BEHAVIOUR OF A SPHEROIDAL GRAPHITE CAST IRON IN HIGH CYCLE MULTIAXIAL FATIGUE UNDER RANDOM LOADING SPECTRUMS IN BLOCKS

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ABSTRACT

Cumulative fatigue damage tests have been carried out on cylindrical smooth specimens under variable amplitude spectrum loadings in blocks. The tested material is a spheroidal graphite cast iron. Several fully reversed solicitations have been applied to the specimens : Plane Bending, Torsion, in phase and out of phase combined Plane Bending and Torsion. The amplitude of stress evolves in short blocks (with a constant duration) randomly arranged with the same apparition frequency such that each amplitude of stress is equi probable. The distribution of lifetimes was found to be log-normal. Comparison of test data was done by using the technique of the statistical hypothesis tests. In Plane Bending and in Torsion the apparition order of blocks had no influence on the median value of the fatigue life. In combined Plane Bending and Torsion the median values of lifetimes were not very different whatever the phase between Plane Bending and Torsion was. In general, predictions of cumulative damage rules are not in good agreement with experiments.

KEYWORDS

Variable amplitude loading, multiaxial, high cycle fatigue, cumulative damage rule, cast iron, experiment, loading history.

INTRODUCTION

The mechanical parts of structures are mostly operated under variable amplitude loadings. That is the reason why damage cumulation in high cycle fatigue is an important field of research for engineers. Many cumulative damage rules have been proposed since the work of Miner (1945) but all of them have been developped from test results under loadings with two blocks. Such loadings are very different from reality. In 1993, Bennebach carried out a few cumulative damage experiments on a spheroidal graphite cast iron. Some of these tests were done under two block loadings (high-low and low-high). Another test series was done in Plane bending under a loading with two sorts of blocks : low and high, both alternatively applied until failure.

The median total lifetime is between the fatigue lifes under high-low and low-high loadings with the same stress amplitudes. The prediction of Miner's rule is in agreement with experiment in this case. In order to test the behaviour of the strictly same material under loadings closer to reality we have carried out tests under spectrum loadings with many blocks of different amplitude randomly arranged. The predictions of several cumulative damage rules can be compared with such experiments.

EXPERIMENTATION

Material and specimens

According to the AFNOR standard, the designation of the tested spheroidal graphite cast iron is FGS 800-2. Its main static characteristics are given in Table 1. The cyclic behaviour of this material is neutral, there is no softening and no hardening. Its cyclic hardening curve can be described by the Ramberg-Osgood equation (1), with n=0.14 and K=1121 MPa ($\Delta\sigma$ is the stress range).

$$\varepsilon_t = \frac{\Delta\sigma}{2\cdot E} + \left(\frac{\Delta\sigma}{2\cdot K}\right)^{1/n} \quad (1)$$

Table 1 : Static characteristics of the tested spheroidal graphite cast iron in tension.

Young modulus (GPa)	Poisson ratio	Yield stress at 0.2% (MPa)	Maximum tensile stress (MPa)	Elongation at failure (%)
164.9	0.275	462	795	9

The fatigue properties of the material under fully reversed constant amplitude loadings were determined in four points Plane Bending, Torsion, in phase and out of phase Combined Plane Bending and Torsion. Bennebach (1993) used 50 specimens to determine each S-N curve. These experiments were carried out on cylindrical smooth specimen with a median torus and a net section of 12 mm in diameter. It involves a theoretical stress concentration factor equal to 1.07 in plane bending and 1.05 in torsion. The roughness of the median torus is such that Ra $\leq$ 0.2 μm. The model of Bastenaire (1960) and the Esope software (1995) have been used to analyse all the Bennebach's test data. Endurance limits are given in Table 2.

Table 2 : Endurance limits, in MPa, of the tested material (S = plane bending stress amplitude, T = torsion stress amplitude, *d* suffix means endurance limit).

Plane Bending	Torsion	Combined Plane Bending and Torsion $\phi = 0°$ S/T = 1.732	$\phi = 90°$ S/T = 1.732	$\phi = 0°$ S/T = 1.35
Sd = 280	Td = 211	Sd = 219 Td = 126	Sd = 234 Td = 135	Sd = 192 Td = 111

Loading spectrums

Nine test series under loading spectrums varying in blocks with different stress amplitudes were carried out. Several types of solicitations were applied to the specimens. All the solicitations were fully reversed. First, the loading spectrums were with simple sollicitations : Plane Bending or Torsion. Secondly, in phase and out of phase combined Plane Bending and Torsion were used.
Five different stress amplitudes were randomly applied to each specimen in blocks of 5,000 cycles. They had the same apparition frequency.

A loading spectrum is characterized by two things :
- the apparition order of five sorts of blocks numbered from 1 (lowest stress amplitude) to 5 (highest stress amplitude). Such a series of blocks is called below "spectrum".
- the solicitation and its five different stress amplitudes corresponding to the five sorts of blocks.
The apparition order of each block is chosen at random by drawing of lots without putting back. Three "spectrums" are determined 'ike this and named spectrums 1, 2, 3. An example of loading spectrum in Torsion is given in Fig. 1.

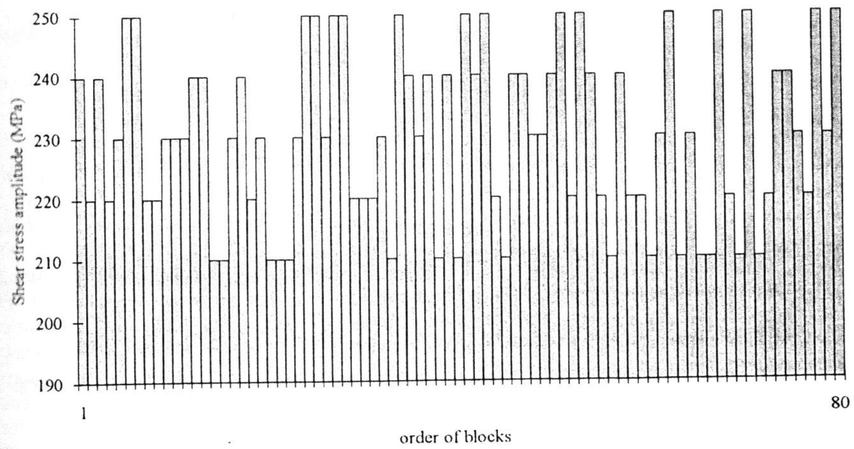

Fig. 1. : Example of loading spectrum number 1 in Torsion.

The stress amplitudes in Torsion were chosen with an equi repartition in stress from the S-N curve of the material between 10^5 cycles and infinite life, see Fig. 2. For all the other solicitations, each stress amplitude σ_i (*i* is the block number) was chosen such that the corresponding life on the S-N curve would have been almost equal to the life in Torsion under the corresponding τ_i stress level as illustrated in Fig. 2.

To be sure that the real loading history applied to each tested specimen be representative of a loading spectrum with equi probable blocks, we verified after each test that the difference between the experimental apparition frequency of each stress amplitude and the theoretical apparition frequency was not significant with a confidence level of 0.99. A statistical hypothesis test was used to do this (Grais, 1992).

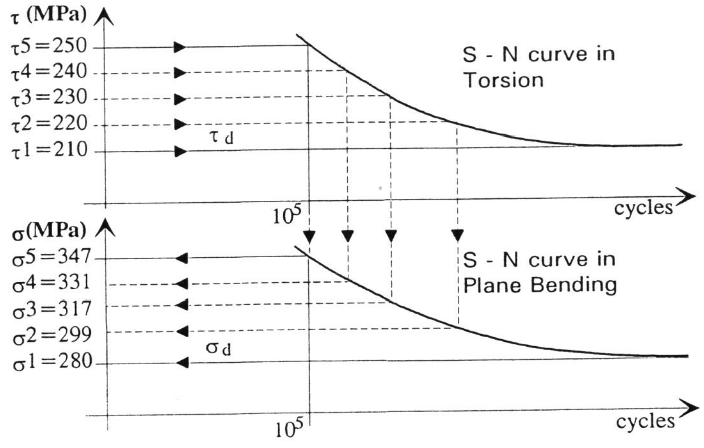

Fig. 2. : Choice of the stress amplitudes for each block in reference to the S-N curve in torsion. Example with Plane Bending solicitation.

Testing equipment

The fatigue testing machine used is servohydraulic. It has been designed and set up by Palin-luc and Lasserre (1994). Fatigue tests are stopped when there is a macro crack of about 0.5 to 1 mm in depth in the specimen. A duration of 5,000 cycles was chosen for each block, it is short enough to have a great number of blocks applied to the specimens when failure occurs, but not too short compared with the duration of a transition between two blocks. For the loading spectrums with one type of solicitation, this transition for passing from the lowest block to the highest one is less than 10 cycles at the operating frequency of 50 Hz. For the loading spectrums with two types of solicitation, the longest transition is less than 200 cycles. It occurs for passing from a block at the lowest amplitude of one solicitation to another block at the highest amplitude of the other solicitation. These transitions can be neglected compared with the block duration. The total duration of a loading spectrum is 400,000 cycles, it is long enough for having failure of all the tested specimens before the end of the spectrum.

EXPERIMENTAL RESULTS

Statistical analysis method

To be sure that the distribution of the fatigue lifes follows a log-normal distribution (Zheng Xiulin 1996) two statistical hypothesis tests were applied on all our test results : Kolmogorov-Smirnov (Schwob and Perache, 1991) and Shapiro-Wilk (AFNOR, 1991 a) with a confidence level of 0.99. These two tests show that the log-normal hypothesis can be accepted for the fatigue life. Thus, the cumulated frequencies method (AFNOR 1991 b) can be used to analyse our test results.

Results

All test data are plotted in Fig. 3 on a normal probability paper.

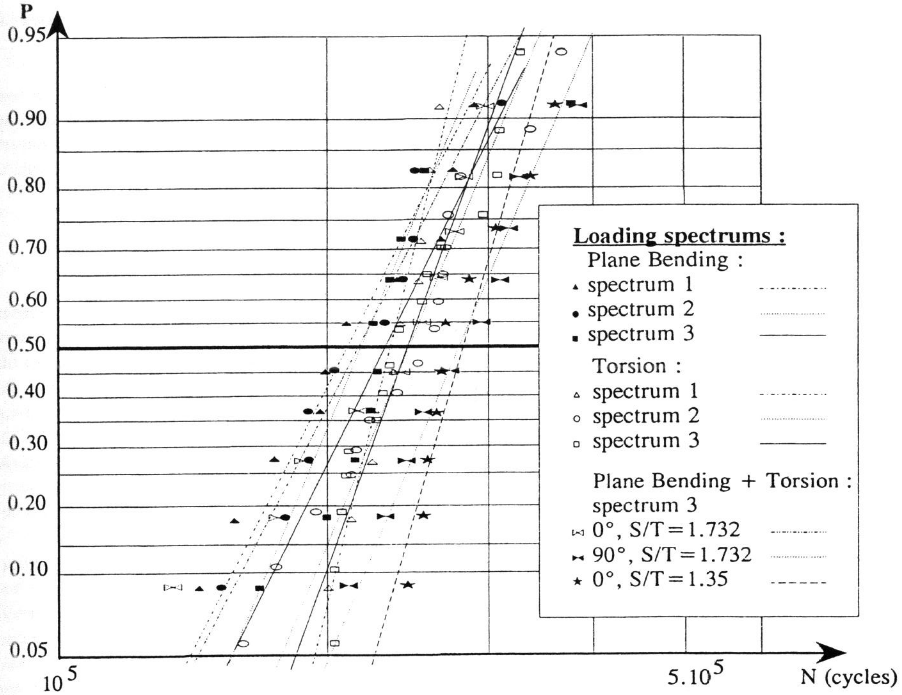

Fig. 3 : Test results under loading spectrums varying in blocks (5,000 cycles by block)

DISCUSSION

Loading spectrums

It can be seen in Fig. 3 that the slopes of all Henry's lines are almost the same. It means that the scatter of the fatigue life is nearly the same under all the loading spectrums. Furthermore, Fig. 3 shows that all the test data look to be around the same median value (lifetime with a probability of 0.5). An hypothesis test proposed by Schwob and Perache (1969) permits to compare two experimental mean values in order to determine if their difference is significant or not. For the fatigue life, mean value and median value are equal because the fatigue life follows a log-normal distribution. According to this test applied to all the test results, it can be concluded with a confidence level of 0.99 that :

- in Plane Bending and in Torsion the median values of lifetimes are not significantly different for each one of the three tested spectrums,

- for each one of the three spectrums (1, 2, 3) the median values of lifetimes can be considered to be the same whatever the simple solicitation is (Plane Bending or Torsion).
With a confidence level of 0.99, the previous statistical hypothesis test applied to the test data in combined Plane Bending and Torsion gives the following conclusion :
- in phase and out of phase, with a stress ratio, S/T, equal to 1.732 or 1.35, the median values of the fatigue lifes are not significantly different for the spectrum number 3.

All these test data seem to prove that the apparition order of the blocks has no influence on the median value of the fatigue life. Of course, this conclusion is available for our testing conditions : all the stress amplitudes are equi probable and the block duration is short compared with the total fatigue life. Nevertheless, this result is in contradiction with two blocks experiments, as well as the predictions of many cumulative damage rules.

Additional test series

Bennebach (1993) investigated the behaviour of this spheroidal graphite cast iron under Plane Bending loadings with two blocks alternatively applied to the specimen. We carried out the same type of tests in Plane Bending and in Torsion. The loading spectrums were with blocks of 5,000 cycles. The amplitude of the low blocks was equal to the third stress amplitude in Fig. 2, the stress amplitude of the high blocks was the fiveth one in Fig. 2. Each one of these test series were done with 10 specimens. The median values of the total lifetimes are : 154,921 cycles in Plane Bending and 183,992 cycles in Torsion. The standard deviations of the decimal logarithm of these lifetimes are respectively 0.073 and 0.052. By taking into account these standard deviations and the number of specimens the statistical hypothesis test of Schwob and Perache (1969) lead us to conclude these two median values are not significantly different.
In Plane Bending with a block duration of 10^4 cycles and with the stress amplitudes of 355 MPa and 325 MPa, Bennebach (1993) found a median lifetime (with 10 specimens) of 175,000 cycles on the same material, with the strictly same specimen geometry. The stress amplitudes used by Bennebach are very closed to those used in our tests but the block duration is twice. Nevertheless, it is always short compared with the total lifetime. The difference between our test results and the data of Bennebach is not significant. It shows that block duration is not an important factor if this duration is an order of magnitude less than the total life under the considered loading.
It has to be noted that these lifetimes are different from the total fatigue life under high-low or low-high loadings. For a loading with a low number of blocks, e.g. if the block duration is not short compared with the total lifetime, an effect of loading history exists. It is the well-known difference between high-low and low-high loadings.

Cumulative damage rules

The predictions of four cumulative damage rules have been compared with experiments under loading spectrums : Miner (1945), Henry (1955), Corten and Dolan (1956), Manson (1967). These rules have been taken from the literature where other rules have been reviewed : the Unified Theory (Dubuc *et al.* 1971), Subramanyan (1976), Lemaitre and Chaboche (Chaudonneret and Chaboche 1986), Papadopoulos (1987). Excepted the Lemaitre and Chaboche rule, all the others can not be applied to predict lifetime under loadings with blocks equal to the endurance limit. It is the case of the tested loading spectrums (Fig. 2). Test of the last proposal of the Lemaitre and Chaboche rule is not presented herein because with the constants identified for the tested cast iron its predictions are very far from experiments.

For each case of loading and each rule, the Relative Error of Prevision, *REP*, defined by expression (2), is given in Table 3. A perfect rule has a *REP* equal to zero and if the *REP* is negative the rule is non conservative, e.g. its use is dangerous in a design department.

$$REP(\%) = \left(\left[(Nmed)_{exp} - N_{pred}\right] / \left[(Nmed)_{exp}\right]\right) \times 100 \quad (2)$$

$(Nmed)_{exp}$: median value of N from experiments N_{pred} : prediction of the rule

Table 3 shows that all rules are generally non conservative for the tested loading spectrums. Under loading spectrums with only one type of solicitation previsions are not in good agreement with experiments excepted for the spectrum number 2 in Torsion. In multiaxial loading spectrums predictions are in better agreement with experiments than in uniaxial loadings. It seems that the more complicated the loading spectrum is, the better the predictions are. It can be noted that the Miner rule is the simplest rule and its predictions are not really badder than the others. Nevertheless, the Henry rule is the best one of the four tested rules.

Table 3 : Relative Error of Prevision (%) of each rule for each loading spectrum.

	Spectrum number	Miner	Henry	Corten Dolan	Manson
Torsion	1	-22.7	-16.0	-12.8	-16.5
Torsion	2	-6.9	3.3	1.9	-2.2
Torsion	3	-25.3	-5.9	-6.9	-22.3
Plane Bending	1	-37.6	-34.1	-36.2	-30.7
Plane Bending	2	-20.2	-11.3	-20.1	-15.2
Plane Bending	3	-32.2	-16.8	-30.0	-29.0
Pl. Bend. + To. :					
0°, S/T = 1.732	3	-33.8	-11.8	-42.8	-30.5
90°, S/T = 1.732	3	-8.5	12.6	3.9	-5.9
0°, S/T = 1.35	3	-9.1	4.8	-6.7	-6.4

Palin-luc *et al.* (1996) carried out cumulative damage tests on smooth specimens. These tests loadings were composed with two sort of blocks in Plane Bending (5000 cycles by block), high and low, alternatively applied to the specimen until failure. The low stress level was equal to the endurance limit. These tests prove that blocks with stress amplitude equal to the endurance limit participate to damage accumulation if there are higher stress levels in the loading to initiate damage. It must be pointed out that all these rules does not take into account the blocks with stress amplitude equal to the endurance limit. In all the tested spectrum loadings the lower stress level is equal to the endurance limit (Fig. 2). This is a possible explanation for the unconservative predictions of the cumulative damage rules.

CONCLUSION

The distribution of the fatigue life under spectrum loadings varying in equi probable blocks is log-normal in uniaxial and multiaxial solicitations. In a spectrum loading in blocks with different stress amplitudes, the order of blocks does not seem to be a significant factor if there are many short blocks (an order of magnitude less than the total lifetime) and many equi probable stress levels. This conclusion has to be confirmed by many other tests on the tested material and on other materials to increase its reliability. The previsions of the tested cumulative damage rules are generally unconservative, but the more complex the loading is the better their predictions are. Henry's rule gives the best predictions. But Miner's rule is the best arrangement between an easy rule to apply and a good quality of prediction. The life prediction of industrial structures needs some understanding of the physical damage mechanisms in multiaxial random loading and the effect of many external factors. As design departments need to caculate the reliability of structures, a probabilistic cumulative damage rule has to be used. Works have to be done in these two ways.

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