HIGH CYCLE FATIGUE ANALYSIS IN MECHANICAL DESIGN

K. Dang Van
Laboratoire de Mécanique des Solides, CNRS URA 317, Ecole Polytechnique
91128 Palaiseau Cedex, France

During these last decades, fatigue research has mainly focused on low cycle fatigue and fatigue crack growth (long cracks as well as short cracks). This trend is favoured by progress done in theories of plasticity and fracture mechanic which provide adequate frameworks for studying those types of fatigue phenomena. Notice that structural analysis based on fatigue crack growth theories are intimately related to Damage Tolerant Design Approach which requires inspections of the various components of a structure at regular time intervals in order to detect possibly existing or newly created cracks. This approach is well adapted to some domains like aircraft industries - design and maintenance -, but is inadequate in many other cases. Unfortunately, these are most frequently encountered in mechanical industries. Take for example the automotive industries dealing with mass market product which have to resist to long term damage. On the other hand, many mechanical structures, even if they are produced in small number - for instance big electric power generators or boat engines...- have to resist a great number of service loading cycles without failure. So that even today high cycle fatigue analysis of structures is clearly still the main industrial need in mechanical design. Despite this persistent goal, modelling of metal behaviour in polycyclic fatigue regime remains restricted to empirical approaches : the resulting criteria which are often obtained by curve fitting of particular ideal experiments are difficult to apply in design. The available relationships in mathematical form between parameters in the system and loading are most of the time untractable when one considers an industrial structure and not only a laboratory test specimen. Consequently engineers use elementary rules drawn mainly from experiences gained by uniaxial testing or previous successful design scheme. Most of the time, engineers have also to test actual structures which may be very expensive and time consuming. Actually, there is a big gap between modern computing tools and the use of the results for fatigue prediction in "automated" design by contrast with other mechanical fields like plasticity, fracture mechanics... Given the increasing reliance upon computer based design methods, it is now very important to make progress in modelling of endurance limit criteria and in high cycle fatigue. In my opinion, the slow progress done in deriving computational methods for fatigue endurance assessment is mainly due to the lack of an underlying coherent theory of metal behaviour under low and moderate external loads (e.g. service loads) that do not introduce visible macroscopic plastic strains. However, abscence of detectable plasticity does not mean that metal behaves in a purely elastic manner at all levels of material description. Even if the material appears elastic at the usual engineer's scale, some metal grains undergo plastic deformations under low/moderate cyclic external loads. These irreversible processes that take place at the scale of the grain size are chiefly responsable (like in low cycle fatigue) for crack initiation in high cycle fatigue regime. Due to local plastic deformation, a residual stress pattern is created which modifies locally the stress cycles. These residual stresses are never taken into account in the existing fatigue theories. Investigation of these phenomena requires the introduction of the mesoscopic scale of material in addition to the usual macroscopic scale. As it will be shown, current developments of the mesoscopic scale approach in high cycle fatigue permits to handle without ambiguity difficult fatigue problems such as multiaxiality, non proportionnal cyclic loadings which are of primary importance for real industrial applications: engineering components are most of the time subjected to complex states of stresses, involving multiaxial stresses.The origin of multiaxialiy comes from different factors such as service loadings, geometry of the structures, residual stresses which are multiaxial by nature. Many industrial

structures combine all these factors so that high cycle fatigue assessment requires a multiaxial fatigue limit computation method.
Some fatigue modellings based on a multiscale approach will be presented paper. To demonstrate their efficiency,they are applied to industrial examples. In particular, the fatigue computation methods are applied to predict damage induced by contact between solids.

Orowan's Model [1]

Fatigue is generally due to stress concentrations and heterogeneities. In high cycle regime,the first fatigue phenomena occur in some grains which have undergone local plastic deformation in characteristic intracrystalline bands. Cracks very often initiate in the shear bands of plastically deformed grains.
Using this image, Orowan proposed in the late thirties the following fatigue nucleation model: The weak plastic element is embedded between two elastic springs which impose their deformation. this element suffers plastic strain and hardens as it is shown on fig.1, if pure isotropic hardening is supposed. Consequently, elastic shake down should occur if fatigue does not appear.The limit loading path oscillates between A (the corresponding shear stress is - τ) and B (resp. τ). If the theorical limit τ is less than some definite threshold value τ_f there is no fatigue. Fatigue limit corresponds to $\tau = \tau_f$.
Another dual interpretation is based on the evaluation of the accumulated plastic strain Γ for the corresponding strain hardening process. If Γ is greater than some critical value, fatigue will occur.
We shall use Orowan's idea to propose multiaxial fatigue criteria.

Macroscopic and mesoscopic scales of material description

Usual engineering computations are based on the assumption that the material is a continuous medium. The mechanical state is described with the help of macromechanical quantities Σ (stress), E (strain) defined at every point M of the structure (Fig.2).This means that in the vicinity of M, an elementary volume V is defined such that V is representative of the considered material. Therefore, the elementary volume V which defines the macroscopic scale of the material is the smallest sample that can be considered as homogeneous and representative of the average properties of the material. Within the volume V, Σ and E are by definition constant and correspond for instance to results obtained with strain gauges. Engineers have only access to these macroscopic quantities. Typical dimensions of a small stain gauge is 1mm x 1mm, so that to give an idea, a representative material volume for moderately fine grained metals contains at least 1000 grains. The mechanical state of each grain is described by the mesoscopic quantities σ, ε which differ from Σ, E because the crystal properties vary from one grain to the other.
One has the well known relations between (σ,ε) and (Σ,E)

$$\Sigma = \frac{1}{V}\int_V \sigma . dV \qquad E = \frac{1}{V}\int_V \varepsilon . dV$$

This last relation is valid only in the volume V if there are no voids (holes, cracks...). If not, one has to add an extra deformation term induced by these defects. Exact relations between macroscopic and mesoscopic quantities (stress, strain, plastic strain and strain rates, energies rates, residual stress rates...) have been established by Bui, Dang Van and Stolz [2]. These relations are rather complex and coupled. One can understand why it is in general incorrect to use directly the macroscopic numerical quantities for the description of mesoscopic phenomena.
The first previous relation, means that even if the mean value of σ corresponds to macrostress Σ, the local stress tensor σ can fluctuate. More precisely, σ and Σ are related by the following relation :

$$\sigma_{ij}(m,t) = A_{ijhk}(M,m).\Sigma_{hk}(M,t) + \rho_{ij}(m,t)$$

In the above equation, ρ is the local residual stress tensor and A(M,m) is the localization tensor which is correlated to the microstructure. In theory, A can be obtained by solving six elastic boundary value problem for the volume V (one for each {h,k} component). If the external loadings are moderate, local plastic strains occur, but the mechanical state will tends toward elastic shaked down state. Then Melan's theorem states that beyond a certain time t, a time independant residual $\rho^*(m)$ exists such that $\sigma(m,t)$ will no more violate the local yield criterion. In the following, it will be assumed that the localization tensor is reduced to identity. This is only a first order approximation of the localization problem, but it is found to provide satisfactory results within the engineering framework of fatigue limit problem of macroscopically isotropic metals. In this problem, *the elastic shakedown assumption at all scales (macroscopic as well as mesoscopic) before the nucleation of fatigue crack is the main hypothesis.*

Origin of the multiscale method

Fatigue is generally due to stress concentrations and heterogeneities. The first fatigue phenomena occur in some grains which have undergone local plastic deformation in characteristic intracrystalline bands. In high cycle fatigue, the rest of the matrix behave elastically in general because the overall plastic strain is negligible. (In some particular cases like contact fatigue, or fatigue on notched specimens, macroscopic flows happen, but tend rapidly toward a stabilized state if the loadings are relevant of high cycle fatigue; these cases will be examined later; for the sake of simplicity, only fatigue of plain smooth specimens is first considered). Fatigue cracks initiation will happen most of the cases in shear bands of plastically deformed grains. Following Orowan's ideas, multiaxial fatigue criteria are derived, based on the description of material behaviour at different scales and on the elastic shake down hypopothesis at all scales.

Multiaxial stress fatigue limit criteria

The first " macro-meso " models for fatigue limit (Dang Van 1973, Papadopoulos 1983) are based on a stress limit approach.
One can give the following image (figure 3) very similar to Orowan's proposal : during the first loading cycles, some misoriented grains undergo plastic deformation. Consequently these grains workharden and a stabilized stress state σ_{stab} is reached after a while. If σ_{stab} is below some threshold σ_{lim}, their is no fatigue (curve A). If σ_{stab} is greater, fatigue will occur (curve B). σ_{stab} corresponds to a pseudo elastic shake down (pure elastic shake down if their is no fatigue). σ_{lim} is the fatigue limit of the material. The main assumption is that near the fatigue limit, the structure shakes down elastically at all scales (macroscopic and mesoscopic).

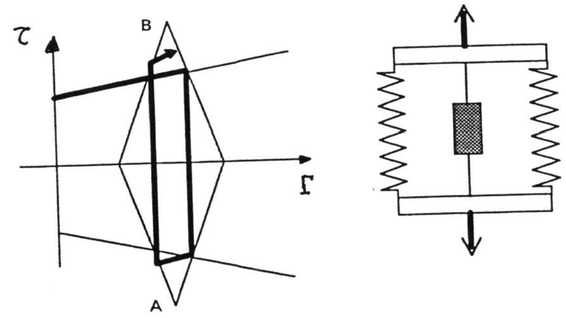

Fig-1- Schematic representation of OROWAN's model

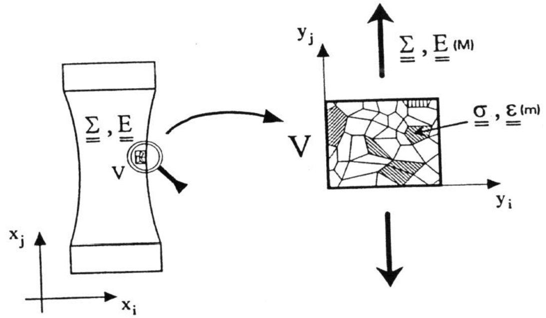

Fig-2- Macroscopic and mesoscopic scales of material description

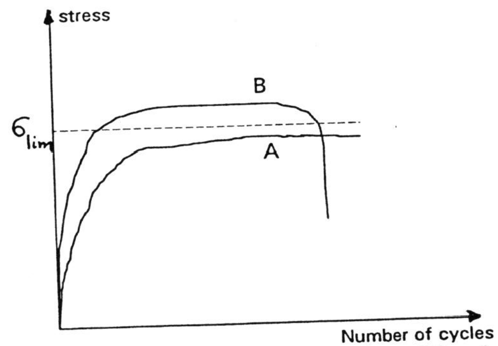

Fig-3- Schematic evolution of strain hardening above (B) and under (A) the fatigue limit

Consequently a stabilizes mesoscopic residual stress tensor ρ^* exist such that the resulting local stress is given by

$$\sigma(m,t) = \Sigma(M,t) + \rho^*(m)$$

The constuction of $\rho^*(m)$ is given in appendix. The local stress tensor $\sigma(t)$ being known at any time t of the cycle, different fatigue criteria can be proposed.

Fatigue limit based on the current stress state (Dang Van 1973)

Fatigue crack initiation will occur in a critically oriented grain which has undergone local plasticity,if, at least for one time t of the stabilized cycle, one has $f[\sigma(m,t)] > 0$ for m belonging to the representative volume V(M).
In such a criterion,the current stress state is considerd and damage happens at a precise instant of the loading path. As crack usually occur in transgranular slip bands, the local shear is an important parameter. In the same way, the normal stress acting on these planes accelerates damage formation. However this parameter is quite difficult to compute in general, because it depends on the considered plane. For this reason, the mean value of the normal stress for all directions, which is precisely the hydrostatic tension p is prefered. Taking these remarks into account, one can particularize $f(\sigma)$ as a relation between τ and p, for instance : $f(\sigma)=\tau + ap - b$. The safety domain delimited by straight lines, can be determined by classical uniaxial laboratory experiments.

Fatigue limit criterion based on the limit of local shakedown state (Papadopoulos 1987)

According to this model the fatigue resistance corresponds to the stabilisation of mesoscopic strain, i.e.:elastic shake down of grains is related to endurance; plastic shake down will enduce damage and finally fracture because of the subsequent cyclic softening.
Let k^* be the limit radius of the Mises norm characterizing the elastic domain at the stabilized state. (See appendix for the construction of k^*). Papadopoulos assumes that the grains will shake down elastically if k^* is less than some limit value k_{lim} which depends of the maximum hydrostatic tension p: $k_{lim} = \beta - \alpha . p_{max}$
The fatigue criterion will be :

$$k^* + \alpha . p_{max} < \beta$$

Many industrial applications have been done with stress based criteria (Dang Van ,Papadopoulos). All these applications which involve local multiaxial loadings need to be done

α and β are two material constants which can be identified by two different tests.
By this method, it is no longer necessary to describe the whole local loading path, once k^* has been determined. In many cases, the obtained predictions are very similar to the current state method.

Model based on accumulated deformation

Recently, Papadopoulos [3] proposed a multiaxial fatigue model based on threshold of accumulated deformation generalizing an possible interpretation of Orowan's model: fatigue criterion mainly implies the application of a bound on the plastic strain that has been accumulated in plastified grains after a great number of load cycles. Papadopoulos considers also that the breaking of the first plastically deforming crystal is not the most critical event and proposes to evaluate an average value of the plastic mesostrain Γ accumulated by all gliding crystals within V and to place a threshold on this average. The average measure chosen to this end is the root mean square of Γ over all directions. Then Papadopoulos estimate this quantity and shows that it is approximatively related to the root mean square of the resolved shear stress $<\tau_a>$. Finally the fatigue criterion takes the form:

$$\sqrt{<\tau_a>^2} = \beta - \alpha . p_{max}$$

Very good agreement (about 2.5% of discrepancy) between experimental results and prediction are obtained for in phase and out phase combined bending and torsion collected in the litterature.

Applications

Example of structural fatigue assessment

in closed collaboration with companies, in order to verify the pertinence of the predicted fatigue limit in comparison with experimental results on structures. For these calculations, precise knowledge of the stress cycles are necessary, so that we have sometimes to use new numerical methods, in particular for the evaluation of the stabilized mechanical parameters induced by repeated contact loadings.

We choose only to present some recent applications:two of these examples are related to automotive industries, and one other is in connexion with damage induced by contact between solids. All these cases involve typical multiaxial loadings. The computed endurance predictions are compared to experimental results.

Fatigue assessment of a suspension arm

(For more detailed presentation of this application see Ref 4)

iA new suspension arm was tested. The loading was a force system following two perpendicular directions X and Y: $F_x = F_a \sin\omega t$, $F_y = F_a (\cos\omega t - 0.5)$. The material is a forged steel. The fatigue limits of the material have been determined in repeated tension (340 ± 278 Mpa), alternate tension (±300 Mpa) and repeated compression (- 365 ± 362 Mpa), on specimen incorporatng the forged skin. Dang Van's criterion is therefore given by:

$$\tau_a + 0.20p < 180.$$

Application of the fatigue analysis requires that the fatigue cycle be described at several discrete times. Six values of ωt have been chosen (0, $\Pi/2$, Π, $3\Pi/2$ and two intermediate values such that $F_x = F_y$). The corresponding stress tensors are calculated by F.E.M. and stress results are analysed with Dang Van's fatigue analysis routine. The calculated value of the criterion displayed on graphic output is the quantity $C_s = (\tau_a + a.p - b) / (b - a.p)$ which measure the danger of fatigue failure. Negative values mean safety, while positive values mean danger. The example, summarized on figure 4 is given for F_a = 8500 N. Two critical areas appear in the component. The corresponding loading paths are drawn in the fatigue diagram. They intersect the material line represented by the above equation . The fatigue limit, $C_s = 0$, is found for F_a = 8000 N. Fatigue tests have been performed on suspension arms The number of cycle to failure is defined by the first visible crack. Below the calculated fatigue limit, no

fatigue crack was detected at the end of the test, after 1.2 10^6 cycles. For loadings above the fatigue limit, the failure of the component always occurs at the most critical point identified by calculation.

Fatigue assessment of a crankshaft reinforced by pressure rolling

(For more detailed presentation of this example see Ref. 4)

This is an interesting example, which shows the influence of residual stress and surface treatment and the way to take into account these factors in the fatigue assessment of an industrial component by the proposed method.

Crankshafts have to be designed for infinite life. The critical areas which are generally filets between bearing and flanges, are reinforced by pressure rolling or thermal treatment. Optimization of fatigue resistance of pressure rolled crankshaft necessitates a global approach taking account the generated initial residual stress field, their evolution to a steady state under service loading and finally fatigue strength calculations to take into account this history. For this purpose, we have developed different computational tools.

The calculations are done in three steps :

1.Simulation of pressure rolling

Pressure rolling is modeled with an original direct stationary method (K. Dang Van and H.Maitournam, 1993 [5], E. Hanus, H. Maitournam and K. Dang Van 1996 [6]). This method proved to be very efficient for studying rolling contact problems. The main feature of the method is that the analysis of the rolling is performed in a (r,θ,z) axis system tied to the moving load applied to the roller. In steady state regime, the physical values related a material point no longer depend on time but on their angular position in the direction of rolling. The calculated residuel stress field, shown on figure 5 are in reasonable agreement with measurements done by X ray at the surface.

2.Relaxation of residual stresses

The initial residual stress may evolve under the effect of service loading. It is thus necessary to estimate this evolution. This calculation is done using a method proposed by J. Zarka (see for instance Ref.7).

3.Fatigue strength assessment under service loading

One considers usually that fatigue improvement by pressure rolling of forged steel can be attributed to compressive residual stresses. Therefore we apply the proposed criterion using the data of the material without treatment, the macroscopic engineering stress cycles being obtained taking account of the two first steps.

More details for this example are presented in Ref 4. The efficiency of the proposed method is illustrated on figure 6, which shows the load path at the critical point for the fatigue limit found experimentally.

Application to modelling of contact fatigue

Fatigue phenomena induced by contact between solids is a major preoccupation in a great number of industries. For instance, railways companies are concerned with different types of fatigue damage due to rolling contacts like " kidney shape cracking ", " squat "or " head checking " defects...; in aeronautics, fretting fatigue failure is a very dangerous phenomenon. It is why contact fatigue is a subject of fundamental importance, much studied but with very little success: at present time: we are most of the time, unable to use results obtained by an experimental set up to predict those resulting from an other one and a fortiori to predict the contact fatigue resistance of an industrial structure. To overcome these difficulties it is

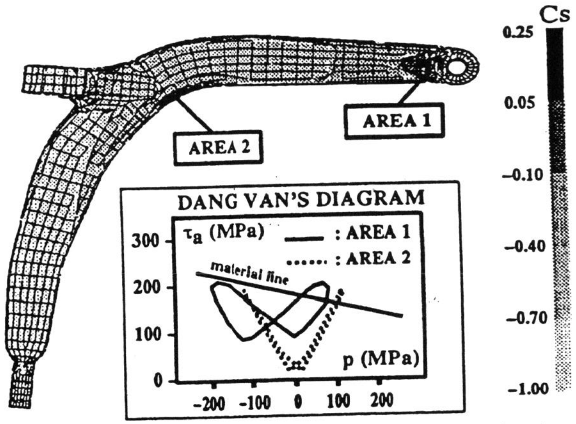

Fig-4- Isovalues of the Dang Van's criterion for Fa=8500 N, and load paths at critical points

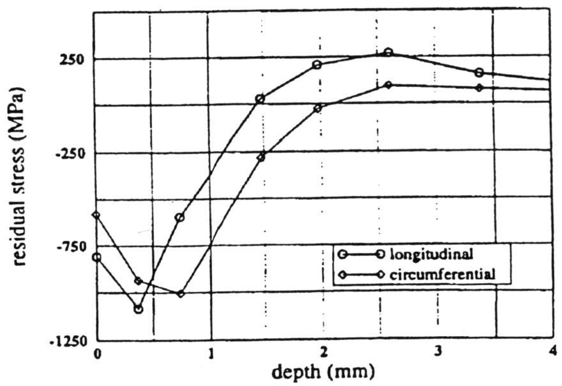

Fig-5- Residual stresses induced by rolling

essential -in my opinion- to separate the structural effects induced by the repeated contact (developpment adequate tools for the evaluation of the corresponding stress and strain cycles is necessary), from the material response, i.e. the material fatigue criterion. This criterion must be multiaxial because the stress state is multiaxial and varies in a complex manner.

Applications of the proposed fatigue multiaxial approaches to rail problems (rolling contact fatigue) can be found in Ref 8

Modelling a fretting fatigue test
(For more detailed presentation see Ref.9 and 10)

Fretting is surface damage induced by small oscillary displacements between metal contacting components. Most of the recent studies have focussed on the link between operating parameters and the damage mechanisms, but the remaining main question is how to transpose these experimental behaviour obtained with a particular set up to avoid fretting on a structure.
To answer to this question it is necessary to be able to modelise the conditions of crack initiation. The studied experimental set up is represented on fig.yy: two cylindrical fretting pads are clamped against the two surfaces of a flat uniaxial fatigue specimen tested under constant amplitude loading. The pads are made of 100C6 steel and the fatigue specimen is made of 3Cr-Mo-V steel. The prescribed oscillatory between the pads are linked to the prescribed oscillatory fatigue stress S in the specimen. The flexible beams are equiped with strain gauge in order to measure the clamping force between pads and specimen and the friction force related to the displacement accommodation. The variations of the tangential force T(t) are recorded for each fatigue cycle and plotted as function of fretting fatigue (fretting fatigue loops). By varying the operating parameters (P, S_{max}), three regimes are established, summarized in the map shown on Fig.7 :
- Stick regime corresponding to no damage appears during the 10^7 cycles of the test ;
- Gross slip regime: In this regime, particle detachment is observed corresponding to wear mechanism ;
- Mixed stick-slip regime: There is partial slip and fatigue crack nucleation are observed.
The limit between the last regime and the two others corresponds to fatigue limit regime.
The question is : are we able to predict this fatigue limit using the proposed multiaxial criterion, identified on the material by classical fatigue tests independently of the fretting fatigue experiment.
For this purpose, it was necessary to evaluate the stress cycles (details of the methods can be found in cited publications) and then to apply the multiaxial stress fatigue criterion (Dang Van). Numerical computations give the stress trajectory of different points of the structure which are compared to the material line. For each case (S_{max}, P_c) the most critical path is plotted (Fig.8). In fact, the most critical point of the structure is always found to be located at the edge of the contact area. The predictions are in good agreement with the experimental observations of fatigue crack initiation.

APPENDIX

Principal steps of the fatigue limit calculations are recalled schematically. These steps are used in the presented fatigue computations.

Step 1: Evaluation of the stabilized macrotress tensor $\Sigma(t,M)$ at different time t representing the fatigue cycle.
For the evaluation of Σ, it is sometimes necessary to perform elastoplastic computations to estimate the macroscopic residual stress field in the shake down state. It is the case of many

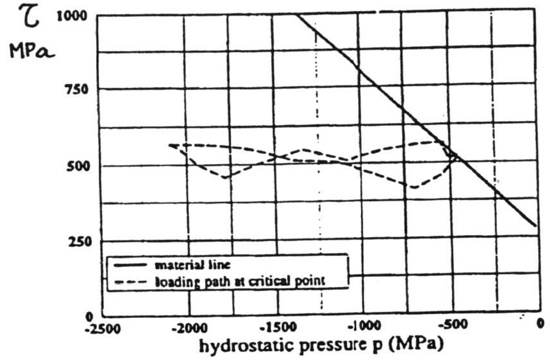

Fig-6- Load path at the critical point of the crankshaft.

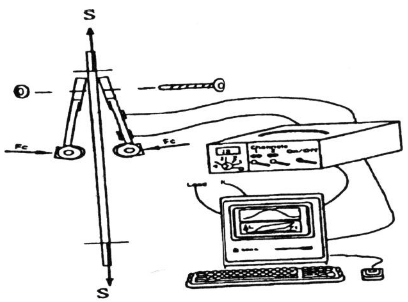

Fig-7- Fretting-fatigue set up

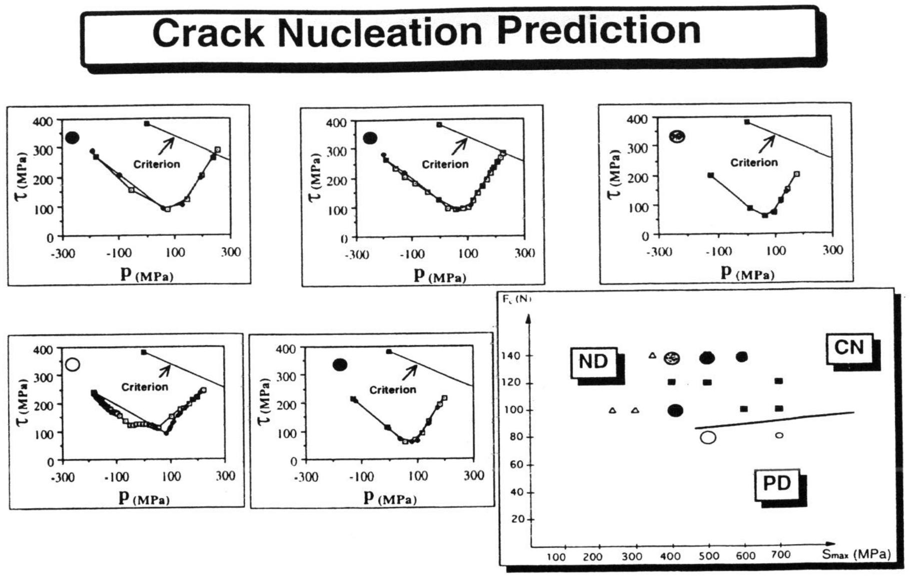

Fig-8- Prediction of fretting fatigue behaviour contact fatigue problems, or when studying fatigue of notched structures; contained plastic flow may occur during the first cycles. In the stabilized state however, the stress cycle is supposed to be purely elastic, because of the shake down hypothesis.

Step 2 : Evaluation of the mesostress tensor $\sigma(m,t)$.
It is based on generalized Melan's theorem as it is given by Mandel and al. studying elatic shake down of elastoplastic structures with combined kinematic and isotropic hardening and on a minimum assumption detailed below.
The elastic domain of the material is defined by

$$g(\sigma - c\varepsilon_p) = k2(p)$$

ε_p is the plastic strain tensor and c a positive constant characterizing linear kinematic hardening and p is the equivalent plastic strain; k is supposed to be a strictly increasing function of p.
Then a sufficient condition for elastic shake down is that a certain fixed (i.e. independent of the time t) residual stress field $\rho^*(m)$ and a plastic strain field $\varepsilon p(m)$ exist such that

$$\forall t > t1,\ g(\Sigma(t)+\rho^* - c\varepsilon_p) < k2(p)$$

For sake of simplicity, we chose the Mises criterion, so that only the deviatoric part of the stress intervenes. Splitting the stress tensor Σ (resp. σ) in deviatoric part devΣ = S (resp. s) and hydrostatic part P = trace $\Sigma/3$ (resp. p), the last condition can be rewritten :

$$\forall\ t > t1,\ J2\,(S(t) - z) < k2(p) \text{ with } z = -\,\mathrm{dev}\,\rho^* + c\varepsilon_p$$

z is thus the center and k is the radius of an hypershere which should contain all the loading path for any time t > t1. However, this condition is not sufficient to determine z, so that an other hypothesis is needed. We make the assumption that the *z solution (near the fatigue limit) corresponds to the smallest hypershere that all the load path.L(t).* Then for a given stress cycle L(t) corresponds a given z and dev ρ^* can be taken approximatively equal to z. In ref (11), an algorithm for evaluating dev ρ^* is given.
Concerning the hydrostatic part, we suppose that the mesoscopic and macroscopic hydrostatic tension are the same.
It is important to notice that the precise knowledge of the local elastoplastic behaviour is not necessary thanks to the shakedown hypothesis.The proposed method gives an approximate way to characterize the local stress tensor in the pseudo stabilized state.

Step 3 - Application of stress based criteria
For Papadopoulos stress criterion, the knowledge of the stabilized radius k* and the maximum hydrostatic tension p_{max} is sufficient.
Dang Van's criterion is more complex. To check automatically the fatigue resistance of a structure can be rather difficult, because at each point,one has to consider the plan on which the loading path $(\tau, p)(t)$ is " maximum " relative to the criterion. This computation can be simplified if one proceed as following (11)
$\tau(t)$ = Tresca $(\sigma(t))$ is calculated over the cycle period. For this, it is important to note that

$$\text{Tresca}\,(\sigma(t)) = \text{Tresca}\,(s(t)) = \text{Max}\,|\,\sigma_i(t) - \sigma_j(t)\,|$$

where σ_i, σ_j are principal stresses;

then, evaluate the quantity d defined by : $d = \text{Max}\,\{\tau(t) / (b - ap(t))\}$
The maximum is to be taken over the cycle period. If d > 1, fatigue will occur. Working this way, all facets which could be involved by crack initiation are automatically reviewed. Couples (τ, p) verifying d > 1 are associated with defined facets and so this criterion gives also the direction of crack initiation.

REFERENCES

(1) OROWAN, E., Proc. Roy. Soc., London, A, 171, 79, 1939.
(2).H.D. BUI, K. DANG VAN, C. STOLZ, Relations entre grandeurs microscopiques et macroscopiques, C.R.A.S., Tome 292, Serie 2, Paris 1981, 1155-1158
(3).Y. PAPADOPOULOS, Exploring the high cycle fatigue behaviour from the mesoscopic scale,J. Mech. Behaviour, Vol. 6,No 2, 1996, pp.93-118.
(4). A. BIGNONNET, Fatigue design in automotive industry,FATIGUE' 96, Berlin, Proc. VIth Int. Fat. Conf., pp.1825-1836.
(5). K. DANG VAN, H. MAITOURNAM, J. Mech. Phys. Solids Vol.41.No 11, pp. 1691-1710, 1993
(6). E. HANUS, H. MAITOURNAM, K. DANG VAN, Int. J. Solids Structures, Vol 33, No.25, pp.3739-3753, 1996
(7). J.ZARKA, J. FRELAT, G.INGLEBERT, K. NAVIDI, "A new approach in inelastic analysis of structures", Martinus Nijhoff pub.1988.
(8). K. DANG VAN, H. MAITOURNAM, B. PRASIL, Elastoplastic Analysis of repeated moving contact: application to railways damage phenomena, To be Published in Wear.
(9). C. PETIOT, L. VINCENT, K. DANG VAN, N. MAOUCHE, J. FOULQUIER, B. JOURNET, Wear 181- 183 (1995) 101- 111.
(10). N. MAOUCHE, H. MAITOURNAM, K. DANG VAN, On a new method of evaluation of inelastic state due to moving contact, Proc. of the 11th Wear of Materials Conf., San Diego , April 1997, To be published in Wear.
(11). K. DANG VAN, Macro-Micro Approach in High Cycle Multiaxial Fatigue, ASTM STP 1191, pp. 120-130.