A HYPOTHESIS OF CUMULATIVE DAMAGE FOR FATIGUE CRACK INITIATION AND PROPAGATION

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ABSTRACT

This paper presents of author's investigations on cumulative fatigue damage. A cumulative damage theory is presented for multilevel cyclic loading. Both the crack initiation and propagation stages of the fatigue failure process are included. The damage accumulated at any stage is evaluated from a knowledge of the fatigue limit in the initiation phase and a reduced limit obtained through fracture mechanics for the propagation phase. The proposed cumulative damage hypothesis is compared with two-, three-level strain cycle data of thin-walled specimens, and is found to be in fairly good agreement.

KEYWORDS

Fatigue, fracture mechanics, cumulative fatigue damage, crack initiation, crack propagation.

INTRODUCTION

During the cyclic loading material is subjected to the fatigue damage process the seriousness of which is related to some damage parameters and the loading history.
The fatigue life, N_f, of a material can be subdivided into two parts:

$$N_f = N_i + N_p \qquad (1)$$

where N_i is the number of cycles to initiate a fatigue crack, and N_p is the number of cycles for an initiationed crack to propagate to final fracture or a critical size.
Many investigations of fatigue crack initiation and propagation have been carried out and some cumulative damage rules including this two stages have been reported.

One of the first approaches is a double linear damage rule, based on the assumption that cumulative fatigue damage in each of this stages can be expressed by linear summation (Manson et al.,1965,1981):

$$\sum_{i=1}^{l} \frac{n_i}{(N_i)_i} = 1 \tag{2}$$

and

$$\sum_{j=1}^{m} \frac{n_j}{(N_p)_j} = 1 \tag{3}$$

where in each case n is the number of cycles applied at the ith or jth load level. According to this theory, the crack propagation period could be expressed as

$$N_p = P(N_f)^p \tag{4}$$

where N_p is the number of cycles to propagate a crack to failure after it has been initiated. P and p are the propagation coefficient and the propagation exponent respectively, both of which are to be determined experimentally.
Miller and Zachariah (1977) proposed to describe the accumulation of damage by a double exponential law. The number of cycles required to initiate a crack was expressed as

$$N_i = N_f \left(\frac{x + y - 1}{y}\right) \tag{5}$$

where x is the fraction of N spent at the initiation strain level ($=n_1/N_{f1}$) and y is the remaining fraction of total life at the second strain level ($=n_2/N_{f2}$).
The assumption used in derivating the above equation imply that the crack length at the end of initiate phase is the same in lower and higher stress levels. Experimental observations suggested that the length of the crack depends on the load level. This approach was recently extended (Ibrahim and Miller,1979).
The mechanisms and kinetics of crack initiation and propagation in the conditions of multilevel cycling, even one as simple at that of two-level test, are difficult and complex. Therefore, in the literature some purely phenomenological hypothesis has been proposed. Recently, Hashin and Rotem (1978) have shown that the phenomenological model of cumulative fatigue damage can be based on families of curves of equivalent damage.The model is defined in terms of residual life and of an equivalent loading postulate. The endurance limit of the material is essential parameter of this hypothesis, and is assumed to be constant for all stages of the fatigue process. In two-level cyclic loading they derived the formula in the form

$$\left(\frac{n_1}{N_{f1}}\right)^{\frac{\log(S_2/S_r)}{\log(S_1/S_r)}} + \frac{n_2}{N_{f2}} = 1 \tag{6}$$

Here S is the non-dimensional stress per cycle, σ/σ_s, where σ_s is interpreted as the cyclic ultimate strength.
Similar approach to this was presented by Subramanyan (1976).
A consequence of the constant fatigue limit, is that the proposed theory cannot take into account the history of loading below the original endurance limit. Experiments have shown that, when the specimen is subjected to cyclic multilevel loading even cycles below fatigue limit produced some damage in the material. As a result has been observed a reduction in S_e.
The amount of accumulated fatigue damage of materials can be associated either with a reduction of the endurance limit or with the size of the propagating fatigue crack. These two ideas are equivalent on the basis of the nature of the nucleation of the critical microcrack size and their subsequent propagation (Fuchs et al., 1980; Lukas et al.,1981).
The aim of this study is to present the a cumulative damage hypothesis, including the initiation and propagation phases, effect of sequence of loading and above mentioned the effect of a reduction of the endurance limit.

A HYPOTHESIS OF FATIGUE FAILURE

The analysis of cumulative fatigue damage consists of two parts:

Description of the fatigue criterion

A number of investigations have been carried out to correlate the fatigue life of a material with either the strain range or stress range (Fuchs et al., 1980). Since the fatigue damage is generally caused by plastic strain, the dissipated plastic strain energy density plays an important role in fatigue phenomena. However, when the number of cycles tends to infinity, the value of plastic strain range as well as plastic strain energy density tends to zero. Therefore in high-cycle fatigue the calculations based only on the plastic strain range or plastic strain energy density may not be accurate.
To unify description of fatigue life in low- and high-cycle fatigue the strain energy density equal to the sum of plastic strain energy density and elastic strain energy density in tensile half-cycle as a damage parameter has been proposed (Golos, 1988; Golos, 1991).
Let the controlling damage parameter will be designated by ϕ.
The specific form of ϕ can as it was discussed earlier stress range, strain range, or deformation energy.

The damage variable in the relation to number of cycles to failure can be expressed through life curve as:

$$\phi = k\,(N_f)^g \tag{7}$$

where k is interpreted as the value of ϕ at which the straight line with the slope g of the life curve intersects the ϕ axis.

Modelling of the cumulative fatigue damage mechanism

The following assumptions are made:

(a) for the crack initiation stage the material has a fatigue limit defined by $\phi_e = C$,

(b) for the crack propagation stage the material has an "reduced fatigue limit" defined by ϕ_r and life time N_r, where N_r is taken as a intersection of the original ϕ-N_f curve with critical damage curve.

(c) the life curve and isodamage curves are assumed to be straight lines on a logϕ vs. logN_f plot and they all converge to the fatigue limit, N_e, for the crack initiation and to N_r for the crack propagation stage.

(d) the transition between crack initiation and propagation stages can be expressed in terms of the critical damage curve which can be associated with French curve.

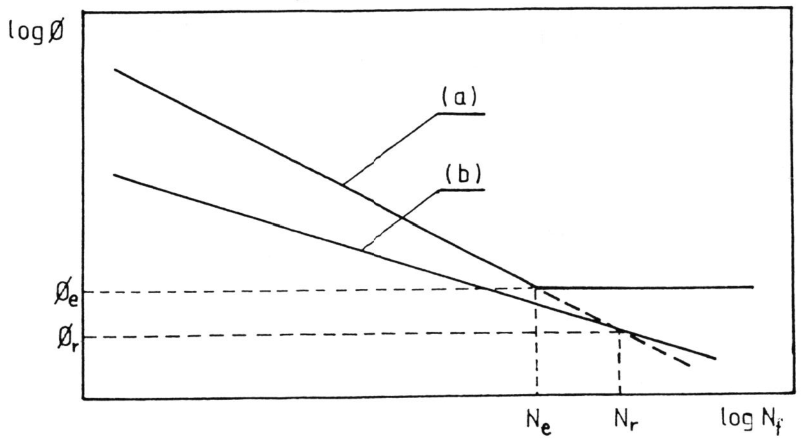

Fig.1. Life curve and critical damage curve.

Now let us consider a specimen which is subjected to two-block cyclic loading. Let damage parameter associated with the first loading block be denoted by ϕ_1 at which is applied n_1 cycles. The damage curve can be expressed as:

$$\frac{\phi_1}{\phi_r} = \left(\frac{n_1}{N_{f1}}\right)^{\zeta} \tag{8}$$

where ζ is a parameter which is constant for a specific damage curve. Changing the applied load and denoting the associated damage parameter ϕ_2 we continue cycling until failure occurs. Application of n_1 cycles at ϕ_1 level will cause same damage in material which can be determined from the damage curve. We could find an equivalent number of cycles n_{12} applied at the level ϕ_2 which would cause the same amount of damage as the first loading block:

$$n_{12} = N_r\left(\frac{n_1}{N_r}\right)^{\frac{\log(\phi_2/\phi_r)}{\log(\phi_1/\phi_r)}} \tag{9}$$

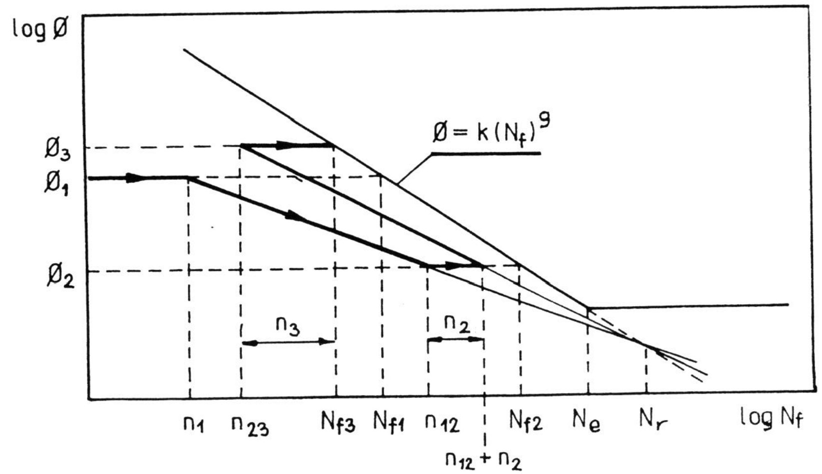

Fig.2 Schematic damage accumulation.

Noting that

$$n_2 = N_{f2} - n_{12} \tag{10}$$

we get the cumulative damage hypothesis for the two-stage loading in the form of

$$\left(\frac{n_1}{N_{f1}}\right)^{\frac{\log(\phi_2/\phi_r)}{\log(\phi_1/\phi_r)}} + \frac{n_2}{N_{f2}} = 1 \tag{11}$$

The method described for two stage loading can be easily generalized to multilevel loading blocks. The extension to the multilevel loading will be demonstrated through a three-level cyclic loading as shown in Fig.2. Let the first block of loading be applied n_1 cycles with the fatigue damage parameter ϕ_1, where N_{f1} is the number of cycles to failure at ϕ_1. Subsequently, the second block with ϕ_2 is applied n_2 cycles with N_{f2} being the number of cycles to failure at ϕ_2. The third block with a controlling damage parameter ϕ_3, is applied until fracture takes place.
Following the procedure for two block loading we can now treat $(n_{12} + n_2)$ at ϕ_2 and n_3 cycles at ϕ_3, as a two stage loading, i.e.,

$$\left(\frac{n_{12} + n_2}{N_{f2}}\right)^{\frac{\log(\phi_3/\phi_r)}{\log(\phi_1/\phi_r)}} + \frac{n_3}{N_{f3}} = 1 \tag{12}$$

Substituting (9) and (11) into (12) we obtain the following damage hypothesis for the three-stage loading,

$$\left(\left(\frac{n_1}{N_{f1}}\right)^{\frac{\log(\phi_2/\phi_r)}{\log(\phi_1/\phi_r)}} + \frac{n_2}{N_{f2}}\right)^{\frac{\log(\phi_3/\phi_r)}{\log(\phi_2/\phi_r)}} + \frac{n_3}{N_{f3}} = 1 \tag{13}$$

We observe from the above that for the increasing value of "reduced" fatigue limit the equation (13) is the same as the linear Palmagren (1924) and Miner rule (1945). Also in the case when the slope of the damage line is constant to that of the life curve we obtained the Palmgren-Miner rule.

COMPARISON WITH TEST DATA

In the analysis the data presented by Miller and Zachariah (1977) for En3A steel have been used. In experiments thin-walled circular specimens were subjected to cyclic torsional straining. Tests have been performed for both low-high and high-low sequences. The corresponding number of cycles to failure at the analyzed stages ranges from $N_f = 720$ to $N_f = 700\ 000$.

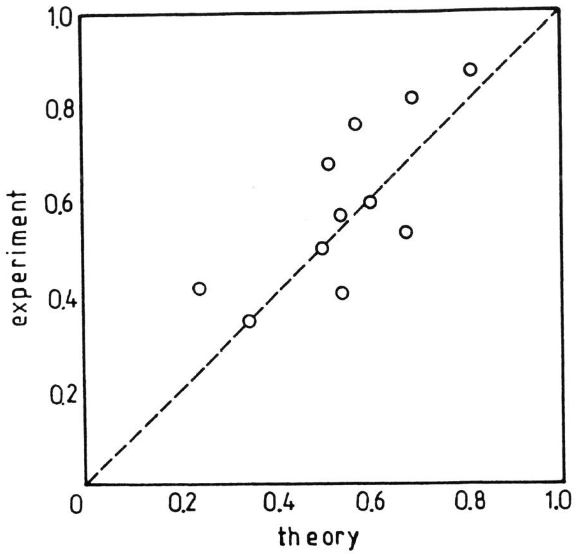

Fig.3. Comparison with test data.

The examples of the obtained results for the two-, and three-stages experiments are shown in Fig.3. In these figure, the ordinate represents the fraction of life spent at the last strain amplitude, predicted by the proposed theory (eqn.(11) or eqn.(13)) and the abscissa is the fraction measured in the experiment.It is seen from the figures, that the investigated theory correctly predicts the trends of experimental results for analyzed strain amplitudes and given sequences.

CONCLUSIONS

A hypothesis to the evaluation of the accumulation of damage for cyclic loading is considered. This concept includes both the crack initiation and propagation stages.The damage accumulated at any stage is evaluated from a knowledge of the fatigue limit in the initiation phase and a reduced limit obtained through fracture mechanics for the propagation phase. The proposed cumulative damage hypothesis is compared with two-, three-level strain cycle data of thin-walled specimens, and is found to be in fairly good agreement.

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