APPLICATION OF THE WEAKEST LINK MODEL TO THE FATIGUE LIMIT OF A HARD BAINITIC STEEL

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ABSTRACT

The influence of notches and loading conditions on the fatigue limit of a bearing steel (SAE 52 100, DIN 100 Cr 6) in a bainitic condition is presented. Since the material behaves elastically at the fatigue limit the influence of notches can be completely described by size effect and multiaxality.

A method based on the weakest link model and the Dang Van criterion for high cycle multiaxial fatigue is shown to predict the fatigue limit of bodies of any geometry. The basic material data of this method are the fatigue limits under torsion and under rotating bending of smooth specimens of similar geometry. Additionally, their fracture probability distribution has to be known. From the geometry of the body and the introduced fatigue load a stress integral is calculated by means of an elastic finite element analysis. With these data the fatigue limits under different zero mean stress proportional loading conditions can be calculated. The predicted fatigue limits of the examined specimens and loading conditions are in good agreement with their measured values. For smooth specimens under push-pull and under rotating-bending the fracture distribution can be predicted by the weakest link theory from the distribution of the fatigue crack initiating inclusions using Murakami's relation between fatigue limit and inclusion size.

KEYWORDS

Fatigue limit, notch, size effect, weakest link model, multiaxiality, Dang Van criterion, finite element analysis, stress integral

INTRODUCTION

Fatigue-limit design of hard steels has still uncertainties concerning the transferability of specimen data to component behaviour. The aim of the present investigation is to apply a transfer concept to a high strength steel, which enables the user to predict the fatigue limit of notched parts under different loadings from the measured fatigue limits of smooth specimens.

For a hard steel which shows no macroscopic plasticity at the fatigue limit, it can be assumed that the notch effect can be reduced to a statistical size effect which can be described by the weakest link model after Böhm and Heckel (1982). A basic quantity of this model is the stress integral which describes the highly stressed volume. Since the stress integral depends on the field of a local stress parameter, an equivalent stress has to be defined for multiaxial cases. A first attempt has been made using the von-Mises criterion for describing the equivalent stress (Bomas *et al.*, 1995). The prediction of the fatigue limit with the von Mises criterion was successful for smooth specimens and notched specimens under torsion. The fatigue limit was overestimated for notched specimens with biaxial hydrostatic notch root stresses. In this paper the Dang Van criterion will be applied, which takes account of the damaging effect of hydrostatic stresses (Dang Van *et al.*, 1989).

WEAKEST LINK MODEL

The weakest link model was developed by Weibull (1959) for the strength of brittle materials and later transfered to fatigue behaviour by other authors. It is based on the premise, that fracture is caused by material or surface imperfections which are distributed in the stressed volume. The fracture is caused by the weakest link, which is the largest imperfection. Thus, the fracture probability near the fatigue limit is not only a function of the load level but also of the size of the stressed volume.

Concerning the size of the largest imperfection the following is assumed: The probability F that this size is smaller than a value a in a given volume obeys a two-parametric Weibull distribution (Scholz, 1988):

$$F = e^{-\left(\frac{a}{a_0}\right)^{-c}} \tag{1}$$

Furthermore it is assumed that there exists a fracture mechanical correlation between the size a of the largest imperfection and the fatigue limit:

$$\sigma_{FL} = b\, a^{-z} \tag{2}$$

Murakami (1989) found that if the fatigue fracture initiates at subsurface inclusions in a hard steel the fatigue limit in N/mm² can be calculated from the inclusion size a in µm and the Vickers hardness of the material:

$$\sigma_{FL} = 1.56\,(HV + 120)\, a^{-1/6} \tag{3}$$

Combining Murakami's relation with equation 2 gives the following values for the constants b and z:

$$b = 1.56\,(HV + 120)\ \mathrm{MPa{\cdot}\mu m^{1/6}} \qquad z = 1/6 \tag{4}$$

The key value of the weakest link model according to Böhm and Heckel (1982) is the so-called stress-integral I which can be interpretated as the highly stressed volume:

$$I = \int_V \left[\frac{\sigma(x,y,z)}{\sigma_{max}}\right]^m dV \tag{5}$$

In the fatigue limit region the fracture probability in this model obeys a Weibull distribution and depends on the maximum local stress and the stress integral I:

$$P = 1 - e^{-\frac{I}{I_0}\left(\frac{\sigma_{max}}{\sigma_0}\right)^m} \tag{6}$$

Assuming that the fracture probability in equation (6) results from the size distribution of imperfections as described in the equations (3) and (4), the following expressions for σ_0 and m are valid:

$$\sigma_0 = b\, a_0^{-z} \qquad m = c/z \tag{7}$$

For a given fracture probability it follows from equation (6) that the expression $I\,\sigma_{max}^{\ m}$ is constant. If the maximum local stress σ_{FL0} at the fatigue limit of a reference specimen with the stress integral I_0 is known, the maximum local stress σ_{FL} at the fatigue limit of any body with the stress integral I can be calculated from

$$\sigma_{FL} = \sigma_{FL0} \cdot \left(\frac{I_0}{I}\right)^{\frac{1}{m}} \tag{8}$$

If the reference specimen is smooth and the body under consideration is notched and has the stress integral I_n, the dependence of the fatigue notch factor K_f from the stress concentration factor K_t can be expressed by the simple relation

$$K_f = K_t \cdot \left(\frac{I_n}{I_0}\right)^{\frac{1}{m}} \tag{9}$$

DANG VAN CRITERION

The Dang Van criterion is a high cycle fatigue criterion which can be applied to any multiaxial stress-time history. If it is applied to proportional loading conditions as in the present context, it can be described by two stress parameters (Flavenot and Skalli, 1989): τ_a, the maximum shear stress amplitude, and the maximum hydrostatic stress p_{max} during one cycle which depends on the principal stresses:

$$p_{max} = 1/3\ \max(\sigma_I + \sigma_{II} + \sigma_{III}) \tag{10}$$

The Dang Van criterion describes the regions of failure and survival in the τ_a-p_{max} plane which are separated from each other by the following straight line:

$$\tau_a + \alpha\, p_{max} = \beta \tag{11}$$

The parameters α and β can be determined by the measurement of the fatigue limits under two loading conditions. In order to have the same highly stressed volumes in these basic tests, it is recommended to measure the fatigue limits of similar specimens under torsion τ_{FL} and under rotating bending σ_{FL}^{RB}. With these experimental values the parameters α and β work out to be

$$\alpha = 3 \cdot \left(\frac{\tau_{FL}}{\sigma_{FL}^{RB}} - \frac{1}{2}\right) \tag{12}$$

$$\beta = \tau_{FL} \tag{13}$$

The equivalent stress σ_{DV} after Dang Van depends on the ratio between the fatigue limits under torsion and under rotating-bending:

$$\sigma_{DV} = \sigma_I + \left(1 - \frac{\sigma_{FL}^{RB}}{2 \cdot \tau_{FL}}\right) \cdot \sigma_{II} + \left(1 - \frac{\sigma_{FL}^{RB}}{\tau_{FL}}\right) \cdot \sigma_{III} \tag{14}$$

EXPERIMENTAL RESULTS

The experimental investigations were carried out on the bearing steel SAE 52 100 (DIN: 100 Cr 6) in a high purity bainitic condition with a hardness of 745 HV. Smooth and notched specimens (Table 1) were manufactured with a net diameter of 6 mm. The notch radius was either 1 mm or 0.2 mm. Due to the final grinding process, compressive residual stresses of about -400 N/mm^2 were induced in a surface region of about 20 µm thickness.

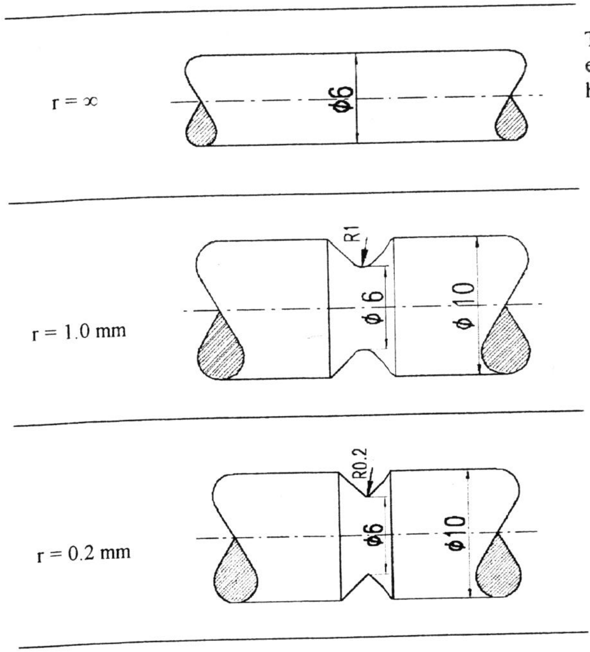

Table 1: Geometries of the examined specimens in the highly stressed regions

The cyclic stress-strain curve was measured on smooth specimens under push-pull loading. It obeys the Ramberg and Osgood relation (1943) with k' = 5 722 N/mm² and n' = 0.13. From these parameters it can be seen that at the fatigue limits described below there is only very small plastic deformation, even in the sharply notched specimens.

The specimens described in Table 1 were used to determine the fatigue limits under push-pull, rotating-bending and torsion loading with a stress ratio of R = -1. The load varied sinusoidally with frequencies from 25 Hz to 100 Hz depending on the test machine used. The fatigue tests were carried out up to $5 \cdot 10^6$ cycles.

The smooth specimens under rotating-bending and push-pull loading failed by crack initiation from Ca-Al-O inclusions (Fig. 1). All other specimens failed by surface crack initiation.

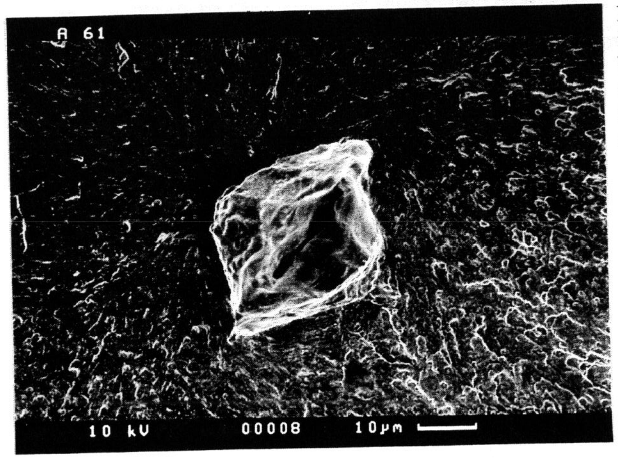

Fig. 1: Fatigue crack initiation at the interface between inclusion and matrix under push-pull loading, SEM-micrograph of the fracture surface

Since the fatigue failure at inclusions can be assumed to occur at the largest inclusion in the highly stressed volume, these inclusions can be analysed in order to get their size distribution according to equation (1). Fig. 2 shows the experimental values and the fitted curve with the parameters $a_0 = 23$ µm and $c = 2.92$. With these values and Murakami's relation, the parameters σ_0 and m of the fracture probability (equation 6) can be calculated (equations 7): $\sigma_0^{cal} = 799$ N/mm², $m^{cal} = 18$.

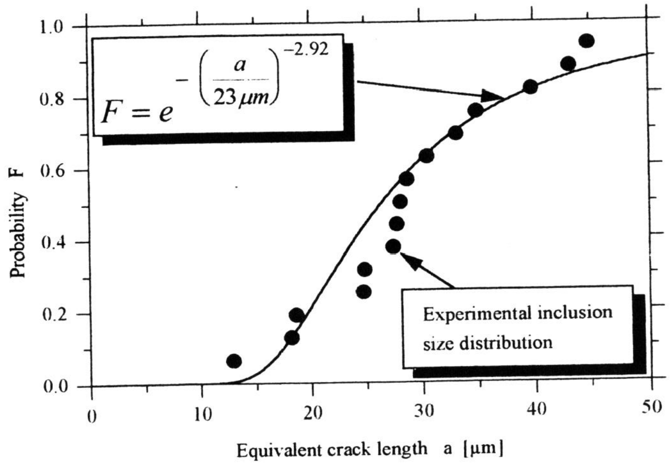

Fig. 2: Distribution of the largest inclusions in the stressed volume of the smooth push-pull specimens according to equation 1

A comparison between the calculated and measured fracture probabilities (Fig. 3) shows a good prediction of the Weibull shape parameter m, indicating the validity of the assumptions between inclusion size and fracture stress distribution. The parameter σ_0 is slightly underestimated.

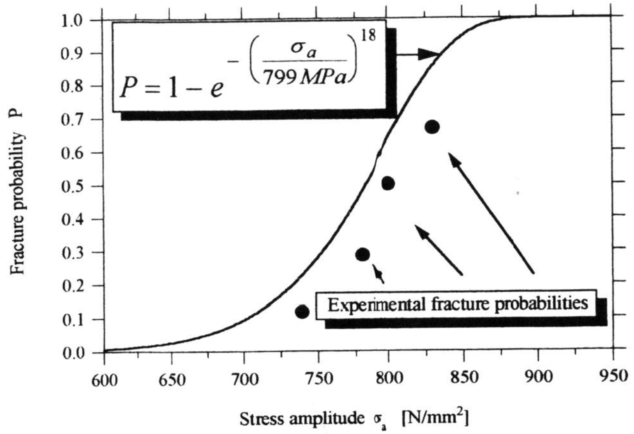

Fig. 3: Calculated and measured fracture probabilities of the smooth specimens under push-pull loading, calculation from inclusion size distribution (Fig. 2) after equation 6

The smooth specimens under rotating-bending and torsion were used to quantify the Dang Van criterion. Their fatigue limits are σ_{FL}^{RB} = 967 N/mm² and τ_{FL} = 576 N/mm². These values give two points on the straight line described by equation (11) from which the Dang Van diagram for the examined steel can be established (Fig. 4).

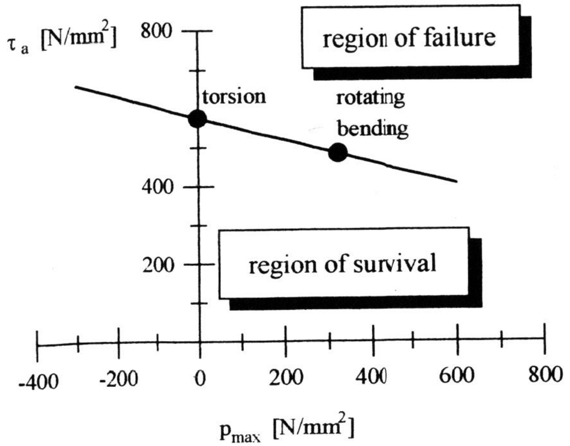

Fig. 4: Dang Van diagram of the examined steel

The local stresses in the specimens were determined by a finite element program. The stress integrals were calculated by replacing the stresses in equation (5) by the Dang Van equivalent stresses (equation 14).

The smooth specimens under rotating-bending and torsion were also taken as reference specimens for the weakest link model. Their mean stress integral and Weibull shape parameter are I_0 = 16 mm³ and m = 21. Equation (8) was used to calculate the equivalent local stresses

σ_{FL}^{DV} in the notch root at the fatigue limit of the specimens. Fig. 5 shows the experimental local equivalent stress amplitudes at the fatigue limit together with their 10 and 90 % confidence intervals and the values calculated after equation (8) (straight line) versus the stress integrals of the specimens. The agreement of the experimental fatigue limits with the calculated ones is very good. At the highest local stress amplitude a plastic strain amplitude of $9*10^{-6}$ can be calculated from the cyclic stress-strain curve of the smooth push-pull specimen. This supports the assumption that the macroscopic behaviour of the material is elastic.

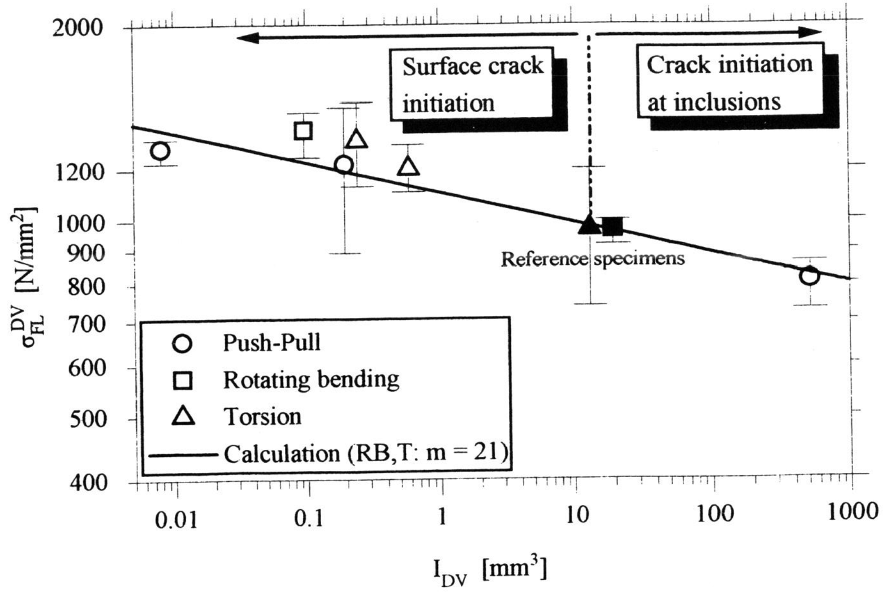

Fig. 5: Equivalent local stress amplitudes (Dang Van) at the fatigue limit as a function of stress-integral with 10 and 90 % fracture probability interval

SUMMARY

In this work, the influence of typical component characteristics like the size of the stressed volume, stress gradients and proportional multiaxiality caused by notches and loading conditions on the fatigue limit of a high purity high carbon steel (SAE 52 100, DIN 100 Cr 6) in a bainitic condition is studied. Near the fatigue limit of this steel no local plasticity is observed. It is possible to predict the influence of notches and loading on the fatigue limit of this steel with a statistical size effect model based on the weakest link theory and the Dang Van criterion. For the smooth specimens under push-pull and under rotating-bending the fracture probability can be predicted by the weakest link theory from the size distribution of the fatigue crack initiating inclusions using Murakami's relation between fatigue limit and inclusion size.

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