A NEW APPROACH TO INTERACTING 3D CRACKS

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ABSTRACT

The phenomenon of aging structures has focused attention on the problems of multiple-site damage (MSD) and wide spread fatigue damage (WFD). In isolation each flaw or crack may be safe. However, the cumulative effect of multiple, interacting cracks may significantly degrade the damage tolerance of a structure. To assist in the damage tolerant evaluation of MSD this paper presents a new finite element alternating technique algorithm suitable for calculating Mode I stress-intensity factors for an arbitrary number of coplanar interacting three dimensional cracks.

KEYWORDS

multi-site damage, finite element analysis, 3D cracks

INTRODUCTION

In 1990 it was reported that 31% of U.S. aircraft fleet exceeded the design goals set by the manufacturers. It was also estimated that by the year 2000, 64% of the worldwide fleet manufactured in the U.S. would be 20 or more years old, Hendricks (1991). At that time the quoted replacement cost of an aircraft was around 55 million dollars. In comparison to this, the cost of a major overhaul of a Boeing 727 was 2 million, whilst for a Boeing 747 this cost was in the region of 4 to 20 million dollars. This trend in operating existing aircraft approaching or operating beyond their intended design life is reflected in an increasing number of structurally significant defects and increases the possibility of a reduction, or loss, of structural integrity due to fatigue. In Australia, the importance of maintaining continued airworthiness was further highlighted by the November 1990 failure of a Royal Australian AirForce (RAAF) Macchi aircraft which suffered a port wing failure whilst in an estimated 6g manoeuvre, Young (1994). A tear down inspection program involving two fuselages, two fins and five horizontal tail planes was subsequently undertaken. Six of the wings showed significant cracking indications and of approximately 1000 holes which were examined, 100 revealed fatigue cracks, including major cracking in the D series rivet holes.

To meet the challenge of ensuring the continued airworthiness it is clear that a methodology for rapidly and accurately assessing damage tolerance of a structural component containing interacting 3D cracks under complex loading is required. This paper describes one such approach.

METHODOLOGY FOR THE ANALYSIS OF INTERACTING THREE-DIMENSIONAL CRACKS.

Swedlow (1988) noted that cracking in structural components is best characterised by three-dimensional modelling rather than by conventional two-dimensional approaches. A part-through crack is a typical example. This type of crack often develops in regions with high stress concentrations such as rivet holes and accounts for many common fatigue and fracture failures. If the conventional finite element method is used to analyse such problems then a detailed numerical modelling of the crack tip singularity is necessary. The relatively fine meshes near the crack front required, to accomplish this, often makes the computation prohibitively expensive and extremely time consuming. To circumvent this problem Annigeri and Cleary (1984) used a surface integral and finite element hybrid method to model fracture problems in finite plane domains. As in the FEAT the crack is modelled independently of the finite element mesh. The required mesh complexity for standard finite element analysis is particularly apparent for problems involving complex geometrical shapes such as fuselage structures with stringers, stiffeners and lap joints, and even more so for those problems involving multiple cracks.

To solve problems involving multiple interacting three-dimensional cracks, in a computationally efficient manner an extension to the finite element alternating technique (FEAT) has been developed. When using the FEAT to solve three-dimensional fracture mechanics problems, extensive use is made of the analytical solution for a single elliptical flaw, embedded in an infinite linear elastic solid, where arbitrary normal and shear crack face loadings are applied to the crack. Historically a complete solution for such a flaw did not exist until 1981 when Vijayakumar and Atluri (1981) considered normal as well as arbitrary shear loading. These authors derived expressions for stress-intensity factors near the flaw border, as well as for stresses in the far field, for these generalised loadings. The key to implementing this solution in the finite element alternating technique is the evaluation of the necessary elliptic integrals. A general procedure for calculating these is detailed in the work of Nishioka and Atluri (1983). When using the finite element alternating technique, the interchange between the finite element model for the uncracked geometry and the complete analytical solution for an embedded elliptical flaw means that accurate results can be obtained using a relatively coarse mesh, without any complex mesh refinement around the crack tips. This greatly improves the computational efficiency for solving three dimensional fracture mechanics problems.

A brief description of the manner in which the finite-element alternating technique operates for a single crack is as follows :-

i) The standard finite-element methodology is used to analyse the uncracked finite body under the prescribed external loads. The geometry of the uncracked body used here is identical to that of the cracked body except that the crack itself is not modelled.

ii) As the crack is not modelled explicitly, non-zero stresses are calculated at the location of the actual crack. These fictitious stresses must be removed in order to create the traction free crack surface that exists in the problem under consideration.

iii) The analytical solution for an embedded elliptical crack subject to an arbitrary distribution of tractions on the crack face is now used. To create the required stress-free crack face the stresses determined in (ii), at the location of the crack faces, are now reversed and a polynomial representation is obtained using a least squares fit. In order to accomplish this the tractions along the crack surface are expressed as a finite order polynomial in x_1 and x_2 as follows

$$\sigma_{3\alpha}^{(0)} = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{m=0}^{M}\sum_{n=0}^{m} A_{\alpha,m-n,n}^{(i,j)} x_1^{2m-2n+i} x_2^{2n+j} \tag{1}$$

where α=1,2,3 and the values of (i,j) specify the symmetries of the load with respect to the axes of the ellipse. The solution in terms of a potential function is then assumed to be of the form

$$f_\alpha = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{M}\sum_{l=0}^{k} C_{\alpha,k-l,l}^{(i,j)} F_{2k-2l+i,2l+j} \tag{2}$$

Using equations for the stress field it is possible to relate the known coefficients A_α in equation (1) to the unknown coefficients C_α of equation (2), see Nishioka et. al (1983) for further details. These crack face stresses are then applied to the infinite geometry cracked problem and the associated stress intensity factors are calculated using :-

$$K_I = 8\mu\left(\frac{\pi}{a_1 a_2}\right)^{1/2} A^{1/4} \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{M}\sum_{l=0}^{k} (-2)^{2k+i+j}(2k+i+j+1)! \\ \frac{1}{a_1 a_2}\left(\frac{\cos\theta}{a_1}\right)^{2k-2l+i}\left(\frac{\sin\theta}{a_2}\right)^{2l+j} C_{3,k-l,l}^{(i,j)} \tag{3}$$

where θ is the elliptic angle and

$$A = a_1^2 \sin^2\theta + a_2^2 \cos^2\theta$$

Thus when all the parameters {C} have been determined, the stress intensity factor at any point along the crack border can be directly evaluated.

iv) The stresses on the external surfaces of the finite body due to these applied loads on the crack faces are now calculated using both (iii) and the solution for an embedded elliptical crack in an infinite body. These boundary stresses are then reversed and applied as external loads on the original uncracked body.

v) This addition to the external loads again creates new fictitious stresses at the location of the crack which must once again be removed to obtain the stress-free crack faces in the actual configuration.

All of the steps (i-v) in the iteration process described above are repeated until the residual stresses on the external boundary are reduced below a user defined tolerance. The overall stress-intensity factor solution is obtained by summing the stress-intensity factor solutions for all iterations.

Note that to analyse the uncracked body, a three-dimensional finite-element model using 20-noded isoparametric elements is used. The mesh used to describe the uncracked body must be sufficiently refined to accurately characterise the stress distribution in the uncracked body. If the geometry and applied loading of the body are relatively simple then not many elements are required as the stress state is not complex. However, if by virtue of the geometry or the loading, there are significant stress gradients in the uncracked body, then a more sophisticated mesh will be required. It is also necessary that there be a reasonable amount of refinement in the region of the crack to enable accurate fitting of a polynomial distribution to the fictitious stresses on the crack face. In most problems this occurs if there are two elements spanning the major and minor axes of the crack.

The extension of the FEAT approach to allow for the interaction effects of multiple interacting three-dimensional cracks requires the incorporation of two additional iterative processes into the overall FEAT procedure. A brief description of this newly developed MSD FEAT algorithm is as follows:-

i) Firstly, a standard finite-element process is used to analyse the uncracked finite structure under the prescribed external loads, σ_1. Once again the geometry of the uncracked structure used here is identical to that of the cracked structure except that the cracks themselves are not modelled, see Figure 1(b). Hence there are considerable savings in mesh generation and CPU time due to the greatly reduced number of nodes and elements required for this model. This also has the advantage that cracks can be made of a different shape or size, or indeed moved to a different location, and the problem re-evaluated with relative ease. Such versatility is not a practical reality using a conventional finite element approach for a multiple crack problem.

ii) As the cracks are not explicitly modelled, the finite element solution calculates nonzero stresses σ'_{12} at the location of each of these cracks. In this notation σ'_{12} is the stress produced on crack face 2 by the original boundary stress distribution σ_1. As in the single crack case these fictitious stresses must be removed in order to create the traction free crack surfaces that exist in the actual cracked problem under consideration. For each of the cracks a polynomial function, for the reverse of these fictitious stresses, is determined using equation (1). These crack face stresses are then applied to the infinite geometry cracked problem. Note that at this stage there is still no allowance for any interaction between the cracks.

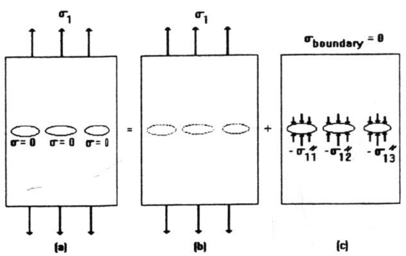

Figure 1 - First part of the MSD FEAT process.

The finite structure to be solved is illustrated in Figure 1.1(a). Using the principle of superposition this is equivalent to the sum of the problems illustrated in Figures 1(b) and 1(c). In Figure 1(b) a coarse mesh is used for a finite element analysis of the uncracked structure. In Figure 1(c) the stresses on the crack faces are the reversed crack face stresses evaluated from Figure 1(b) plus the interaction stresses (which will be evaluated in step iii).

iii) The crack interaction effect is now evaluated in the following manner :-

a) Using the stresses calculated in step (ii), coupled with the full analytical solution for an embedded elliptical crack in an infinite body, the stresses caused by the presence of the first crack on each of the other cracks can be evaluated in turn.

b) This procedure is repeated for each of the other cracks present.

c) The final tractions on each crack are then the original tractions calculated in step (ii) plus the sum of all the 'interaction' stresses evaluated in this step, see Figure 2. In the case of crack two for instance : $-\sigma''_{12} = -\sigma'_{12} + -\sigma'_{112} + -\sigma'_{132}$.

This particular part of the process produces an 'error', in that the boundary forces on the finite body are now no longer zero as they were in part (ii). This shortcoming is dealt with as the cracks are progressively unloaded in stage (v).

iv) By employing equations (1) and (2) and the final tractions on each crack face the associated stress intensity factors for each crack for this particular iteration can be calculated.

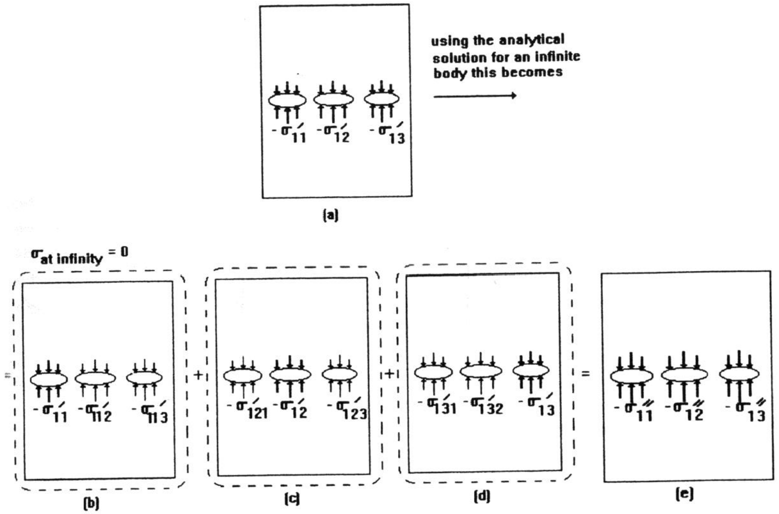

Figure 2 - Accounting for the crack interaction effects.

v) In the next stage the cracks are progressively unloaded to ultimately determine the new stress distribution on the external boundary surfaces of the finite geometry structure. This procedure is performed as follows :-

a) Firstly the tractions on the crack number one, as evaluated in step (iii), are reversed in order to make that crack surface stress free, see Figure 3. This process has two effects namely :-

- the stresses at the positions of the other cracks are slightly altered. The size of this change is calculated utilising the analytical solution for the first crack at the locations of the other cracks. Thus the new tractions for each crack are updated and recorded.

- it produces an additional stress on the external surface of the finite structure. The distribution of this stress on the external surface depends upon the actual location of the crack in the structure and the stresses on that crack. This crack's contribution to the total stress on the external surface of the body is again evaluated using the infinite body solution.

b) The tractions on the second crack are now reversed. Once again this alters the tractions on every other crack, and hence updated tractions for each of the cracks must now be recalculated. At the end of this stage the faces of crack number two are stress free and a new 'residual' stress has been placed on crack one. Crack two's contribution to the stresses on the external surface of the body are now added to those produced by unloading of the first crack.

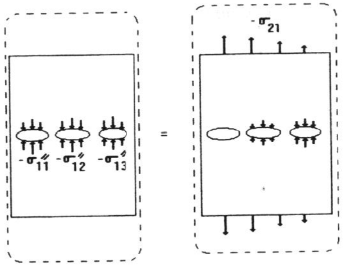

Figure 3 - "Unloading" the stresses on the first crack face.

c) Each subsequent crack is then progressively unloaded, in turn, by reversing the tractions acting on it. As each crack is unloaded the tractions at every other crack face are updated. At the end of this procedure the 'last' crack is the only one for which the crack faces will be stress free. Every other crack will have a small 'residual' traction acting on it. The final stress on the external surface of the body is the sum of all the contributions from each crack, see Figure 4.

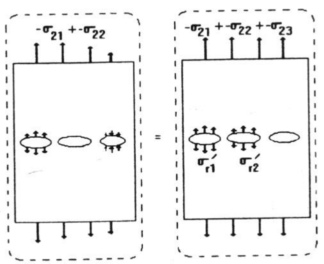

Figure 4 - Illustration of the problem remaining after "unloading" all crack faces.

vi) This total boundary surface stress field is now fed back into step (i) above, and the iterative procedure repeats itself except that the 'residual' stresses on each crack are now added to the crack face stresses evaluated from the finite element analysis of the uncracked structure. For each crack, the resultant crack face stresses are checked against some user defined tolerance. If they are below this then the solution is deemed to have converged and the iterative process stops. If not then the iterative process, described in steps (i -vi) continues until convergence has been reached. The final stress-intensity factors for each crack are the sum of the stress-intensity factors obtained during each iteration for each individual crack.

It will be noticed that the criterion for convergence of a solution for the MSD FEAT is necessarily different to that for a single flaw problem solved using the FEAT. For a single flaw, the boundary stresses produced during each iteration can be used to check for convergence. However, for multiple flaws consideration of the situation of two interacting cracks in an infinite body shows that a boundary stress criterion would result in convergence

to a solution which has not properly accounted for the interaction effects between the cracks. In this case the crack face stresses would not produce boundary stresses and the algorithm would stop after the first iteration, even though there were residual stresses on the crack faces. These residual stresses would be produced by crack interaction effects, and in the present example would be the only interaction forces present. Consequently by stopping at the first iteration the crack interaction would not be included in the solution.

VERIFICATION OF THE ALGORITHM FOR MULTIPLE INTERACTING CRACKS.

To illustrate this technique let us consider the problem of two interacting quarter elliptical cracks. This particular problem has been chosen to illustrate this technique because I) the close proximity of the boundary surfaces to each crack will require the solution algorithm to cycle through several iterations and ii) the presence of only one quadrant for each crack requires the generation of fictitious crack quadrants in order to correctly utilise the analytical solution. Diagrams illustrating the meshes used are shown in Figure 5 below. In this instance a set of 5 different crack configurations were considered. The crack aspect ratio (a_2/a_1) was fixed at 0.8, and the distance between the crack centres was held at 400mm. A remote tensile stress of 100 MPa was applied in the z-direction. To obtain different values of the crack separation ratio, lambda ($\lambda=2a_1/d$), the semi-major crack length (a_1) and the semi-minor crack length (a_2) were subsequently altered. The Mode I stress-intensity factor was evaluated at 5° intervals around the crack front. The values obtained using the MSD FEAT algorithm are compared with those obtained by symmetry in Figure 6. This figure shows that there is very close agreement between the two sets of results. The maximum difference is ~3.5%. The steady increase in the size of the K_I values can be attributed to the increase in crack size necessary to obtain a larger value of the crack separation value λ.

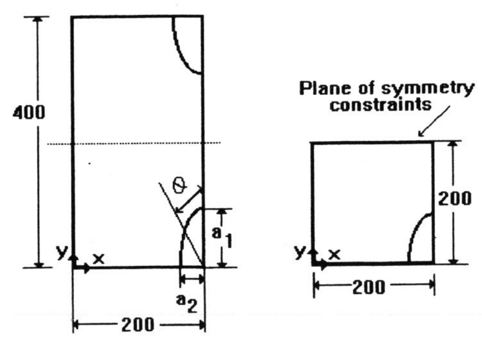

Figure 5- Cross-sectional view of the two meshes used to compare the interaction of two quarter elliptical cracks in a finite body. The diagram on the left indicates the mesh used for the new MSD FEAT algorithm, whilst the diagram on the right indicates a similar mesh employing symmetry arguments used for comparison purposes.

CONCLUSIONS

This paper has presented a new methodology for the analysis of multiple interacting cracks. The cracks may be fully embedded ellipses, semi-elliptical surface flaws, or quarter elliptical cracks. One significant potential advantage of this approach is its ability to utilise the recent advances in parallel and multi-processing. In this case the calculations for each individual crack could be performed on a separate chip or processor. This has the ability to further increase the computational efficiency of the FEAT approach.

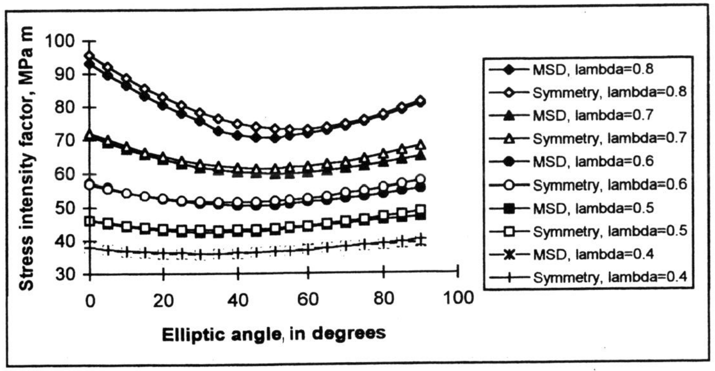

Figure 6 - Comparison of Mode I stress-intensity factors for quarter elliptical cracks in a finite body. One set of results was obtained using the conventional FEAT, whilst the other were produced using the new MSD FEAT algorithm.

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