A SIMPLE MODEL TO PREDICT FATIGUE STRENGTH WITH OUT-OF-PHASE TENSION-BENDING AND TORSION STRESS CONDITION.

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ABSTRACT

Most biaxial fatigue research has been conducted under proportional in-phase loading. In service, many aircraft structures are subjected to cyclic biaxial out-of-phase stress conditions. However, very little experimental work has been undertaken to establish the effects of out-of-phase loading on fatigue properties of materials and components. The aim of this paper is to present a simple high-fatigue criterion suitable for multiaxial out-of-phase stress loading. Estimations are compared with experimental results carried out on tension/bending-torsion specimens. Analysis of the results shows a highly satisfactory correlation between predictions and experimental data.

KEYWORDS

Fatigue strength, multiaxial criterion, out-of-phase loading, aircraft structures.

INTRODUCTION

A vast number of high fatigue-criteria are given in the documentation available. A criterion providing a general behaviour model must be consistent with the tendencies observed through simple conventional tests.
It must be independent of the reference system linked to the structure, and be consistent with Haigh diagrams under tension-compression and torsion. Several criteria are in compliance with these conditions, e.g. those of Sines (1981), Crossland (1956) or Dang Van (1973). These criteria give good estimations in the case of in-phase loading (Papadopoulos, 1987). However, experience demonstrates that they can not process out-of-phase loading. It is possible to formulate this loading as follows :

$$\sigma_{ij} = \sigma_{ij_{moy}} + \sigma_{ij_{alt}} \cdot \sin(\omega t - \alpha_{ij})$$

where : σ_{ij} component i,j of stress tensor,
$\sigma_{ij_{moy}}$ mean value of σ_{ij},
$\sigma_{ij_{alt}}$ maximum half-amplitude of σ_{ij}, $\sigma_{ij_{alt}} > 0$,
α_{ij} phase difference between the stresses σ_{ij},
ω frequency of loading

Phase difference between the stresses considerably reduces fatigue strength. Thus, predictions are inclined to be over-optimistic and non conservative.

The purpose of this paper is to present a criterion which could correctly integrate the effect of out-of-phase loading. It is derived from Crossland formula : results are identical in the case of in-phase loading.

INITIAL HIGH-FATIGUE CRITERION

The initial formula proposed by Crossland is expressed as a linear combination of the equivalent shear stress amplitude and the maximum hydrostatic stress reached during the cycle:

$$T_{eqa} + B_N.P_{max} \leq A_N$$

where:
A_N and B_N are positive constants defined for fatigue life N
T_{eqa} the equivalent shear stress amplitude,
P_{max} maximum hydrostatic stress.

Failure occurs when ($T_{eqa}+B_N.P_{max}$) equals A_N.

Definition of P_{max} and T_{eqa}

Hydrostatic stress p(t) equals a third of the trace of the stress tensor $\Sigma(t)$.

Expressed within the principal reference system, $\Sigma(t)$ is expressed on a single point of the structure:

$$\Sigma(t) = \begin{bmatrix} \sigma_I(t) & 0 & 0 \\ 0 & \sigma_{II}(t) & 0 \\ 0 & 0 & \sigma_{III}(t) \end{bmatrix}$$

An iteration over time is used to define the instant in time corresponding to maximum hydrostatic stress:

$$P_{max} = \frac{1}{3} \max_{(t)}[\mathrm{trace}\{\Sigma(t)\}] = \frac{1}{3} \max_{(t)}[\sigma_I(t)+\sigma_{II}(t)+\sigma_{III}(t)]$$

For any periodic load, the point representing the stress tensor $\Sigma(t)$ describes a closed curve (C_Σ) which represents a load trajectory.

For radial (or proportional) loads, the load trajectory is a line segment passing through the origin. The principal axes of the stress tensor $\Sigma(t)$ are fixed during the cycle.

In this case, the equivalent shear stress amplitude for a point on the structure is expressed as:

$T_{eqa} = \sqrt{J2_a}$ where:

$$J2_a = \frac{1}{6}[(\sigma_{Ia}-\sigma_{IIa})^2+(\sigma_{Ia}-\sigma_{IIIa})^2+(\sigma_{IIa}-\sigma_{IIIa})^2]$$

$$\sigma_{Ia} = 1/2.(\sigma_{Imax}-\sigma_{Imin})$$

$$\sigma_{IIa} = 1/2.(\sigma_{IImax}-\sigma_{IImin})$$

$$\sigma_{IIIa} = 1/2.(\sigma_{IIImax}-\sigma_{IIImin})$$

$\sigma_{Ia}, \sigma_{Ia}, \sigma_{Ia}$: amplitude of principal stress $\sigma_I(t)$, $\sigma_{II}(t)$ and $\sigma_{III}(t)$ respectively.

In the most general case of periodical loads, the principal stress axes vary over time, as the load is not proportional. T_{eqa} is homogeneous at a distance in the hyperplane of the deviatoric tensor.

The projection of the load trajectory (C_Σ) onto the hyperplane of the deviatoric tensor is a closed curve (C_S) :

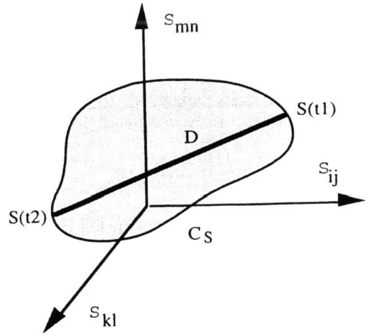

Figure 1. Load trajectory (C_S).

T_{eqa} is then expressed as: $$\boxed{T_{eqa} = \frac{1}{2}\frac{D}{\sqrt{2}}}$$

D is the length of the biggest segment intercepting (C_S). It is calculated as follows:

$$D = \max_{(t1,t2)} \sqrt{\mathrm{trace}([S(t1)-S(t2)].[S(t1)-S(t2)])}$$

where: $S(t) = \Sigma(t)-p(t).Id$ $\quad Id = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

MODIFICATION OF THE FATIGUE CRITERION

Considering an out-of-phase tension-bending and torsion stress load, stress tensor $\Sigma(t)$ is formulated as follows :

$$\Sigma(t) = \begin{bmatrix} \sigma_{11}(t) & \sigma_{12}(t) & 0 \\ \sigma_{12}(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where : $\sigma_{11}(t) = \sigma_{11moy} + \sigma_{11alt} \cdot \sin(\omega t)$

$\sigma_{12}(t) = \sigma_{12moy} + \sigma_{12alt} \cdot \sin(\omega t - \alpha)$

In the stress space, point M representing stress tensor $\Sigma(t)$ is revealed as a closed curve which is an ellipse. The projection of this ellipse in the deviatoric plane also results in an ellipse of long segment D/2 and short segment d/2.

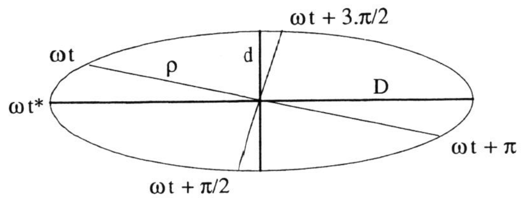

Figure 2. Projection of the load trajectory in the deviatoric plane

D et d are calculated with the following forms : $D = \max_{(t)} \rho(\omega t)$

$d = \min_{(t)} \rho(\omega t)$

with $\rho(\omega t) = \sqrt{\mathrm{trace}\left([S(t)-S(t+\pi)].[S(t)-S(t+\pi)]\right)}$

The deviatoric tensor is defined by the following relation : $S(t) = \Sigma(t) - p.Id$

with $p = \dfrac{\sigma_{11}(t)}{3}$

$$S(t) = \begin{bmatrix} S_{11}(t) & S_{12}(t) & 0 \\ S_{12}(t) & S_{22}(t) & 0 \\ 0 & 0 & S_{22}(t) \end{bmatrix}$$

where : $S_{11}(t) = \frac{2}{3}\sigma_{11moy} + \frac{2}{3}\sigma_{11alt} \cdot \sin(\omega t)$

$S_{22}(t) = S_{33}(t) = -\frac{1}{2} S_{11}(t)$

$S_{12}(t) = \sigma_{12moy} + \sigma_{12alt} \cdot \sin(\omega t - \alpha)$

$$\rho(\omega t) = \sqrt{4.[(2/3.\sigma_{11alt})^2.\sin^2(\omega t) + 2.\sigma_{11alt}{}^2.\sin^2(\omega t-\alpha) + 2.(1/3.\sigma_{11alt})^2.\sin^2(\omega t)]}$$

The maximum or the minimum of $\rho(\omega t)$ is obtained expressing the relation :

$$\frac{d}{dt}[\rho(\omega t)] = 0$$

$$\sin(\omega t^*).\cos(\omega t^*).[(2/3.\sigma_{11alt})^2.+2.(1/3.\sigma_{11alt})^2]+2.\sigma_{11alt}{}^2.\sin(\omega t^*-\alpha).\cos(\omega t^*-\alpha)=0$$

$$\omega t^* = \frac{1}{2}\arctan\left[\frac{2.\sigma_{12alt}{}^2.\sin(2.\alpha)}{2/3.\sigma_{11alt}{}^2+2.\sigma_{12alt}{}^2.\cos(2.\alpha)}\right]$$

Long segment D/2 is equal to the maximum of the two terms $\dfrac{\rho(\omega t^*)}{2}$ and $\dfrac{\rho(\omega t^*+\pi/2)}{2}$, short segment d/2 corresponds to the minimum.

In Crossland's original formula, only D is used in the calculation of the equivalent shear stress amplitude.

In order to take into account the totality of the phase difference (characterized by D and d), it is judicious to replace D by the half-perimeter of the ellipse : $p_e/2$.

T_{eqa} is therefore formulated : $T_{eqa} = \dfrac{1}{2}\dfrac{p_e/2}{\sqrt{2}}$

where : $\dfrac{p_e}{2} \approx \dfrac{\pi}{2}\dfrac{D+d}{2}\left[1+\frac{1}{4}\lambda^2+\frac{1}{64}\lambda^4+\frac{1}{256}\lambda^6\right]$ and $\lambda = \dfrac{D-d}{D+d}$

In the case of in-phase loading, $p_e/2$ is equal to D.

For out-of-phase tension-bending and torsion stress load, maximum hydrostatic stress becomes : $P_{max} = \dfrac{1}{3}(\sigma_{11moy}+\sigma_{11alt})$

DEFINITION OF CONSTANTS A_N AND B_N

The two constants can be defined by means of two simple uniaxial tests. The tests selected are generally an alternating tension test (mean stress equal to zero) on an unnotched test specimen and an alternating torsion test on a thin tube.

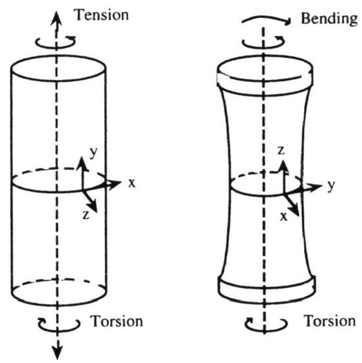

Figure 3. Test specimens.

By applying Crossland's formula to both these tests, the result is :

- for the alternating tension test:

$$\frac{\sigma_{Ia}}{\sqrt{3}} \leq A_N - B_N.\frac{(\sigma_{Imoy}+\sigma_{Ia})}{3}$$

$$\sigma_{Ia} \leq \frac{\sqrt{3}}{1+\frac{1}{\sqrt{3}}.B_N}(A_N - \frac{B_N}{3}.\sigma_{Imoy})$$

On failure: $\sigma_{-1}(N) = \frac{\sqrt{3}}{1+\frac{1}{\sqrt{3}}.B_N} A_N$

$\sigma_{-1}(N)$ is the maximum stress corresponding to failure at N cycles for the alternating tension test.

- for the alternating torsion test : $\sigma_{12a} \leq A_N$

On failure: $\tau_{-1}(N) = A_N$

$\tau_{-1}(N)$ is the maximum stress corresponding to failure at N cycles for the alternating torsion test. $\tau_{-1}(N)$ is independent of a mean torsion value (in conformity with experimental observations).

hence: $A_N = \tau_{-1}(N)$ and $B_N = 3.(\frac{\tau_{-1}(N)}{\sigma_{-1}(N)} - \frac{1}{\sqrt{3}})$

Positive B_N implies that $\frac{\sigma_{-1}(N)}{\tau_{-1}(N)} < \sqrt{3}$. This is verified for the materials used in aeronautics.

Crossland's formula is finally expressed as:

$$T_{eqa} + 3.(\frac{\tau_{-1}(N)}{\sigma_{-1}(N)} - \frac{1}{\sqrt{3}}).P_{max} \leq \tau_{-1}(N)$$

APPLICATIONS

The experimental data presented in this paper originate from scientific literature. These data concern bending-torsion tests (Froustey and Lasserre, 1989) and tension-torsion tests (Mielke, 1980).

For the results of each test, the calculation of the K ratio ($K = (T_{eqa}+B_N.P_{max}) / A_N$) enable the quality of the predictions to be appreciate. If K is equal to 1, the prediction is perfect. If it is higher, the prediction is conservative.

Error is quantified as follows : I=(K-1)*100.

Table 1. Test results and predictions with out-of-phase tension-torsion stress condition.

Material : CK45, Out-of-phase Tension-Torsion, Life to failure : 10^5 cycles τ_{-1}=287 MPa, σ_{-1}=423 MPa, σ_0=712 MPa								
σ_{11moy} (MPa)	σ_{11alt} (MPa)	σ_{12moy} (MPa)	σ_{12alt} (MPa)	α	Initial criterion K	Initial criterion I%	Modified criterion K	Modified criterion I%
250	250	0	144	90°	0,68	-32	0,97	-3
0	288	165	165	90°	0,68	-32	1,01	1
0	292	0	167	60°	0,76	-24	0,95	-5
0	304	0	174	90°	0,72	-28	1,06	6

The error never exceeds the value ±10% and for most of the data this value is less than ±5%. The calculation method provides excellent correlation with test results.

Table 2. Test results and predictions with out-of-phase bending-torsion stress condition.

Material : 30NCD16, Out-of-phase Bending-Torsion, Life to failure : 10^6 cycles τ_{-1}=415 MPa, σ_{-1}=695 MPa, σ_0=1040 MPa								
σ_{11moy} (MPa)	σ_{11alt} (MPa)	σ_{12moy} (MPa)	σ_{12alt} (MPa)	α	Initial criterion K	Initial criterion I%	Modified criterion K	Modified criterion I%
0	480	0	277	90°	0,69	-31	1,07	7
300	222	0	385	90°	0,95	-5	1,06	6
300	480	0	277	45°	0,85	-15	1	0
300	470	0	271	60°	0,77	-33	0,99	-1
300	473	0	273	90°	0,69	-31	1,07	7
300	565	0	141	45°	0,86	-24	0,93	-7
300	540	0	135	90°	0,79	-21	0,92	-8
300	465	200	269	90°	0,68	-32	1,05	5
450	405	0	234	90°	0,60	-40	0,93	-7
600	390	0	225	90°	0,59	-61	0,90	-10

CONCLUSION

A simple high-cycle fatigue criterion has been presented in this work. It is suitable for in-phase and out-of-phase conditions. Implementation of this criterion is extremely simple and requires no special numeric calculations.

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