A MORE CONCISE SYSTEMATIZATION OF UNIFIED FRACTURE THEORY

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ABSTRACT

It has been attempted to express more concisely the systematization of the unified fracture theory proposed in terms of non-linearly atom, nano, meso and macroscopic fracture mechanics, based on the concept of simultaneous compatibility of energy(global) balance requisite and local critical stress requisite. Furthermore, the conventional fracture mechanical theory by Griffith and Irwin and the Neuber's hypothesis and also the dislocation theory for fracture, such as, by Stroh and Cottrell are all contained, respectively, as a special case of this theory proposed.

KEYWORDS

Combined atom, nano, meso and macroscopic, The interaction between crack-like stress raisers, Overall stress intensity factor, Simultaneous compatibility of energy(global) requisite and local critical stress requisite.

INTRODUCTION

Previously it has been attempted to systematize the unified fracture theory proposed in terms of non-linearly atomic, nano, meso and macroscopic fracture mechanics, namely, Zairyo Kyodogaku or Fractology, for brittle fracture(Yokobori, 1994) and fatigue fracture(Yokobori, 1995), respectively. On the other hand, it is necessary to compare the results by this new concept with those by the conventional fracture mechanics.

From this point of view, in the present paper a more concise systematization of this new concept has been attempted concerning the two critical requisites for brittle fracture. Furthermore, it is shown that the results by the conventional fracture mechanics by Griffith(1920) and Irwin(1958) and, also, by the Neuber's(1937) hypothesis for fracture based on the local critical stress requisite are both contained as a special case of this theory proposed. Also, the result by the dislocation theory for fracture by Stroh(1957) and Cottrell(1958) is included as a special case of this proposed theory.

SOME DIFFICULTIES IN THE CONVENTIONAL FRACTURE MECHANICS

For ideal or perfect crystals the fracture stress σ_{th} is given by as:

$$\sigma_{th}=\sqrt{\frac{\gamma_s E}{b}}, \tag{1}$$

where γ_s = specific surface energy, E = Young's modulus, b = atomic spacing. The fracture stress of real materials is far less than σ_{th}, say, 1/10~1/100 of σ_{th}. Then this predicts that high stress concentration will be caused near by the fracture origin by some defects, and the studies on such mechanism have been studied.

For perfect elastic body containing crack, the fracture stress σ_F has been given by Griffith(1920) base on the energy balance condition as follows:

$$\sigma_F=\sqrt{\frac{2\gamma_s E}{\pi a}}, \tag{2}$$

where a = half length of the slit type crack with infinite width in plane strain. Or Eq.(2) is rewritten as:

$$\sqrt{\pi a}\,\sigma_F=\sqrt{2\gamma_s E}. \tag{2a}$$

For the body with crack accompanying small scale plastic deformation, Irwin(1948) and Orowan(1949) have proposed that free energy required for extending a crack is available from the plastic surface work γ_p also and that fracture stress σ_F is given by:

$$\sigma_F=\sqrt{\frac{2(\gamma_s+\gamma_p)E}{\pi a}} \tag{3}$$

using $\gamma_s+\gamma_p$ instead of γ_s. Or Eq.(3) is rewritten as:

$$\sqrt{\pi a}\,\sigma_F=\sqrt{2(\gamma_s+\gamma_p)E}. \tag{3a}$$

Afterward, Irwin(1958) has shown that the stress σ_y near by the crack is expressed in general throughout the specimens with various shape and the crack dimension and shape as follows:

$$\sigma_y\simeq\frac{K}{\sqrt{2\pi x}} \tag{4}$$

where x = the distance from the crack tip, K is named as stress intensity factor. For instance, $\sqrt{\pi a}\,\sigma$ corresponding to the left-hand side $\sqrt{\pi a}\,\sigma_F$ in Eq.(3a) is expressed in general form as:

$$\sqrt{\pi a}\,\sigma\equiv K. \tag{5}$$

At the critical stage for crack extension, Eq.(5) is written as:

$$\sqrt{\pi a}\,\sigma_F\equiv K_c. \tag{6}$$

K_c = the critical value of K at the instant of fracture, and is called fracture toughness

Then using Eq.(6), we can express Eq.(3a) as:

$$K_c=\sqrt{2(\gamma_s+\gamma_p)E} \tag{7}$$

For usual case such as steel, $\gamma_p \gg \gamma_s$ and therefore Eq.(7) reduces to:

$$K_c=\sqrt{2\gamma_p E}. \tag{7a}$$

However, the value of γ_p is experimental one determined case by case, and dependent of the materials and the conditions. For instance, γ_p is not specified by atomic, nano and meso structure.

For nearly perfect crystal , say, one which contains only lattice defects such as dislocations, the dislocation theory fracture has been proposed by Stroh(1957) and Cottrell(1958) and colleagues. For this case, high stress concentration required for fracture is caused by the dislocation pile-up against the obstacles such as grain boundary. The configuration is shown as in Fig.1. Fracture stress σ_F is given, for instance, by Stroh(1957) based on energy balance condition as follows:

$$\sigma_F=\sqrt{\frac{2\gamma_s E}{d}}, \tag{8}$$

where d = slip band length on which piling-up dislocations line up.

THE INTERACTION BETWEEN ELASTIC CRACKS AND/OR CRACK-LIKE STRESS RAISERS

Materials contain cracks and other crack-like stress raisers such as slip bands and any other different phases in terms of atom, nano, meso and macro materials structures. Therefore, in mechanical treatment of fracture, we should consider the crack growth under the interaction between these crack-like stress raisers. Thus it is necessary to solve the interaction between cracks and/or crack-like stress raisers.
For the various configurations of cracks and slip bands, systematic analysis has been made based on singular integral equations and physico-computational mechanics. The stress intensity factor and the interaction factor have been derived for the following typical cases:

Interaction between parallel elastic cracks. (Ichikawa, Ohashi, Yokobori 1965; Yokobori & Ichikawa 1967b)
Interaction of two asymmetrical elastic cracks. (Yokobori et al 1965)
Interaction between non-coplaner parallel staggered elastic cracks. (Ohashi, Ichikawa & Yokobori, 1965; Yokobori et al 1971)
Interaction between parallel slip bands. (Ichikawa & Yokobori 1965;

Yokobori & Ichikawa 1967a)
Interaction between slip band and obstacle as crack. (Yokobori 1962; Yokobori 1963; Yokobori & Yoshimura 1966)
Interaction between crack and near by slip band. (Yokobori 1968a,b; Yokobori et al 1975a,b; Yokobori et al 1976)
Interaction between crack and slip induced by dynamically emitted dislocation from the crack tip. (A.T.Yokobori,Jr., Isogai and Yokobori, 1993; A.T.Yokobori,Jr. et al 1994)

OVERALL STRESS INTENSITY FACTOR BY CRACK AND/OR OTHER DEFECTS INTERACTIONS

On the basis of combined atom, nano, meso and macro stress concentraters interactions, the overall stress intensity factor K_{In} is derived (Yokobori et al 1975a,b; Yokobori et al 1976; Yokobori 1977) as follows:

$$K_{In}=\alpha_{11}\alpha\sqrt{\frac{d}{h}}\sqrt{a}\sigma \qquad \text{for } \frac{d}{h}\gg 1 \tag{9}$$

or

$$K_{In}=\alpha_{11}\sqrt{\frac{d}{h}}K_I \ , \tag{9a}$$

where α_{11} = the interaction coefficient $=\psi(a/d)$, K_I = stress intensity factor by the crack itself (as given by conventional fracture mechanics $\equiv\alpha\sqrt{a}\sigma$)
a = half of the crack length, σ = applied stress, α = non-dimensional coefficient, d = grain radius in crystalline phase, or the effective crack length, say, the length of equivalent crack of various inclusions and voids contained in the materials, h = effective length, that is, the nearest distance between the tip of the crack concerned and the tip of the various defects as stress raisers.

The typical relation of α_{11} versus $\sqrt{a/2d}$ is derived as shown in Fig.2. As can be seen from the argument mentioned above, the overall atress intensity factor K_{In} is applicable not only to crystalline solids like metals in which dislocations play an important role, but also inorganic materials in which micro cracks or cavities have a considerable influence. This concept (Yokobori 1955) and the approach may be called as the combined nano, meso and macro fracture mechanics. (Yokobori 1974)

SIMULTANEOUS COMPATIBILITY OF ENERGY(GLOBAL) REQUISITE AND LOCAL CRITICAL STRESS REQUISITE.

Either energy balance criterion as Griffith criterion (Griffith 1920) or so-called local stress criterion as attributed to Neuber criterion (1937) is usually used for fracture criterion in the present fracture mechanics. However, either of them is only one requisite for fracture, but not enough condition. For the brittle fracture with small scale plastic deformation under unidirectional and single loading in which the

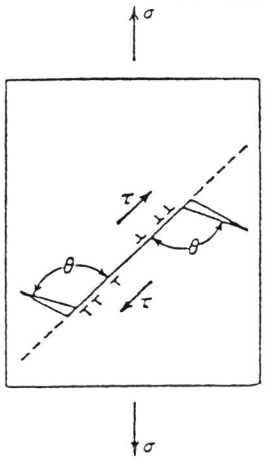

Fig. 1 Dislocation model for fracture (Stroh 1957)

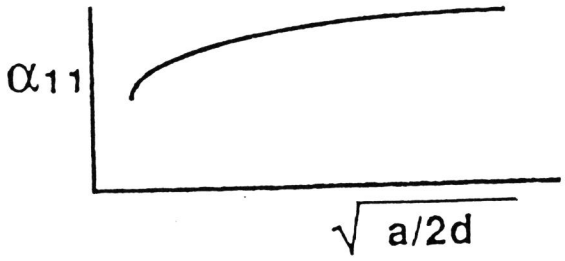

Fig. 2 The typical relation of α_{11} versus $\sqrt{a/2d}$ (T. Yokobori et al 1975a)

applied stress plays a governing role and is not so much affected by thermal activation, it is proposed (Yokobori 1974, 1981) that the following two requisites should be satisfied simultaneously at the same instant. Next let us concern this matter.

THE REQUISITE FOR GLOBAL ENERGY BALANCE FOR THE CRACK EXTENSION

When the crack extends, the strain energy of the body containing this crack will change. The energy balance requisite corresponds to thermodynamic first low in terms of the strain energy, work by applied load and the increase of the crack surface energy. The critical requisite for energy balance becomes as follows:

$$K_{1n}=\sqrt{2\gamma_s E}\ , \tag{10}$$

where γ_s = specific surface energy.

Therefore the applied stress σ in Eq. (9) at the critical condition corresponds to the critical stress σ_{ecr}, and Eq. (9) at this condition becomes to

$$K_{1nc}=\alpha_{11}\sqrt{\frac{d}{h}}\,\alpha\sqrt{a}\,\sigma_{ecr}\ . \tag{11}$$

Substituting Eq. (11) into Eq. (10), we get the critical stress σ_{ecr} as

$$\sigma_{ecr}=\frac{1}{\alpha}\sqrt{\left(\frac{h}{\alpha_{11}^2 d}\right)\gamma_s E}\ . \tag{12}$$

Assuming that fracture is controlled only by the energy requisite as the conventional fracture mechanics assumes, then the energy balance critical stress σ_{ecr} given by Eq. (12) is described as fracture stress σ_{F1} by:

$$\sigma_{F_1}=\frac{1}{\alpha}\sqrt{\left(\frac{h}{\alpha_{11}^2 d}\right)\gamma_s E}\ . \tag{13}$$

(Actually the fracture is not controlled by this requisite only as shown in the following.) On the other hand, the conventional fracture mechanics uses only the energy balance requisite, and gives fracture stress σ_F as Eq. (7a), using $K_c\equiv\alpha\sqrt{a}\,\sigma$ as follows:

$$\sigma_{F_1}=\frac{1}{\alpha}\sqrt{\frac{2\gamma_p E}{a}}\ . \tag{14}$$

Comparing Eqs. (13) and (14), we get

$$\gamma_p=\frac{1}{\alpha_{11}^2}\frac{h}{d}\gamma_s\ . \tag{15}$$

We can see that by using this proposed concept, γ_p and thus fracture stress σ_F described by the conventional fracture mechanics can be expressed explicitly in terms of atomic, nano, meso, macrostructure and macroscale as well as the critical stress σ_{ecr} can be.

Furthermore, it is understood that fracture mechanics proposed includes the conventional fracture mechanics as a special case.

THE REQUISITE FOR BREAKING THE ATOMIC OR MOLECULAR BONDS AT OR NEAR THE CRACK TIP

The extension of the crack tip means the breaking of atoms or moleculs at the tip. That is, it is necessary for one pair or a few pairs of atoms to be broken. Thus the local stress at or near the tip of the crack should exceed the ideal strength, that is, the breaking stress of the atomic bond. This requisite is the one in terms of atomic scale or nano scale. (Fig. 3) Notice that this requisite, that is, local critical stress requisite is not the same as so-called local stress criterion in the present fracture mechanics. In the latter approach, the local critical stress $\sigma\ell$ is assumed as the value belonging to macroscopically applied stress distribution at or near the crack tip. $\sigma\ell$, however, cannot exceed about ten times the yield strength, σ_Y, and, therefore, $\sigma\ell$ is far less than the atomic bonding strength, say, ideal strength, σ_{th}.

For the case of energy balance requisite, it is to be noted that the concerned size of the region is macroscopic far much larger (Fig. 4) than atomic or nano scale for the critical local stress requisite. It can be easily seen from the following: For instance, the fundamental equation in linear elastic fracture mechanics is expressed as :

$$\sigma_y=\frac{K_I}{\sqrt{2\pi x}} \tag{16}$$

where $a \gg x \gg \rho$ or $a \gg x > s$. a = half of the crack length, ρ = crack tip radius, s = plastic region size, x is larger than plastic region size s. Thus, the energy balance requisite concerns macroscopic scale.
From the considerations mentioned above, it can be seen easily local critical stress requisite and energy (global) requisite are quite different in nature, and each of them is only one requisite. Therefore, it is concluded that for the crack to extend and for the fracture to occur, both energy (global) requisite and local critical stress requisite should be satisfied at the same instant. (Yokobori 1994, Yokobori 1968, 1974, 1981, 1988; Yokobori et al 1979)

Local critical stress $\sigma_{\varepsilon,c}$ at ε distant from the crack tip is given as:

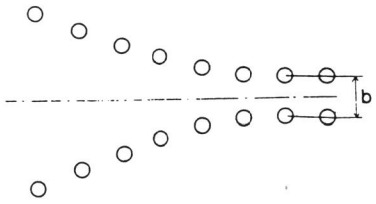

Fig. 3 Requisite for breaking of atomic bonds at the crack tip

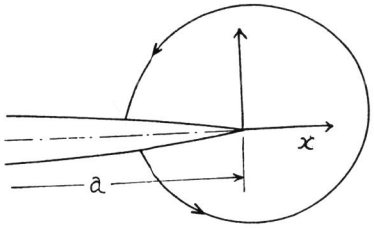

Fig. 4 Requisite for energy (global) balance

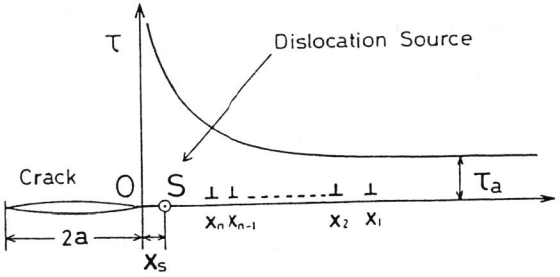

Fig. 5 The model in which the source emitting the dislocations group is located at the tip of the crack under the stress singularity (A.T. Yokobori, Jr., T. Isogai & T. Yokobori; 1993)

$$\sigma_{\varepsilon,c} = \frac{K_{Ic}}{\sqrt{\varepsilon}} \equiv \frac{1}{\sqrt{\varepsilon}} \alpha_{11} \sqrt{\frac{d}{h}} \alpha \sqrt{a}\, \sigma_{lcr} \tag{17}$$

at the critical instant of breaking the atomic bonds at the crack tip. The critical requisite for breaking the atomic bonds at the crack tip is the condition that $\sigma_{\varepsilon,c}$ reaches the atomic bonding force(strength) σ_{th} and it is expressed as:

$$\sigma_{\varepsilon,c} = \sigma_{th}. \tag{18}$$

From Eqs.(17) and (18), we get

$$\sigma_{lcr} = \frac{1}{\alpha} \sqrt{\left(\frac{h}{\alpha_{11}^2 d}\right) \frac{\varepsilon}{a}}\, \sigma_{th} \tag{19}$$

If we assume fracture is controlled only by local critical stress requisite as Neuber(1937), Eq.(19) corresponds to fracture stress, σ_{F2}. (Actually fracture stress is not controlled by this requisite only as shown in the following.) That is, Eq.(19) is expressed as:

$$\sigma_{F_2} = \frac{1}{\alpha} \sqrt{\left(\frac{h}{\alpha_{11}^2 d}\right) \frac{\varepsilon}{a}}\, \sigma_{th}. \tag{20}$$

On the other hand, the conventional local critical stress criterion uses only the local critical stress requisite such as by Neuber(1937), and gives fracture stress σ_{F2} as

$$\sigma_{F_2} \simeq \sqrt{\frac{\varepsilon_0}{a}}\, \sigma_{th}, \tag{21}$$

where ε_0 = so-called Neuber's critical distance, the value of which is assumed case by case according to the materials and the experimental conditions.

Comparing Eqs.(20) and (21), we get

$$\varepsilon_0 = \left(\frac{h}{\alpha^2 \alpha_{11}^2 d}\right) \varepsilon. \tag{22}$$

In Eq. (20) proposed, ε may be given, for instance, in the case rate-controlled by the crystalline phase. That is, the dislocation group will emit dynamically from the stress concentrated site such as the crack tip. The dislocation will pile up inversely (A.T. Yokobori, Jr. et al 1991, 1993, 1994) against the site specifically distant from the crack tip. (Fig. 5) Thus coupled high stress concentration by both crack tip and the inversed pile up dislocations will be caused within this specific distance, ε (A.T. Yokobori, Jr. et al 1993, 1994), which can be designated as ε described in Fq. (20). In this way ε can be expressed explicitly in terms of atomic, nano and meso structures, in the following paper. In this way, the local critical stress σ_{lcr} can also be expressed explicitly in terms of atom, nano and meso structures and macroscopic factors.

UNIFIED CRITERION FOR BRITTLE FRACTURE

Based on both the concept of simultaneous compatibility of energy (global) requisite and local critical stress requisite and the concept of the combined atom, nano, meso and macro mechanics mentioned above, unified criterion for brittle fracture is obtained. It is concluded that fracture stress σ_F is the higher value of critical stress σ_{ecr} (Eq. (12)) by energy balance requisite and critical stress σ_{lcr} (Eq. (19)) by local critical stress requisite. Simply, it can be expressed as:

$$\sigma_F = \sigma_{ecr} \qquad \text{for the case } r > 1 \qquad (23)$$

$$\sigma_F = \sigma_{lcr} \qquad \text{for the case } r < 1 \qquad (24)$$

where

$$r = \frac{\sigma_{ecr}}{\sigma_{lcr}} = \frac{\sqrt{2\gamma_s E}}{\sqrt{\varepsilon}\,\sigma_{th}} \qquad (25)$$

The explanations have been already well made (Yokobori 1994) on the critical experimental characteristics which cannot be explained by any other theories or criteria ever presented.

CONCLUSIONS

(1) A more concise systematization of the fracture theory proposed has been attempted in terms of non-linearly combined atomic, nano, meso and macroscopic measures, and based on the proposed concept of simultaneous compatibility of energy (global) balance requisie and local critical stress requisite.

(2) The explanation can be well made on the critical experimental characteristics which cannot be explained by any other theories or criteria ever presented.

(3) The conventional fracture mechanical theory by Griffith and Irwin and the Neuber's hypothesis for fracture and also the dislocation theory for fracture by Stroh and Cottrell are respectively all contained as a special case of this theory proposed.

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