A SIMPLE METHOD OF J-INTEGRAL ESTIMATE FOR BIAXIALLY STRESSED MODE I CRACKS

M. SAKANE*, T. ITOH** and M. OHNAMI*

* *Department of Mechanical Engineering, Ritsumeikan University, 1916, Noji-machi Kusatsu Shiga, Japan*

** *Department of Mechanical Engineering, Fukui University, 9-1 3-chome Bunkyo Fukui, Japan*

ABSTRACT

This paper describes a simple method of estimating J integral for biaxially stressed Mode I cracks. Finite element analyses were made to obtain the relationship between J integral and crack opening displacement by changing constitutive relation, crack length and stress/strain biaxiality. A simple method was proposed to estimate J integral based on the equivalent strain based on crack opening displacement for Mode I cracks under biaxial stress states.

KEYWORD

J integral, Crack opening displacement, FEM analysis, Mode I crack, Multiaxial stress.

INTRODUCTION

Multiaxial stress has an influence on crack propagation behavior when a relatively large plastic zone exists ahead of crack. In torsion low cycle fatigue tests using a tubular specimen, for example, enhanced crack propagation rate was observed by the assistance of parallel stress to Mode I crack (Sakane *et al.*, 1988), but there have been few experimental and theoretical studies related to the inelastic stress intensity for biaxially stressed cracks.

J integral and crack opening displacement (COD) are typical inelastic fracture mechanics parameters which express the stress/strain intensity, and a linear relationship holds between these parameters under a uniaxial stress state (Paranjpe and Banerjee, 1979; Paris *et al.*, 1979). However, there exists no relationship proposed between the two parameters under multiaxial stress states.

This paper carried out finite element analyses for Mode I cracks stressed biaxially, and J integral and COD values were calculated by changing crack length and material constants, *i.e.*, yield stress (σ_Y), strain hardening coefficient (A) and strain hardening exponent (n), which characterize the inelastic deformation behavior. A simple method of estimating J integral is proposed based on the equivalent strain based on COD (COD strain) (Hamada *et al.*, 1991; Itoh *et al.*, 1994).

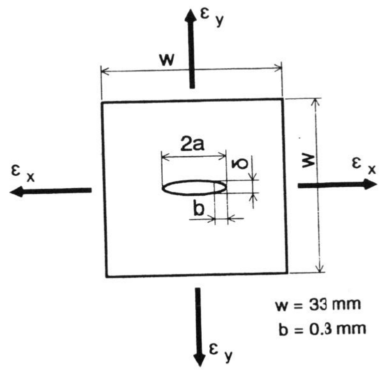


Fig.1 Model for FEM analyses.

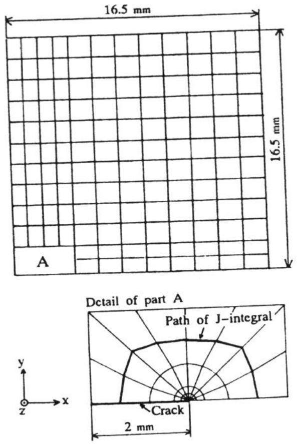


Fig.2 FEM mesh used.

Table 1 Values of σ_Y, A and n for 25 cases.

Case	σ_Y (MPa)	100					200	400	800
	A (MPa)	300	600	1200	2500	5000		2500	
n	0.10	–	3	–	10	–	–	19	–
	0.15	1	4	8	11	15	17	20	24
	0.20	–	5	–	12	–	–	21	–
	0.30	2	6	9	13	16	18	22	25
	0.50	–	7	–	14	–	–	23	–

FEM ANALYSES

The FEM model analyzed is a center cracked plate of 33mm × 33mm in dimension as shown in Fig.1. In FEM analyses, a quarter of the model was analyzed considering the symmetry of the model. This paper employs ϕ (=$\varepsilon_x/\varepsilon_y$) to express the strain biaxiality, where ε_x and ε_y are the principal strains parallel and normal to crack, respectively. Figure 2 shows a FEM mesh with a 4mm center crack. The number of nodes and elements of the FEM mesh are 180 and 589, respectively. The FEM code used was MARC K-5.

Values of Young's modules and Poisson's ratio used in the analysis were 180 GPa and 0.3, respectively. The following equation was used to express the plastic deformation.

$$\varepsilon_p = \left(\frac{\sigma - \sigma_Y}{A}\right)^{\frac{1}{n}} \qquad (1)$$

ε_p is plastic strain, σ_Y yield stress, A strain hardening coefficient and n strain hardening exponent. Twenty five constitutive relations were applied to FEM analyses where the

combination of the values of σ_Y, A and n is listed in Table 1.

Values of J integral and crack opening displacement away 0.3mm from the crack tip were calculated using 25 constitutive equations. In each analysis, the principal strain ratio was ranged from −1 to 1 and crack length from 2mm to 6mm.

The authors (Itoh *et al.*, 1994) proposed the COD strain (ε^*) below for correlating the multiaxial low cycle fatigue lives.

$$\varepsilon^* = \beta\,(2-\phi)^{m'}\varepsilon_y \qquad (2)$$
$$\beta = 1.83, \quad m' = -0.66$$

where β and m' are the constants independent of the material. The COD strain physically expresses the COD amplitude of Mode I crack under biaxial stress state.

RESULTS AND DISCUSSION

Relationship Between J and ε in Small Scale Yielding*

Figures 3 (a)–(d) show the typical results of the J–ε^* relationship obtained by FEM. A unique

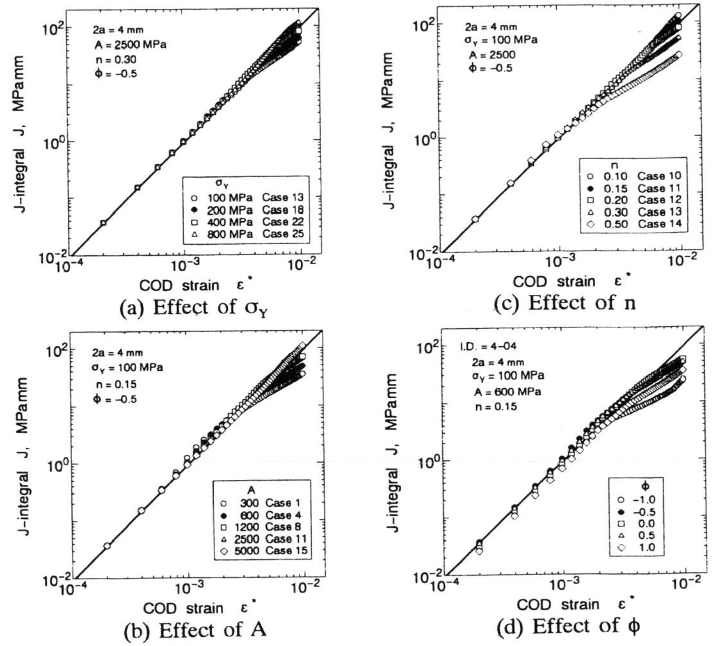
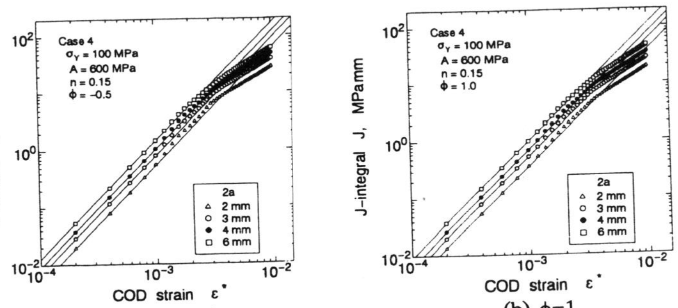


Fig.3 J–ε^* relationships at 2a=4 mm.



(a) ϕ=−0.5 (b) ϕ=1

Fig.4 J−ε^* relationships at ϕ=−0.5 and 1.

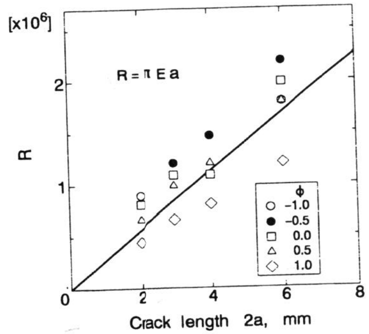


Fig.5 R *vs* 2a.

relationship holds between J and ε^* in the region of ε^* being less than 0.2% for all the cases shown in Fig.3. The relationship does not depend on σ_Y, A, n and ϕ, so J can be equated as a function with only ε^*,

$$J = R(\varepsilon^*)^r \tag{3}$$

Figures 4 (a) and (b) show the effect of crack length on the J−ε^* relationship. Linear relationships also hold in the small strain region but the J integral value increases with increasing crack length. Thus, R in equation (3) is a function of crack length, but the exponent r is a constant of 2 independent of crack length.

Figure 5 shows the variation of R with crack length, 2a. A linear relationship holds between R and 2a, whereas there is a small scatter in the correlation.

$$R = \pi E a \tag{4}$$

where E is the Young's modules and takes the value of 180 GPa. In small scale yielding, therefore, J integral can be equated with only ε^* as equations (3) and (4) independent of the material constants.

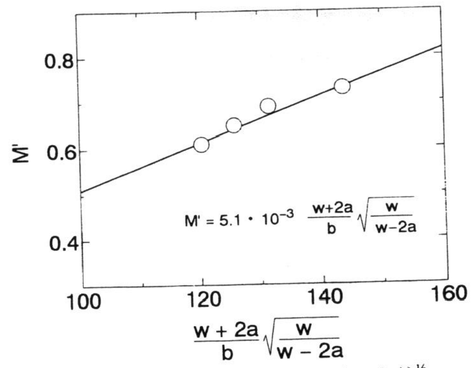


Fig.6 M' *vs* 1/b (w+2a) (w/(w−2a))$^{1/2}$.

Relationship Between J and ε^* in Large Scale Yielding

In large scale yielding, ε^* being larger than 0.2%, the J−ε^* relationship is not simply equated as equations (3) and (4), and it depends on the material constants and the principal strain ratio as shown in Figs.3 and 4. Theoretical and numerical analyses showed that the relationship between J integral and the crack opening displacement, δ, in a uniaxial stress state is expressed as

$$J = M \sigma_Y \delta \tag{5}$$

M is a parameter of plastic constrain parameter and takes unity in uniaxial plane stress. The equation holds for not only in small scale yielding but also in large scale yielding, which was proved analytically and numerically (Paranjpe and Banerjee, 1979).

We will modify equation (5) to develop equation available in biaxial stress. The modified equation is

$$J = M' k \left(\frac{A}{\sigma_Y}\right)^{1-n} \sigma_Y \delta \tag{6}$$

M' is a geometrical factor which is a function of specimen width (w), crack length (2a) and the distance (b) between crack tip and the location where COD is estimated. M' is explicitly expressed as;

$$M' = 5.1 \times 10^{-3} \frac{w+2a}{b} \sqrt{\frac{w}{w-2a}} \tag{7}$$

The above equation is obtained based on the relationship shown in Fig.6

Since the stress biaxiality has an influence on the J−δ relationship, k is set to the ratio of COD stress (Hamada *et al.*, 1991) to the Mises' equivalent stress, *i.e.*;

$$k = \frac{\alpha\,(2-\lambda)^m}{\sqrt{\lambda^2-\lambda+1}}\ ,\qquad \lambda = \frac{\sigma_3}{\sigma_1} \tag{8}$$

where λ is the principal stress ratio defined as $\lambda=\sigma_3/\sigma_1$; σ_1 and σ_3 are the maximum and minimum principal stresses, respectively. α and m are the constant taking the values of 0.707 and 0.5 in the range of $-1\leq\lambda\leq0$ and 0.95 and 0.075 in the range of $0\leq\lambda\leq1$, respectively.

The COD strain (ε^*) expresses crack opening displacement under biaxial stress state but it does not take account of the crack length. The crack opening displacement (δ) under biaxial stress is equated with the COD strain by multiplying L;

$$\delta = L\,(\varepsilon^*)^{1.25} \tag{9}$$

L is a function of crack length 2a, the material constants σ_Y, A and strain biaxiality ϕ.

$$L = L'\,b\sqrt{\frac{2a}{w-2a}}\,\left(\frac{\sigma_Y+A}{E}\right)^{-0.3}\ ,\qquad L' = -20\,(\phi+0.1)^2+66 \tag{10}$$

These equations are obtained by approximately the FEM results shown in Fig.7.

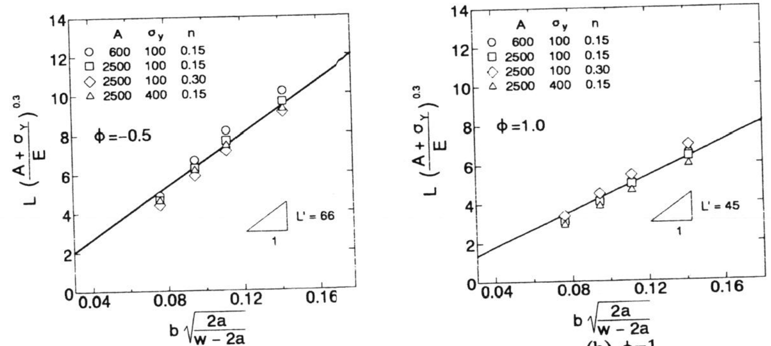
Combining the above equations leads to

$$J = 5.1\cdot10^{-3}\,k\,L'\,\frac{w+2a}{w-2a}\,\sqrt{2a\,w}\;\sigma_Y^n\,A^{(1-n)}\left(\frac{A+\sigma_Y}{E}\right)^{-0.3}(\varepsilon^*)^{1.25} \tag{11}$$

To simplify the equation, kL' is approximated as

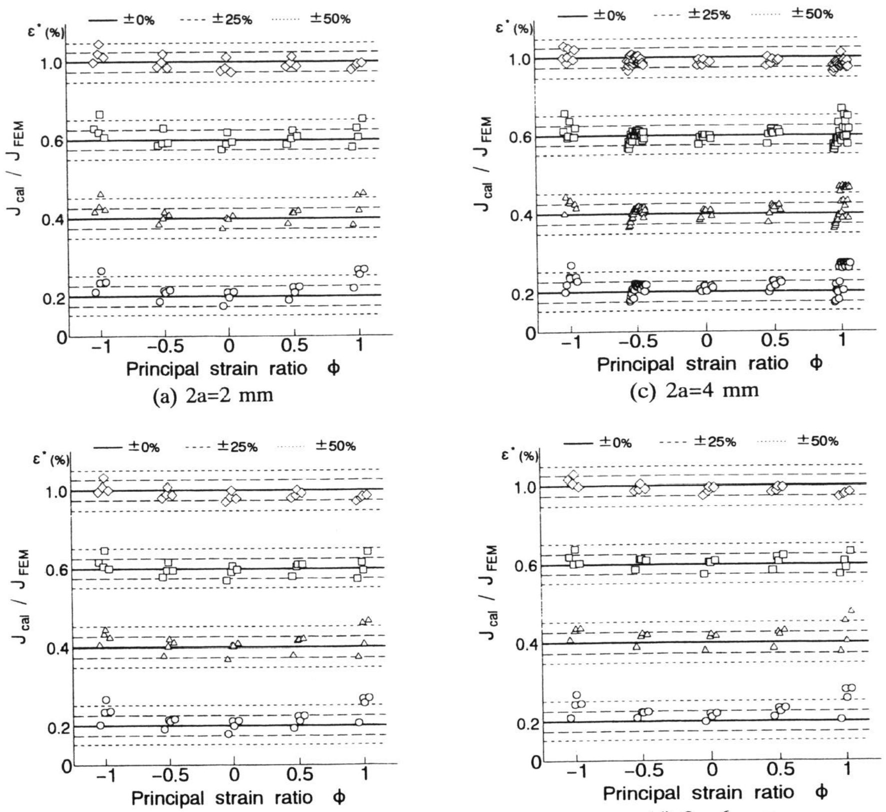
$$k\,L' = -33\,\phi^2+75 \tag{12}$$

where the graphical representation is omitted here.



(a) ϕ=−0.5 (b) ϕ=1

Fig.7 $L\,((A+\sigma_Y)/E)^{0.3}$ *vs* $b\,(2a/(w-2a))^{1/2}$.



(a) 2a=2 mm (c) 2a=4 mm (b) 2a=3 mm (d) 2a=6 mm

Fig.8 Comparison between the J integral predicted and that calculated by FEM.

The final form of the relationship between J integral and ε^* is

$$J = 5.1\times10^{-3}\times f_1\times f_2\times f_3\times(\varepsilon^*)^{1.25}$$

$$f_1 = -33\,\phi^2+75\ ,\qquad f_2 = \frac{w+2a}{w-2a}\sqrt{2a\,w}\ ,\qquad f_3 = \sigma_Y^n\,A^{(1-n)}\left(\frac{A+\sigma_Y}{E}\right)^{-0.3} \tag{13}$$

The values of J integral of Mode I crack under biaxial stress states can be calculated for the various material constants, only knowing the principal strain ratio (ϕ), material constants (σ_Y, A and n) and crack length (2a).

The J integral values (J_{cal}) calculated by equations (3) and (13) are compared with those of FEM analysis (J_{FEM}) in Fig.8. Almost all the J integrals estimated by the two equations are within a factor of 0.25 scatter band. Thus, equations (3) and (13) are available to estimate the J integral under biaxial stresses. However, J integrals at ϕ=1 estimated are beyond a factor of ±50% in the small strain region at all the crack length. Improvement of the estimation is necessary for these data in future.

CONCLUSION

A simple method of estimating J integral for Mode I crack under biaxial stress states is proposed based on the COD strain. In small scale yielding, the equation proposed is a function of only crack length, but in large scale yielding, it is a function of σ_Y, A, n, ϕ and 2a. Both the equations estimates the J integral within a factor of ±0.25 except for that at ϕ=1.

REFERENCES

Hamada, N., M. Sakane and M. Ohnami (1991). In:*Fatigue Under Biaxial and Multiaxial Loading*, ESISIO (K. F. Kussmaul, *et al.*), Mech. Engng. Pub., London.

Itoh, T., M. Sakane and M. Ohnami (1994). High Temperature Multiaxial Low Cycle Fatigue of Cruciform Specimen, *ASME, J. Engng. Mater. Tech.*, **116**, 90–98.

Paranjpe S. A. and S. Banerjee (1979). Interrelation of Crack Opening Displacement and J–integral, *Engng. Fract. Mech.*, **11**, 43–53.

Paris, P. C., H. Tada, A. Zahoor and H. Ernst (1979), In:*Elastic Plastic Fracture*, STP668 (J. D. Lardes *et al.*, ed.), 5–36, American Society for Testing and Materials, Philadelphia.

Sakane, M., M. Ohnami and N. Hamada (1988). Biaxial Low Cycle Fatigue for Notched, Cracked, and Smooth Specimens at High Temperature. *ASME, J. Engng. Mater. Tech.*, **110**, 48–54.