. The model of the structural transformation with discontinuities along a moving front which is close to that considered here was recently developed by Pradeilles-Duval and Stolz (1995). They considered the quasi-static process of transformation only. However in the all considered examples they obtained the stress-strain diagram with a descent segment. This unavoidably must lead to the loss of stability.

To demonstrate mechanical behaviour of a solid with abrupt damage we consider two problems: stretching of a rod and propagation of a semi-infinite longitudinal shear crack.

Stretching of a Rod

Consider quasi-static uniaxial stretching of a rod. The elastic potentials and stresses σ in the parental and damaged solids may be written in the form

$$W_p = \frac{1}{2}E_p\varepsilon^2, \qquad \sigma = E_p\,\varepsilon$$

$$W_d = W_* - E_d\varepsilon_*\varepsilon + \frac{1}{2}E_d\,\varepsilon^2, \qquad \sigma = E_d(\varepsilon - \varepsilon_*) \qquad (20)$$

Here the strain ε is referred to the natural stress-free configuration of the parental material, E_p and E_d are Young's moduli of the parental and damaged solids, respectively, ε_* and W_* are the kinematic and energy characteristics of abrupt damage, Fig.2.

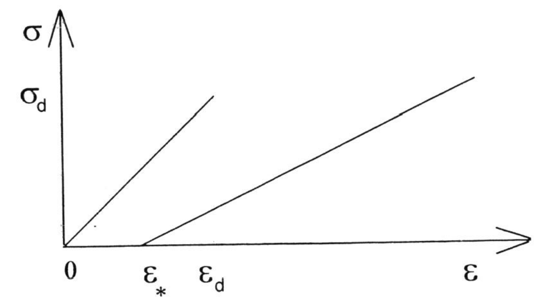

Fig. 2. Stress-strain diagram for a solid with abrupt damage

As it was mentioned above as soon as the stress reaches the threshold value $\sigma = \sigma_d$ homogeneous stress and strain states in a rod lose stability, strains are localised at some cross-sections and the dynamic process starts. Equation of motion in terms of displacement u is

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}, \qquad a^2 = \frac{1}{\rho}\frac{d\sigma}{d\varepsilon} \qquad (21)$$

Velocity of wave propagation a takes value $a = a_p \equiv (E_p/\rho)^{1/2}$ for the parental solid and $a = a_d \equiv (E_d/\rho)^{1/2}$ for the damaged one. Location and number of places of localisation can not be found. The dynamic process in the vicinity of one of them, say at $x = 0$, will be studied. The problem is self-similar since it does not contain any scale length or time. Hence the wave fronts radiating from $x = 0$ are straight and stress and particle velocity $v = \partial u/\partial t$ are constant between the fronts. Due to the symmetry we consider the region $x > 0$ only. The fronts of unloading $x = a_p t$ and abrupt damage $x = bt$ start simultaneously, Fig.3. Velocity b of the front and the stress drop behind it are to be found. The region $x < a_p t$ is at rest

$$v = 0, \qquad \sigma = \sigma_d \tag{22}$$

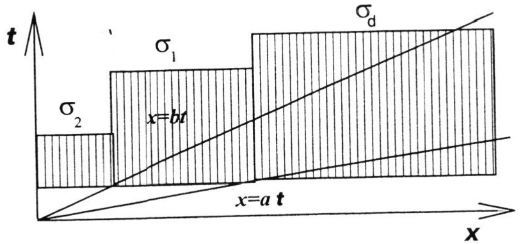

Fig. 3. The wave fronts and stress distribution in the dynamic process of abrupt damage

In the region $a_p t < x < bt$ between the fronts of elastic unloading and abrupt damage both the stress σ_1 and the velocity v_1 are unknown. Behind the front of abrupt damage $v = 0$ due to the symmetry while the stress σ_2 is to be found. At the front of unloading the principle of momentum gives

$$\sigma_d - \sigma_1 = a_p \rho v_1, \qquad x = a_p t \tag{23}$$

At the front of abrupt damage the condition of displacement discontinuity along with the principle of momentum and the energy equation give

$$b(\varepsilon_1 - \varepsilon_2) + v_1 = 0, \qquad \sigma_1 - \sigma_2 + b\rho v_1 = 0$$
$$b\left(W_p - W_d - \frac{1}{2}\rho v_1^2\right) + \sigma_1 v_1 = 0, \qquad x = bt \tag{24}$$

From (20)

$$\sigma_1 = E_p \varepsilon_1, \qquad \sigma_2 = E_d(\varepsilon_2 - \varepsilon_*) \tag{25}$$

Eqs(22)-(25) constitute the system of six simultaneous equations for determination of six unknowns $\sigma_1, \sigma_2, v_1, \varepsilon_1, \varepsilon_2$ and b. Numerical analysis of the system shows that for small W_* it always has solution with $0 < b < a_d$. With growth of W_* velocity b approaches zero and discontinuity at the front $x = a_p t$ vanishes. Solution of this system, for instance for $E_d = \frac{1}{4}E_p$, $\varepsilon_* = \frac{1}{2}\sigma_d/E_p \equiv \frac{1}{2}\varepsilon_d$, $W_* = \frac{1}{2}\sigma_d^2/E_p$ give for the shock front velocity of abrupt damage $b \cong 0{,}30\, a_p$, for the stress drop behind the front of unloading $\sigma_1 \cong 0{,}67\sigma_d$ and for the strain localisation $\varepsilon_2 \cong 2{,}60\varepsilon_d$, Fig.3. Velocity b in this case vanishes when $W_* \cong \frac{3}{2}\sigma_d^2 / E_p$.

Propagation of a Longitudinal Shear Crack

Crack propagation is controlled by the energy absorbed near the crack tip. Broberg (1964) was the first who calculated the energy flux into the crack tip for an elastic medium. The commonly used expression for the energy release rate explicitly, in terms of the stress intensity factors was published by Kostrov *at al.*, (1969) in Russian and Kostrov and Nikitin, (1970) in English. On the base of this expression the Griffith energy balance was extended on the case of dynamics and gave dependence of the crack tip velocity on the stress intensity factors. However this dependence was not always confirmed experimentally (Knauss and Ravi-Chandar, 1985). Discrepancy between theory and experiment may be due to non-elastic behaviour of material near the crack tip. Formation of numerous defects ahead the crack tip may be modelled as abrupt damage.

Consider the stationary propagation of a semi-infinite longitudinal shear crack. Refer a crack to the Cartesian co-ordinates x_1, x_2 with the origin at the moving crack tip and the axis x_1 directed along a crack Fig.4.

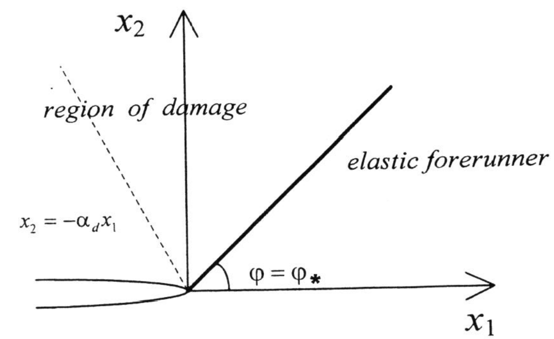

Fig.4. Longitudinal shear crack.

The elastic potentials and shear stresses τ_i for the antiplane stress state are

$$W_p = \frac{1}{2}\mu_p\gamma_i\gamma_i, \qquad \tau_i = \mu_p\gamma_i$$

$$W_d = W_* - \mu_d\gamma_i^*\gamma_i + \frac{1}{2}\mu_d\gamma_i\gamma_i, \qquad \tau_i = \mu_d\left(\gamma_i - \gamma_i^*\right) \tag{26}$$

Here $\gamma_i = \partial w/\partial x_i$ is the shear strain and w is the only nonzero component of the displacement. Since motion is stationary the derivative with respect to time t reduces the derivative with respect to x_1

$$\frac{\partial}{\partial t} = -v\frac{\partial}{\partial x_1}$$

Equations of motion for the parental and damaged solids take the form

$$\left(1 - \frac{v^2}{b_p^2}\right)\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} = 0, \qquad b_p = \left(\frac{\mu_p}{\rho}\right)^{1/2} \tag{27}$$

$$\left(\frac{v^2}{b_d^2} - 1\right)\frac{\partial w}{\partial x_1^2} = \frac{\partial^2 w}{\partial x_2^2}, \qquad b_d = \left(\frac{\mu_d}{\rho}\right)^{1/2} \tag{28}$$

The upper half of the plane $x_2 = 0$ only may be considered due to the symmetry. The crack banks are assumed to be stress free

$$\tau_2 = 0, \qquad x_2 = 0, \qquad x_1 < 0 \tag{29}$$

The case of trans-sonic velocity will be studied

$$b_d < v < b_p$$

Then Eq.(27) is elliptic and describes the non-singular state of the elastic forerunner in the sector $0 < \varphi < \varphi_*$, where φ is the polar angle $\varphi = \operatorname{arctg} x_2 / x_1$. Taking in mind the study of energetics of fracture the singular part of solution only will be considered. Therefore consider solution of Eq.(28) of the form

$$w = c_1\left|x_2 + \alpha_d x_1\right|^{1/2} + c_2\left|x_2 - \alpha_d x_1\right|^{1/2} \tag{30}$$

where $\alpha_d = b_d/\left(v^2 - b_d^2\right)^{1/2}$ and c_1, c_2 are constants. Stresses and strains in this case have the square root singularity what is needed for non-zero energy flux into the crack tip.

Abrupt damage takes place along the ray $\varphi = \varphi_*$. Continuity of the displacement and the principle of momentum give along $\varphi = \varphi_*$ the following equations for the case under consideration (Mukhamediev and Nikitin, 1989)

$$\gamma_1\cos\varphi_* + \gamma_2\sin\varphi_* = 0 \tag{31}$$

$$\gamma_1\sin\varphi_* + \alpha_d^2\gamma_2\cos\varphi_* = 0 \tag{32}$$

The energy release G_φ along $\varphi = \varphi_*$ is assumed to be finite. Then

$$G_\varphi = \frac{v\mu_d\sin\varphi_*}{2\alpha_d}\left(\alpha_d^2\gamma_2^2 - \gamma_1^2\right) = 0 \tag{33}$$

Solution in the region adjoining the crack bank $\pi > \varphi > \varphi_*$ with taking into account (29) is

$$w = K\left(-x_2 - \alpha_d x_1\right)^{1/2} + K\left(x_2 - \alpha_d x_1\right)^{1/2} \tag{34}$$

Here K plays a role of the stress intensity factor. If φ_* lies to the left from the characteristic $x_2 = -\alpha_d x_1$ of Eq(28) the condition (29) can not be met. Hence $\varphi_* < \pi - \operatorname{arctg}\alpha_d$ and solution in the region to the right of the characteristic $x_2 = -\alpha_d x_1$ is

$$w = K\left(x_2 - \alpha_d x_1\right)^{1/2} \tag{35}$$

The conditions (32) and (33) lead to

$$\varphi_* = \operatorname{arctg}\alpha_d \tag{36}$$

The condition of displacement continuity (31) was not used what seems to be admissible in the case of abrupt damage. However this condition appears to be met automatically.

Although solutions (34), (35) possess the square root singularity the energy flux into the crack tip calculated on the base of them vanishes. For the first glance the result is disappointing. However the experimental observations show that the stress intensity factors do not always control crack velocity (Knauss and Ravi-Chandar, 1985). Energy release as in elasto-plastic solids is diffused and is not concentrated at the crack tip as in elastic solids.

CONCLUSIONS AND DISCUSSION

Two different problems of fracture, namely gradual and abrupt damage are considered using the Griffith type energy approach. The developed energy theory of gradual damage describes a number of observed effects: existence of the threshold stress and strain for incipience of fracture, dilatancy, fracture under both tension and shear but not under compression, possibility of a falling portion on the stress-strain diagram.

The model of abrupt damage is alternative to the questionable model of the elasto-plastic softening solid. Even quasi-static loading of a structure made of such solids leads to the dynamic process with strain localisation and formation of shock waves of unloading and abrupt damage. In spite of the square root singularity of stress and strain the energy release rate at the crack tip of a fast moving longitudinal shear crack in a medium with abrupt damage is absent.

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