# INTERRELATION BETWEEN INELASTICITY AND HIGH-CYCLE FATIGUE OF METALS

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General regularities of inelasticity and high-cycle fatigue of metals have been considered. The criteria of metals fatigue fracture based on the consideration of cyclic inelastic strains and inelastic strain energy have been formulated. The interrelation between inelasticity and high-cycle fatigue has been considered with the account taken of temperature, stress gradient, stress concentration, size effect, load biaxiality, scatter of fatigue characteristics, program and bifrequency loading, surface conditions.

## KEYWORDS

Inelasticity, high-cycle fatigue, cyclic stress-strain diagrams, fatigue limits, fatigue damage.

## INTRODUCTION

In high-cycle loading, the most part of metals lifetime is generally spent for the stage of macroscopic crack initiation. To evaluate the accumulation of fatigue damage in this stage the effect related to inelastic deformation of local volumes of the material are often used. These are the occurrence of a hysteresis loop in the "stress-strain" coordinates, the phase shift between the stress and strain signals, heating up of specimens during cyclic loading, damping of free vibrations, Kimball's effect (Lazan, 1950) and others. The earliest investigations into the interrelation between fatigue and inelasticity of metals were performed by Hopkinson et al. (1912), Voropayev (1914), Haigh (1928) and others. The majority of those investigations were performed under simple types of loading (tension, solid, specimens, harmonic loading). In the cases when the tests were carried out under conditions of nonuniform stress state (bending, torsion), the averaged values of the characteristics of metals inelasticity were used which do not define real processes in the most stressed surface layers of the metal. All these facts give no way of analyzing the interrelation between high-cycle fatigue and inelasticity of metals considering all the variety of structural, service and technological factors whose impact upon high-cycle fatigue is determining. The paper presents the analysis of interrelation between high-cycle fatigue and inelasticity of metals with the account taken of the factors affecting the fatigue strength. The analysis has been made with the use of primarily the results obtained at the Institute for Problems of Strength of the National Academy of Sciences of Ukraine.

## EXPERIMENTAL PROCEDURE

Inelasticity of metals is studied on the basis of measurements of various effects. One of such effects is the phase shift between the stress and strain signals (Figs 1a,b). This effect was used in conducting the main part of the investigations described in the present paper.

In the case of elastic deformation (Fig. 1a), a linear relationship is observed between stresses and strains. When the material deforms inelastically (Fig. 1b), a closed hysteresis loop is observed whose width is equal to inelastic strain per cycle, $\Delta\varepsilon_i$, ($\Delta\gamma_i$ in torsion), and the area is equal to the energy dissipated per cycle, ΔW.

Simple relations exist between the values $\varepsilon_a$, $\Delta\varepsilon_i$ and ΔW:

$$\varepsilon_a = \frac{\sigma_a}{E} + \frac{\Delta\varepsilon_i}{2} \qquad (1) \qquad\qquad \Delta W = k\Delta\varepsilon_i\sigma_a \qquad (2)$$

where E is the elasticity modulus, k is the coefficient of the hysteresis loop shape.

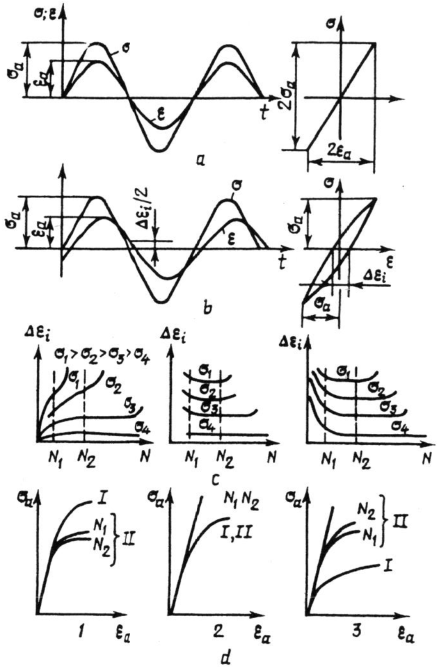

Fig.1. Variation of stress and strain signals and their processing

The width of the hysteresis loop can vary depending on the number of load cycles. Figure 1c illustrates this variation for cyclically softening (1), cyclically stable (2) and cyclically hardening (3) materials tested under conditions $\sigma_a$ = const. Figure 1d presents static (I) and cyclic (II) stress-strain diagrams for the numbers of load cycles $N_1$ and $N_2$ for materials of various classes. For some materials more complicated dependences than those shown in Fig. 1e may take place. The procedures for measuring inelastic strains are described elsewhere by Troshchenko (1981).

The loading frequency in the investigations varied within 20...50 Hz. The magnitudes of inelastic strains were determined by scanning the stress and strain signals with subsequent processing of the information obtained by the computer. In some investigations inelastic strains were measured directly from the hysteresis loops represented on the screen of the oscilloscope. The analysis of the results of investigations revealed that with the use of those procedures one can measure inelastic strains from $1\cdot10^{-5}$ mm/mm with sufficient reliability.

The information obtained from testing specimens under conditions of nonuniform stress state was processed in order to recalculate nominal stress values $\sigma_a^n/\sigma_a = M_b/W$ (where $M_b$ is the bending moment, W is the section modulus) and nominal inelastic strains $\Delta\varepsilon_i^n$ into their true values $\sigma_a^t$ and $\Delta\varepsilon_i^t$ occurring in the material surface layer, as is shown in Fig. 2 taking bending as an example. In this figure, 1 is the $\sigma_a^n - \varepsilon_a$ dependence for a certain number of load cycles, 2 is the calculated $\sigma_a^t - \varepsilon_a$ dependence, 3 is the fatigue curve in the coordinates "$\sigma_a^n - N_f$", 4 is the fatigue curve in the $\sigma_a^t - N_f$ coordinates ; $\sigma_{pr}^c$, $\sigma_e^c$ are the proportionality and elasticity limits, respectively, with the tolerance for residual strain *e*.

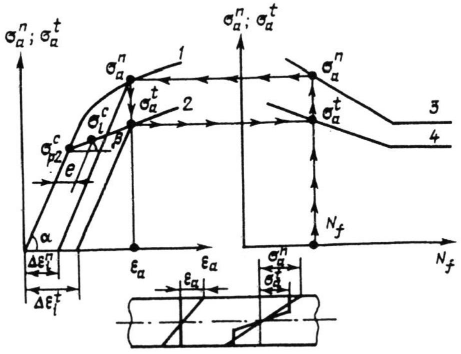

Fig.2. Plotting of true stress-strain diagrams and fatigue curves for bending.

The scheme indicated can be used in the calculations for solid specimens in torsion and for specimens with stress concentrators. The procedures for these calculations are described in the monograph by Troshchenko (1981).

## INELASTICITY CHARACTERISTICS USED IN THE ANALYSIS OF FATIGUE DAMAGE.

The investigations performed revealed that at stresses equal to the fatigue limit even for materials of the same class (e.g. steels) different values of inelastic strains occur at the stage of stabilization (or at $N = 0.5N_f$). The ranges of those strain values $(\Delta\varepsilon_i)_{\sigma_{-1}}$ for some classes of materials studied in fully-reversed tension-compressin are given in Table 1. At the same time, the experimental data reveal that for the same material the $\Delta\varepsilon_i$ value can be used as a characteristic of the intensity of fatigue damage accumulation, which is very convenient in a comparative evaluation of the influence of various factors upon fatigue. The investigations of physical processes which define the regularities in the $\Delta\varepsilon_i$ value variation were performed by Troshchenko (1960), Bega et al. (1979). Those investigations revealed that at the stage of stabilization inelastic cyclic strains result from plastic deformation of local volumes of metals, which is accompanied by the manifestation of Bauschinger's effect and other effects, and at stresses above the fatigue limit determine the instant of macroscopic fatigue crack initiation.

Table 1. The $(\Delta\varepsilon_i)_{\sigma_{-1}}$ values for materials of various classes

| Class of material | The quantity of the materials studied | $(\Delta\varepsilon_i)_{\sigma_{-1}} \cdot 10^5$ |
|---|---|---|
| Carbon and low-alloy steels | 12 | 0.35...13.4 |
| Austenitic steels and steels with a physical yield strength | 7 | 23.2...50.0 |
| Copper and its alloys in a deformed state | 4 | 0.86...1.56 |
| Aluminium and its alloys | 3 | 5.0...6.0 |
| Grey cast iron | 1 | 5.6 |
| Nickel alloys:<br>at ambient temperature | 3 | 0.5 |
| at high temperatures | 3 | 0.34...2.8 |

The transition from elastic to inelastic deformation and the rate of inelastic strain increase are convenient to analyze by plotting a cyclic stress-strain diagram.

Figure 3 compares static 1 and cyclic 2 stress-strain diagrams and fatigue curves 3 for single crystals of molybdenum (a), coarse-grain nickel (b) and steels (c,d,e,f) in tension-compression and steels (g,h,i) in torsion for thin-walled specimens. For the case of the absence of static stress-strain diagrams, the figure shows the yield stress $\sigma_{02}$ for corresponding materials. As it follows from this figure, the cyclic stress- strain diagrams in no way coincide with the static ones and can be considered as an independent characteristic of the material properties under cyclic loading.

As was shown in the works of Tanaka et al. (1981) and Troshchenko (1971), there is a correlation between the degree of cyclic hardening (softening) and the ratios of yield stresses (or proportionality limits) to the ultimate strengths of the materials studied.

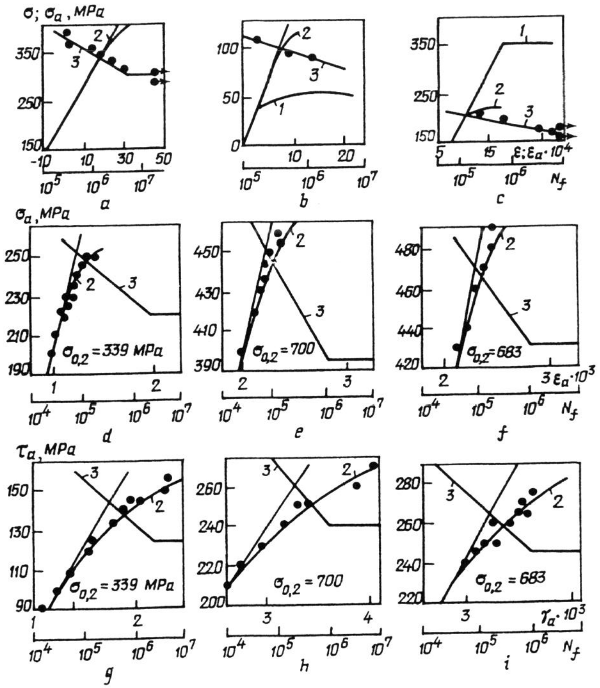

**Fig. 3.** Cyclic stress-strain curves and fatigue curves

It is appropriate to use a part of the total dissipated energy spent for fatigue fracture as a criterion of the accumulated fatigue damage. Troshchenko (1966) proposed such a criterion in the following form:

$$\sum_{1}^{N_f}\left[\Delta W_i - \Delta W_R\left(\frac{\Delta W_i}{\Delta W_R}\right)^{\alpha}\right] = W_f = const \tag{3}$$

where $\Delta W_i$ is the energy dissipated per cycle, $\Delta W_R$ is the energy dissipated per cycle with a fatigue limit, $\alpha$ is a parameter. The use of this criterion makes it possible to estimate fatigue fracture energy which is independent of the number of cycles to the fatigue crack initiation.

Makeyev et al. (1975) showed that criterion (3) corresponds to the hypothesis of fatigue damage accumulation of Corten-Dolan (1956). Recall that the criterion of Miner (1945) suggests that all the dissipated energy is spent for fatigue fracture.

## INELASTICITY AND FATIGUE OF METALS

Below is given the analysis of interrelation between the regularities of inelastic deformation and fatigue fracture of metals with the account taken of various factors.

Temperature. Troshchenko in his work (1981) demonstrated that with an increase in the test temperature up to 973 K and 1153 K for nickel alloys a correlation is observed between a decrease in the stress of transition from elastic to inelastic deformation and the value of the fatigue limit for $10^7$ cycles. In the same work it was shown that a decrease in the test temperature of an aluminium alloy from ambient down to 77 K induces a decrease in the intensity of inelastic deformation and accordingly an increase in the fatigue limits.

Table 2. The values of cyclic proportionality limits and fatigue limits in bending

| Steel | Yield stress $\sigma_{02}$ MPa | Ultimate stress $\sigma_u$ MPa | Proportionality limit under cyclic loading $\sigma_{pr}$, MPa | Fatigue limit $\sigma_{-1b}$ MPa | $\sigma_{pr}/\sigma_{-1b}$ |
|---|---|---|---|---|---|
| 25 | 279 | 518 | 260 | 255 | 1.06 |
| 45 | - | 618 | 280 | 282 | 0.99 |
| 45 | 476 | 668 | 285 | 365 | 0.91 |
| 15G2AFDps | 410 | 532 | 275 | 300 | 0.92 |
| 20Kh | 268 | 500 | 245 | 245 | 1.00 |
| 40Kh | 675 | 840 | 455 | 430 | 1.11 |
| 1Kh13 | 614 | 746 | 432 | 400 | 1.08 |
| EI612 | 582 | 896 | 336 | 310 | 1.08 |
| EI726 | 560 | - | 215 | 205 | 1.10 |

Stress gradient. Investigations into the influence of stress gradient upon the regularities of surface layers deformation of specimens from cyclically softening materials in bending and in torsion were performed by Troshchenko et al. (1970, 1981). The basic conclusions made on the basis of those investigations are as follows. Cyclic stress-strain diagrams for surface layers of specimens in bending and for solid specimens in torsion, calculated according to the procedure presented in Fig. 2, differ appreciably (shift to the region of higher stresses) from similar diagrams for uniform stress state. This is explained by deceleration of cyclic softening of these layers in the presence of stress gradient and can also be explained considering a sequential mechanism of plastic deformation wherein the grain size and stress gradient play an essential role.

A clear correlation is observed between cyclic proportionality limits of the steel surface layers, calculated in accordance with the procedure presented in Fig. 2, and fatigue limits. Table 2 compares those characteristics for different steels studied in bending (relative stress gradient $\overline{\eta} = 0.2\,\mathrm{mm}^{-1}$).

Stress concentrators. As it was shown by Troshchenko et al. (1972), the deceleration of the cyclic plastic strain growth in the presence of stress gradient also shows up significantly in the presence of stress concentrators. On the basis of the calculated and experimental methods it was shown that the use of cyclic stress-strain diagrams obtained for materials in tension-compression does not lead to the agreement between the calculated and experimental data on fatigue strength of specimens with stress concentrators in bending. This agreement can be obtained only if the dependence of cyclic stress-strain diagrams on the stress gradient is taken into account.

Figure 4 shows $\sigma_{-1}^{*}$ versus relative stress gradient in the stress concentration zone $\overline{\eta}$ relations plotted from the test results for specimens ( 1-6, 12-14 - alloy steels; 7 - aluminium alloy; 8-11, 15 - nickel alloys) in tension-compression and for solid and notched specimens in bending. Here $\sigma_{-1}^{*} = \alpha_{\sigma}[\sigma_{-1b}]_n$, where $\alpha_{\sigma}$ is the theoretical stress concentration factor, $[\sigma_{-1b}]_n$ is the fatigue limit of specimens with stress concentrators in bending. With the account taken of the data listed in Table 2, the results presented in Fig. 4 characterize a decrease in cyclic proportionality limits at large stress gradients.

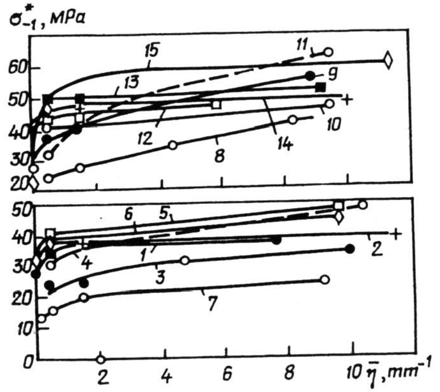

Fig. 4. Variation of $\sigma_{-1}^{*}$ depending on the $\overline{\eta}$

Size effect. Taking into account the above results, a reduction in the fatigue limits of metals with a decrease in the specemen size can be related to the dependence of the cyclic proportionality limit on the stress gradient. This suggestion correlates well with the fact that under uniform stress conditions (tension-compression) the fatigue limit is independent of the specimen size.

Load biaxiality. Figure 5 shows cyclic stress-strain diagrams obtained by Troshchenko et al. (1984) for a carbon steel (0.45 % C) under fully-reversed tension-compression(1) and torsion of thin-wall specimens (2) plotted in the coordinates which correspond to the theory of maximum relative strains $\sigma_{II}$, $\varepsilon_{II}$ (a), maximum shear stresses $\sigma_{III}$, $\varepsilon_{III}$ (b), shape changing energy $\sigma_{IV}$, $\varepsilon_{IV}$ (c), and octahedric stresses and strains $\tau_{oct}$, $\gamma_{oct}$ (d). As it follows from those results, the best correspondence between stress-strain diagrams takes place in the coordinates $\tau_{oct}$ - $\gamma_{oct}$. As is known, in this case the ratio between fatigue limits for thin-wall specimens in torsion and in tension-compression is 0.57. This value can be much different if the analysis involves the data obtained on specimens in testing of which the stress gradient differs from zero (bending, torsion of solid specimens).

Scattering of fatigue characteristics. Figure 6 presents fatigue curves for a carbon steel in tension-compression in the coordinates "$\sigma_a - \lg N_f$" and "$\lg \Delta\varepsilon_i - \lg N_f$". As it follows from this figure, the scattering of the test results presented in the "$\lg \Delta\varepsilon_i - \lg N_f$" coordinates is much lower. This suggests that the specimen lives depend not only on the acting stress, but also on the intensity of fatigue damage accumulation in each specimen which is defined by the $\Delta\varepsilon_i$ value. As it was indicated by Troshchenko et al. (1984), the scatter of the fatigue limits can be estimated from that of the stress values corresponding to the magnitudes of inelastic strains in the specimens under study which, for the given class of materials, correspond to the fatigue limit.

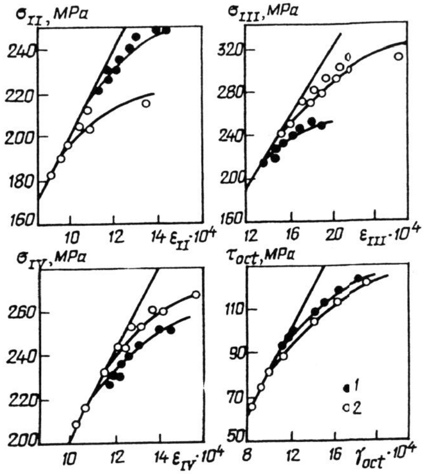

**Fig. 5.** Cyclic stress-strain diagrams for tension compression and torsion

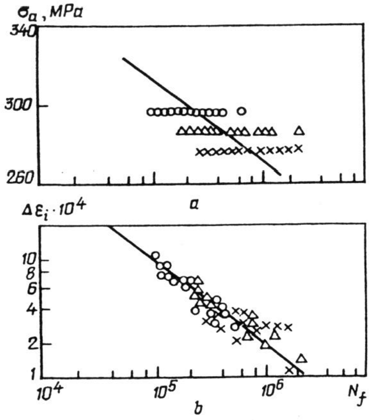

**Fig. 6.** Fatigue curves for carbon steel

Program loading. The knowledge of the laws of inelastic deformation of metals makes it possible to enhance the reliability of life prediction in program and block loading. Consider those possibilities taking linear damage summation hypothesis as an example and using the results obtained by Troshchenko et al. (1973).

$$\sum_{1}^{s} \frac{N_i}{N_{fi}} = 1 \text{ or } a \qquad (4) \qquad\qquad \sum_{1}^{s} \frac{N_i}{b_i \left(N_{fi}\right)_t} = 1 \qquad (5)$$

Formula (4) corresponds to the classical linear hypothesis where $s$ is the number of load steps, $N_i$ is the number of cycles of operation at $\sigma_{ai}$, $N_{fi}$ is the number of cycles to fracture at $\sigma_{ai}$ determined generally from the fatigue curve corresponding to 50% of the fracture probability. In eqn (5), $\left(N_{fi}\right)_t$ is the number of cycles to fracture which corresponds to the fatigue curve of an individual specimen tested at program loading, $b$ is the value which takes into account inelastic strain variation under program loading. The calculation procedure for these values is described by Troshchenko et al. (1984).

Figure 7 presents the experimental data in the coordinates "$a$ - $a_m$" ($a_m$ is the mean value of the accumulated damage for a batch of specimens tested at the same regime) for a carbon steel tested at different regimes of two-step loading ($\sigma_1$ and $\sigma_2$). Dark symbols correspond to the results from eqn (4).

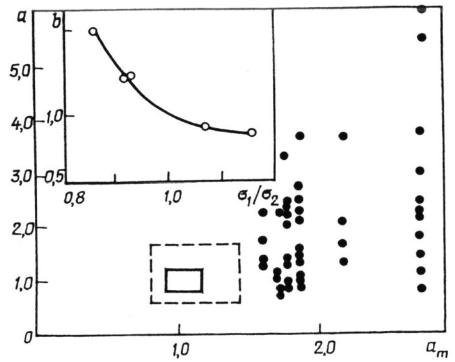

**Fig. 7.** Test results for two-step loading

The symbols corresponding to eqn (5) fall within the rectangle bounded by solid lines. Symbols corresponding to eqn (5), assuming $b = 1$, fall within the rectangle bounded by dashed lines.

Bifrequency loading. Figure 8a shows cyclic stress-strain diagrams for an austenitic steel of different thermal treatment and Fig. 8b shows corresponding fatigue curves for one-frequency (1) and bifrequency (2) loading obtained by Khamaza et al. (1989). The loading frequencies were 0.082 Hz and 34 Hz, the low-frequency stress amplitude was constant, $\sigma_m$= 50 MPa. Figures 8a and 8b were constructed using high-frequency stress amplitudes. As it follows from the figures, in bifrequency loading the transition from elastic to inelastic deformation shifts to the region of lower stresses. A similar shift occurs with the fatigue curves. Considering the interrelation between fatigue and inelasticity in bifrequency loading and using criterion (3), Khamaza et al. (1989) obtained the dependences which allow prediction of life under bifrequency loading.

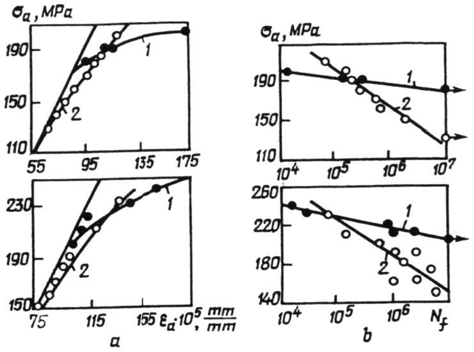

**Fig. 8.** Cyclic stress-strain diagrams (a) and fatigue curves (b) for bifrequency loading

Loading frequency. It was shown by Forrest et al. (1954) that with an increase in the loading frequency inelastic cyclic strains decrease at the same stresses. This corresponds to an increase in the fatigue limits with the loading frequency, the latter fact being confirmed experimentally. An increase in the loading frequency results not only in a reduction of cyclic plastic strains with an increase of the loading frequency, but also in dissimilarity of the parameters in the equation relating stresses ($\sigma_{a_1}, \sigma_{a_2}$) and inelastic strains ($\varepsilon_i = \Delta\varepsilon_i/2$) which was demonstrated by Khamaza et al. (1989). Figure 9 gives $\sigma_{a_1}\left(\sigma_{a_2}\right)$ vs $\varepsilon_{i_1}\left(\varepsilon_{i_2}\right)$ relations for an austenitic steel of two thermal treatments and for a carbon steel.

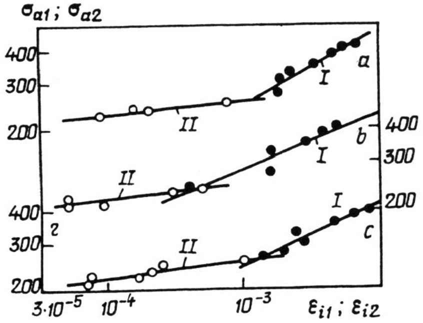

**Fig. 9.** Cyclic stress-strain diagrams at low (I) and high (II) loading frequencies

Surface conditions. Lyalikov (1989) determined that cyclic inelastic strains in specimens of carbon and alloy steels in torsion decrease when considerable residual compressive stresses are initiated in a surface layer due to plastic surface deformation. The results of those investigations for a carbon steel are given in Fig. 10 as the dependences of nominal inelastic shear strains upon the number of load cycles $N$ for polished specimens (1) and for specimens after surface plastic deformation (2). This figure also shows the distribution of residual stresses $\sigma_r$ through the specimen depth for the surface conditions studied.

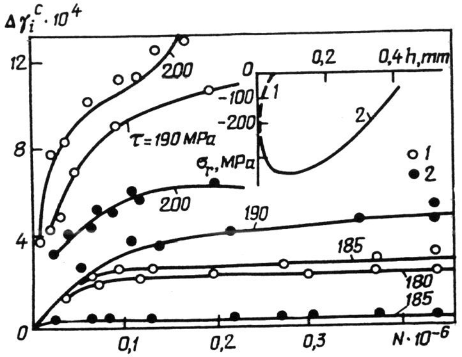

**FIg. 10. Cyclic strains after polishing (1) and surface plastic deformation (2)**

## CONCLUSIONS.

1. A method has been developed for studying inelastic cyclic strains in metals under high-cycle loading which allows inelastic cyclic strains to be investigated in tension - compression, bending and block loading. True values of stresses and inelastic strains are calculated under nonuniform stress conditions.
2. Strain and energy criteria of fatigue fracture of metals under high-cycle loading have been formulated and justified.
3. Interrelation between inelasticity and fatigue of metals under high-cycle loading has been analyzed with the account taken of the temperature, stress gradient, stress concentration, size effect, load biaxiality, scatter of fatigue characteristics, program and bifrequency loading, loading frequency and surface layer conditions. A good correlation has been demonstrated between the laws of fatigue fracture and specific features of inelastic deformation.

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