THE CORRELATIONS AMONG EACH PARAMETER IN SOME EQUATIONS ON CRACK GROWTH STAGE

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ABSTRACT

The general regularities and some features in evolving process for material behaviours on crack growth stage are analyzed and investigated in the paper, as a result the simplified double direction curves in a double-direction coordinate system have been established and several new equations have been suggested with respect to relations among residual life N_{f2} and the stress intensity factor range ΔK, the J-integral range ΔJ_{eff}, the crack opening displacement range $\Delta\delta_t$. Hence the relationship between opposite direction curves, both equations, and both material parameters can all be illustratively presented, and some material parameters can also be directly calculated from the material constants K_{fc}, J_{fc}, δ_{fc} and m_2, λ_2, n_2.

KEYWORDS

Double direction coordinate, curves; equations; material parameters.

NOTATIONS

da/dN—Crack growth rate;

a^{mca}, N^{mac} —size and life corresponding with forming macro-crack, respectevely;

a_c, N_{f2} —critical size and life corresponding with crack starting fast growth,respectively;

$K_{fc}, J_{fc}, \delta_{fc}$ —critical value of stress intensity factor amplitude, J-integral amplitude and crack tip opening displacement amplitude corresponding with crack starting fast fracture, respectively.

m_2, n_2, λ_2 —material constants in crack growth rate equations expressed with varied form, respectively.

INTRODUCTION

The famous Paris's equation has successfully solved the calculations to the crack growth rate and its residual life of components and materials with cracks under high-cycle or low-strain loading. It is well know that some materials shown up plasticity usually produced plastic strain fatigue under low cycle or high-strain loading, at the time, the Paris's equation no longer holds. Some of scientists had imitated the form of the Paris's equation introduced into one's equation with the crack opening displacement range $\Delta\delta_t$, or the J-integral range ΔJ_{eff} and had usefully carried on research, obtained some satisfactory answers. But when the positive results are used into engineering ,due to it lacks some material parameters; due to the equations gained through tests even more need lots of experimental set-up, materials capitals, and times, therefore, many fruits obtained in scientific research can not be quick used to production engineering.

If can establish a number of new equations derived in the form of the double direction curves, and derive their relations between new equations and original equations, between new material parameters and original material parameters, then it can speedily make the research fruits to have a great economy benefit. This paper proceeds from the aim mention above, has suggested establishing the double direction curves in a double direction coordinate system, proposed a number of equations between the ΔK-N,the $\Delta\delta_t$--N, ΔJ_{eff}--N, in which it can be made with the Paris's equation to have connection, thereby it is arrived at a goal as stated above . So that the present paper will have practical significance, when it is carried out the experimental researches and engineering applications, and it is used for sparingly manpower and material resources.

A COORDINATE SYSTEM OF DOUBLE DIRECTIONS AND THE CURVES OF DOUBLE DIRECTIONS

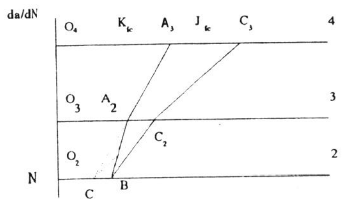

Fig. 1 Double logarithmic coordinate system with two directions

The figure 1 is a double logarithmic coordinate system shown with simplified two directions curves, it consists of three abscissa axes 2,3 and 4 and a double directions ordinate axis o_2o_4 , Upward direction along the ordinate axis is presented as crack growth rate da/dN, and downward direction ,presented as each history residual life up to fracture, which are all described as evolving process of material behavior on crack growth stage. In the double direction system, distance on the ordinate axis below abscissa axis 2 is shown as region from un-crack to micro crack initiation; the distance o_2o_3 between axis 2 and 3, as region to be relative to life $N^{min\text{-}mac}$ from micro crack growth until macro crack generation, when the stress intensity factor range ΔK-values arrive at the threshold value ΔK_{th}; the distance o_3o_4 between axis 3 and 4, as region relative to life $N^{mac\text{-}}$ to be macro crack stable growth stage, in which as was also that region on crack growth curve applied to the Paris equation. Above axis 4, it is shown as region to be crack fast fracture . The curve 1 in Fig.1 is a crack growth curve , it shows varying regularities of crack behavior of linear elastic strain material (like subjected to high-cycle fatigue) ;positive direction A_2A_3 presents relation between da/dN - ΔK; negative A_3A_2, between ΔK-N or ΔJ_{eff}-N ($\Delta K/2$-2N or $\Delta J/2$-2N); The curve 2 it shows varying regulars of crack behavior applied to plastic strain material (like, subjected to low-cycle fatigue): positive C_2C_3 shows relation between da/dN-$\Delta\delta_t$ or da/dN- ΔJ_{eff} ; negative C_3C_2, between $\Delta\delta_t$- N or Δ J_{eff}-N ($\Delta\delta_t/2$-2N or $\Delta J_{eff}/2$ -2N). In order to qualitatively analyze and state problems, the material constants in each equation corresponding with double direction curves can be properly made by derivation, So assuming that:(1) The point B on axis 2 in Fig.1 is one relating to threshold value ΔK_{th} ;(2) the culminating points A_2 and C_2 on axis 3 are respectively ones

homologised to macro-crack-generation and just started steady propagation stage ;(3) the points A_3, C_3 on axis 4 are ones homologised to the critical values K_{fc},$\delta_{fc}$and J_{fc} . As long as it can be based upon the several points of assumption, this paper will no difficulty derive varies relationship among each equations or each parameters, which are relating to two directions curve between axis 3-4.

CORRELATIONS AMONG CRACK GROWTH RATE AND RESIDUAL LIFE EXPRESSIONS WITH THEIR MATERIAL PARAMETERS

For components under high-cycle loading, for materials shown out linear elastic strain, the relation between the da/dN-ΔK in the form of positive direction A_2A_3 of the straight line 1 in Fig. 1 can be described by the Paris's equation

$$da/dN = A_2\Delta K^{m_2} ,m/cycle \qquad (1)$$

Where, m_2 is a material parameter, also a slope of the curve A_2A_3. A_2 is a comprehensive material parameter in connection with K_{fc} and m_2, it equals

$$A_2 = 2(2K_{fc})^{-m_2}(a_c - a^{mac})/(N_{f_2} - N^{mac}),(MPa\sqrt{m})^{-m_2}m/c \qquad \text{(Yu Yangui, 1991)} \qquad (2)$$

and lgA is presented with distance o_3o_4 on ordinate axis. the A_3A_2 is a form of negative direction curve of A_2A_3,It can be fitted to an equation

$$\Delta K N_{f}^{-b_2'} = C_1 \qquad (3)$$

Afpssuming that the life N^{mac} of forming macro crack equals 0, the eq.(3) should be become

$$\Delta K^{-1/b_2'}(N_{f2} - N^{mac}) = C_1^{-1/b_2'} = C_2 \qquad (3a)$$

Here, b_2', C_1, and C_2 are all material parameters. If equation (3) and (4) is presented by stress intensit; y factor amplitude, then the eq.(3) should be rewritten as follows

$$K_a = \frac{\Delta K}{2} = K_{fc}'(2N_{f_2})^{b_2'} \qquad (4)$$

if simultaneous equations consisted of eq.(3) and (4) are solved , it follows

$$C_1 = 2^{(1+b2')}K_{fc}' \qquad (5)$$

$$C_2 = \frac{1}{2}(2K_{fc}')^{-1/b_2'} \qquad (6)$$

It is apparent that the relation between the equation (2) and (6) is a that

$$C_2 = \frac{a_c - a^{mac}}{A_2} \qquad (7)$$

$$m_2 = -1/b_2' \qquad (8)$$

For components under low cycle loading, for materials shown out plastic strain, the relation between the da/dN-$\Delta\delta_t$ in the form of positive direction C_2C_3 of the cure 2 in Fig.1 had been described by equation as follow (П о к р о в с к и й , 1987)

$$da/dN = B_2(\Delta\delta_t)^{\lambda_2'}, mm/cycle \quad (9)$$

Where , the B_2 and m_2 are also material parameters, that their physical and geometry meanings are same with the A_2 and m_2 mentioned above. The reference (Е . А . Г Р И Н Ь et al.,1987) had experimentally proved the eq. with steel 15X2MфA, that it is all stable as well elastic strain materials as also plastic strain materials for the unsymmetry of loading. But, the negative direction curve C_3C_2 between $\Delta\delta_t$-N_f ($\Delta\delta/2$-$2N_f$) can be fitted to following equation

$$\Delta\delta_t N_{f_2}^{-c_2'} = D_1 \quad \text{or} \quad (10)$$

$$\Delta\delta_t^{-1/c_2'}(N_{f_2} - N^{max}) = D_1^{-1/c_2'} = D_2 \quad (11)$$

here c_2' C_1 C_2 are all also material parameters. The equation (10) can be also become

$$\frac{\Delta\delta_t}{2} = \delta_{f_c}(2N_{f_2})^{c_2'} \quad (12)$$

where δ_{fc} is a critical value of crack opening displacement. Same, it can be derived out from eq.(9),(10), (11) as following relations

$$B_2 = 2(2\delta_{fc})^{-\lambda_2}(a_c - a^{max})/(N_{f2} - N^{max}),(mm)^{-\lambda_2} mm/cycle \quad (13)$$

$$D_2 = \frac{1}{2}(2\delta_{fc})^{-1/c_2'} \quad (14)$$

$$B_2 = \frac{a_c - a^{max}}{C_2} \quad (15)$$

$$m_2 = -1/c_2' \quad (16)$$

It should be pointed out that the parameter A_2 indicated above can be yet expressed as δ_c, if the size of crack tip plastic zone is only small compared to the size of the crack, that it is

$$A_2 = 2(2\sqrt{2E\sigma_y'\delta_c})^{-m_2} \quad (17)$$

Where, σ_y'and E' are respectively the yield stress and young's modulus under cyclic loading. Because the plastic zone size of crack tip under negative direction loading is only one quarter of plastic zone size under monotone loading, in which the σ_y' is substituted for $2\sigma_y$'.

It must be yet admitted that, as well elastic materials, as also plastic material, the crack growth rate da/dN can be all expressed by J-integral (Lang Fuyuan, 1991)

$$da/dN = C_2(\Delta J_{eff})^{n_2}, m/cycle \quad (18)$$

correspondently, the relation between the ΔJ_{eff}-N of curve A_3A_2 or C_3C_2 in negative direction coordinate system is same fitted to following form

$$\Delta J_{eff} N_{f2}^{b_2'} = E_1, \quad (19)$$

$$\Delta J_{eff}^{-1/b_2'}(N_{f2} - N^{max}) = E_1^{-1/b_2'} = E_2 \quad (20)$$

where, b_2', E_1 and E_2 are all material parameters

$$\frac{\Delta J_{eff}}{2} = J_{fc}(2N_{f_2})^{b_2'} \quad (21)$$

It will be seen from this that the correlations among their each parameters are

$$C_2 = 2(2J_{f_c})^{-n_2}\frac{a_c - a^{max}}{N_{f2} - N^{max}} \quad (22)$$

$$E_2 = \frac{1}{2}(2J_{f_c})^{-1/b_2'} \quad (23)$$

$$n_2' = -1/b_2' \quad (24)$$

$$C_2 = \frac{a_c - a^{max}}{E_2} \quad (25)$$

DISCUSSIONS

1). In regard to scientific basis of equations (3), (10) and (19) :The equations (1),(9) and (18) related to positive direction curve A_2A_3 and C_2C_3 in the double direction system had been proved by lots of experiments to be scientific. So it may be also believed that the equations(3), (10) and (19) described by their negative direction curve A_3A_2, C_3C_2 will must be right. In fact, one of they had been already proved by some experiments (Lang Fuyuan 1991), Only due to this equations having not been found and understood by men before now. Author of present paper has used the double direction coordinate system, and established the double direction curves in it, so that the correlations among the da/dN-ΔK and ΔK-N, da/dN-$\Delta\delta_t$ and $\Delta\delta_t$-N, da/dN-ΔJ and ΔJ-N can be illustratively made by shown expression and the new equations (3), (10) and (20) have been derived out from simplified curves. In reality, author of the reference (Lang Fuyuan, 1991) had tested in specimen made of steel 45 with initial crack size a_0=2mm, and obtained such curve as the A_2A_3. Its result is in the following,

$$N_{f2} = 3.5503 \times 10^{15}(\Delta K)^{-6.40187} \quad (26)$$

It is evident that the result is in agreement with the eq.(3),which as has been fitted in present paper. Where, $C_1 = 3.5503 \times 10^{15}, b_2' = -6.4, N^{max} = 0$.

2). In regard to the physical and geometrical meaning of the several parameters. It may be clear seen from the double direction coordinate system and its diagram, that the parameter A_2,B_2 and C_2 are respectively distance O_4A_3 and O_4C_3 on ordinate axis $O_2O_4$for varied curve on geometry. But on the physical meaning, if the $\Delta K^{m_2}, \Delta\delta^{\lambda_2}, \Delta J^{n_2}$ are regarded as a crack resistance which is equivalent to a kind of energy, that it corresponds with the crack growth size, then A_2, B_2,C_2 should be defined to be the consumed energy per a cycle dN and per a crack extension da. It should be pointed that, their dimensions should respectively be:

$$(2K_{fc})^{-m_2}(a_c - a^{max})/(N_{f2} - N^{max}),\ (Mpa)^{-m_2} m/cycle;\ (2J_{fc})^{-n_2}(a_c - a^{max})/(N_{f2} - N^{max}),(KN/m)^{-n_2} m/cycle;$$

$$(2\delta_{fc})^{-\lambda_2}(a_c - a^{max})/(N_{f2} - N^{max}),(mm)^{-\lambda_2} mm/cycle.$$

Secondary, the $m_1(-1/b'_2)$, $\lambda_2(-1/c'_2)$,$n_2(=-1/b''_2)$ are the slopes of curve 1, 2 on the geometry , but on the physical meaning, they are shown as stiffness constants of materials. The K_{fc}, J_{tc}, δ_c in equation(4), (12)and(21) are the distances O_4A_3 and O_4 C_3 ,on abscissa axis O_44 on the

geometry, and on physical, they are the critical values of material fracture under cyclic loading.
3). In regard to relationship of several parameters. The reference (Я рема et al.1981) had suggested, the parameter A (= A_2) and m (= m_2) in the Paris's equation are correlative between each other. This predication is correct answer, (like lgA= -1.28m-7.21) but, this opinion is only obtained from mathematical treatment. The paper recognizes that the parameters A_2, B_2 and C_2 are the function-values to bear a relation to K_{1c} and $m_2(-1/b_2')$, δ_c and $\lambda_2(-1/c'_2)$, ΔJ_c and $n_2(-1/b_2'')$. So, they are some comprehensive property parameters of materials.

CONCLUSIONS

1). This paper suggests establishing the simplified double-direction curves in a double-direction coordinate system, in order that the relations of opposite directions between the ΔK, $\Delta\delta_t$, ΔJ_{Fe}-N and the da/dN-ΔK, $\Delta\delta_t$, ΔJ_{Fe} are illustratively expressed, and their materials parameters and expressions are defined and connected with each other on the physical and geometrical meaning .

2)The equations $\Delta K/2 = K_{fc}(2N_{f2})^{b'_2}, \Delta J/2 = J_{fc}(2N_{f2})^{b''_2}, \Delta\delta_t/2 = \delta_{fc}(2N_{f2})^{c'_2}$ and their parameters $K_{fc}, b'_2; J_{fc}, b''_2; \delta_{fc}, c'_2$ suggested in the paper are respectively corresponded with the equations $da/dN = A_2\Delta K^{m2}$, $da/dN = C_2\Delta J^{n2}$, $da/dN = B_2\Delta\delta_t^{\lambda 2}$ and their parameters A_2, m_2; B_2, n_2; C_2, λ_2. The correlation between them are opposite relations on direction of curves, so that it can be derived from each other and they are agreement.

3). The material parameters $A_2, B_2 and C_2$ are not independent ones, they are ones of comprehensive property in connection with $K_{fc}, m_2(b'_1); J_{fc}, n_2(b''_2); \delta_{fc}, \lambda_2(c'_2)$.

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