DETERMINATION OF RESIDUAL STRESSES AND THE RESULTING STRESS INTENSITY FACTORS IN THE LIGAMENT OF PRE-CRACKED PLATES

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ABSTRACT

In general, the ligament of cracked components contains residual stresses, resulting either from the crack forming process or from a prior overload. The crack compliance method allows not only to determine their distribution in the ligament, but also offers the possibility to obtain quite easily the stress intensity factor that is produced when the original crack is growing. The latter affect particularly the subsequent sub-critical growth behaviour and, thereby, the safety and residual life of the considered component. Residual stress fields in pre-loaded cracked components exhibit some peculiarities regarding the high level and the local concentration of the residual stresses, which give rise to local plastic yielding in the vicinity of the crack tip that affects the stress measurement. An approximate correction for this effect is suggested. Furthermore, a simple influence function for deeply cracked plates is developed. As demonstrated by examples the crack compliance method is a promising tool to analyse the residual stresses near stress concentrations and their effect on the crack behaviour.

KEY WORDS

Residual stress, crack compliance method, influence function, stress intensity factor, subcritical crack growth.

INTRODUCTION

Stress raisers like cracks or sharp notches cause inhomogeneous plastic deformation in an elastic-plastic solid when it is mechanically loaded. Therefore, after unloading, residual stresses are left in the ligament of cracked components, mainly in the region of the crack tip. Nearly all cracked components, thus, contains residual stresses in their ligament. Either they originate from the crack forming processes, or from previous mechanical loads that were applied in the already cracked state. These residual stresses contribute to the stress intensity factor as the crack extends, and can affect, thus, the subsequent behaviour of the cracked component significantly. A typical example is an overload applied to a fatigued structure, which is well known to cause crack retardation and fatigue life extension. The phenomenon of crack closure of fatigue cracks, which is known to be in general inherently present and to affect significantly the subsequent growth behaviour of the crack, can also be considered as a special type of compressive residual stresses. Actually, when performing a fracture mechanics analysis to predict the mechanical behaviour of a cracked component, the residual stresses should be known and taken into account. The fact that they are usually disregarded might be a reason why theoretical residual life predictions often are in poor agreement with reality.

In the present paper it is shown that the residual stresses in the ligament of cracked plates as well as the stress intensity factors that result from the residual stresses in case of further crack growth can be obtained relatively easily by the crack compliance method (abbreviated in the following as CC-method). However, these systems exhibit some peculiar features such as extremely high magnitudes and high gradients of residual stresses, which can cause additional local yielding when the CC-method is applied. Therefore, the CC-method as developed by Cheng and Finnie (1987, 1994) (see in the latter for further references) and the corresponding procedure for direct determination of the stress intensity factors (SIF) as pointed out by Schindler (1995a) and Schindler et al. (1996) require some special modifications and corrections.

THE PRINCIPLE OF THE CRACK COMPLIANCE METHOD

The basic idea of the crack-compliance-method is to introduce a narrow slit into the considered body by progressive cutting along the plane where the residual stresses are to be measured. Thereby the corresponding residual stresses are released, causing a re-distribution of the residual stress-field in the entire body. The strain change at a suitable location, which can be measured by means of a strain gage, contains information about the released stresses at the cut plane. If the theoretical relation between the released stresses and the strain change at the measurement point is known, then it is possible to determine the former from the latter. Since a narrow slit, in an overall consideration of the system, is nearly equivalent to a perfect crack, and since essentially elastic behaviour (or small scale yielding, resp.) prevail, the basic equations of linear elastic fracture mechanics can be used to establish the required mathematical relations.

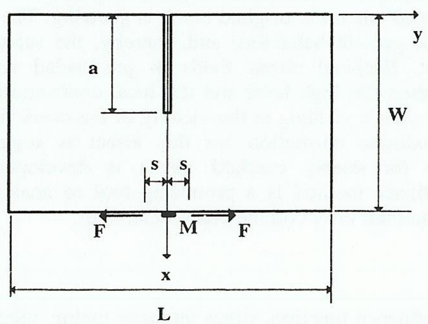


Fig. 1: Principle of the CC-method

In the present case we consider a rectangular plate of length L and width W (Fig. 1). To obtain the normal stresses on the plane y=0 the most suitable strain measurement point is y=0 at the rear surface (point M in Fig. 1), which turned out to cover more than 90% of the cross section. Following the derivation of Cheng and Finnie (1987, 1994) the strain increment $\delta\varepsilon_M$ as measured at M due to an incremental cut prolongation δa can be expressed in terms of the SIF due to the residual stresses, $K_{Irs}(a)$ as

$$\delta\varepsilon_M = \frac{B}{E'F} K_{Irs} \left.\frac{\partial K_{IF}}{\partial s}\right|_{s=0} \delta a \qquad (1)$$

K_{IF} denotes the SIF due to a pair of virtual forces F (Fig. 1) that are introduced in order to calculate the strain by means of Castigliano's theorem (Timoshenko 1970), B the thickness of the component (which should be essentially plane) and E' the generalised Young's modulus,

which means E' = E for plane stress conditions (i.e. thin plate with respect to the distance between the cut tip and M) and $E' = E/(1-\nu^2)$ for plane strain. Replacing $\delta\varepsilon/\delta a$ by the corresponding derivative $d\varepsilon/da$, one readily obtains the following relation between the strain change at M and the actual SIF at the cut tip:

$$K_{Irs}(a) = \frac{E'}{Z(a)} \cdot \frac{d\varepsilon_M}{da} \qquad (2)$$

where

$$Z(a) = \frac{B}{F}\left(\left.\frac{\partial K_{IF}}{\partial s}\right|_{s=0}\right) \qquad (3)$$

According to eq. (2), the SIF of a surface crack in a residual stress field can be experimentally determined by measuring the gradient of strain $d\varepsilon_M/da$ that results from progressive cutting. The function Z(a) defined by (3) and referred to as the "influence function" is a unique function that depends on the component geometry, on the cut plane and on the location of the measurement point M, but not on the residual stress distribution (Schindler, 1995a, Schindler et al., 1996). It characterises the sensitivity of the measurement point M with respect to the stress intensity factor and the stresses on the cut plane: the higher the value of $|Z(a)|$, the more sensitive the measurement. There are analytical as well as numerical methods to determine Z(a), e.g. by using eq. (2) and calculating the required functions $K_I(a)$ and $\varepsilon_M(a)$ numerically by the finite element method for an arbitrary reference load case (Schindler and Landolt 1996).

From the function $K_{Irs}(a)$ as delivered by (2) it is possible to calculate the initial distribution $\sigma_{rs}(x)$ of the residual normal stresses that acted prior to cutting on the cross section y=0 by inversion of the general relation

$$K_{Irs}(a) = \int_0^a h(x,a) \cdot \sigma_{rs}(x) \cdot dx \qquad (4)$$

where h(x,a) denotes the so-called weight function as introduced by Bueckner (1970). Emphasis must be laid on an adequate accuracy of h(x,a), which is essential to obtain accurate $\sigma_{rs}(x)$. In the present investigation, weight functions determined by the estimation procedure as described by Schindler (1994) are used. The inversion of (4) can be achieved by the step-by-step-procedure suggested by Schindler (1995b).

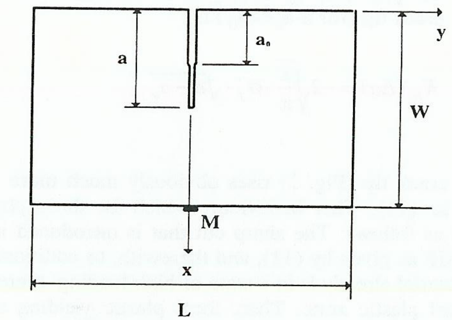
DETERMINATION OF THE INFLUENCE FUNCTION

Consider a rectangular plate or beam of length L and width W that contains an initial crack or sharp notch of a depth a_0 (Fig. 2). To measure the residual stresses in its ligament a cut of progressively increasing length a is introduced. The strain is measured at y=0 on the rear surface. If the initial crack is relatively deep (which means, roughly, about $a_0 > W/3$) and the plate slender (about $L > 3W/2$), then the only relevant geometrical parameter is the ligament width W-a. Thus, from a dimensional analysis of (2) or (3), it follows that Z(a) is of the form

$$Z(a) = \frac{A}{(W-a)^{3/2}} \qquad (5)$$

where A is introduced as a non-dimensional constant, which can be determined analytically by considering the exact solution for a radial cut in a circular disk (Fig. 3). According to Schindler (1995a) and Schindler et al. (1996), the latter is given by

$$Z(a) = -\frac{7.952}{\pi \cdot D^{3/2}} \cdot \sqrt{\frac{a/D}{(1-a/D)^3}} \tag{6}$$



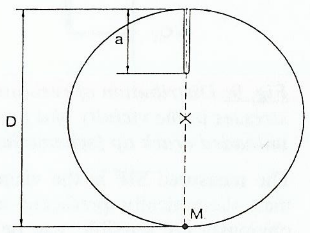


Fig. 2: Cutting and strain measurement in the case of a pre-cracked rectangular plate or beam

Fig. 3: Cutting and measurement in the case of a circular disk

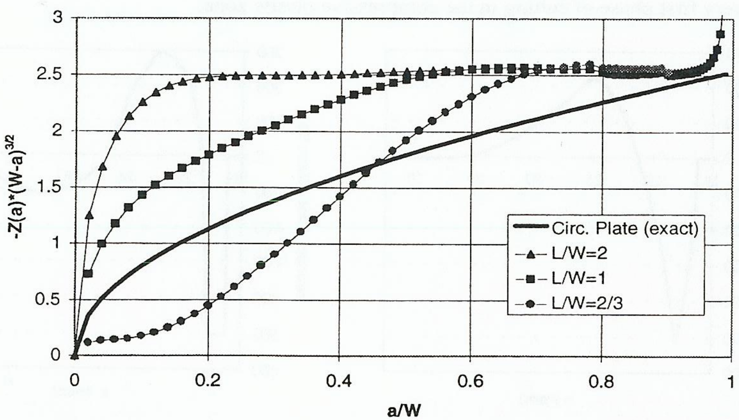


Fig. 4: Numerically determined influence functions for rectangular plates, in comparison with the analytical one for the circular disk with a radial crack (D=W).

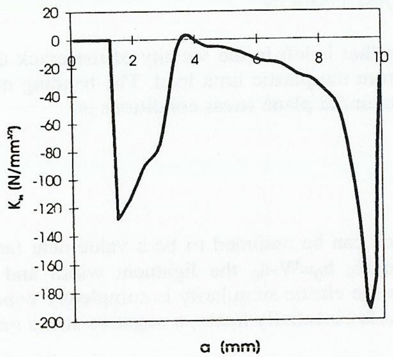
It is physically obvious that (5) and (6) must coincide as a→W and a→D, respectively. From this condition one finds A= 2.532, thus

$$Z(a) = \frac{2.532}{(W-a)^{3/2}} \tag{7}$$

Numerical results obtained by Schindler and Landolt (1996) for rectangular plates (Fig. 4) confirm the validity of the simple influence function (7), indicating that it holds as a good approximation for cut depths in the surprisingly large range 0.2W<a<W and (W-a)<L/2. For less deep cracks or cuts the numerical solutions given by Schindler and Landolt (1996) (see Fig. 4) have to be used, where the one for L=2D holds also for longer plates, i.e. beams.

EXPERIMENTAL RESULTS

To demonstrate the potential of the described method two typical examples are presented in the following. The first one is a Charpy-type specimen (W=10mm, initial notch depth 1.2 mm) made of mild structural steel that had been cyclically loaded to produce a fatigue crack depth of about 2.5 mm. The measured stress intensity factors due to the residual stresses are given as a function of crack length in Fig. 5, and the residual stress distribution calculated therefrom in Fig. 6. Actually the stresses in the range of 1.2mm<x<2.5mm is compression due to crack closure. Unexpectedly, their peak occurs at the notch root rather than at the fatigue crack tip. This result can be explained on one hand by the notch machining and on the other by the history of the cyclic load ΔK_I, which was decreasing from about 550N/mm$^{3/2}$ in the initiation phase to 260N/mm$^{3/2}$ at the end of fatiguing.



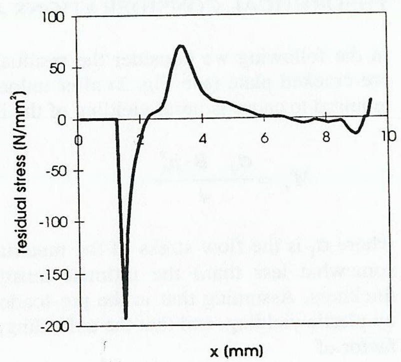
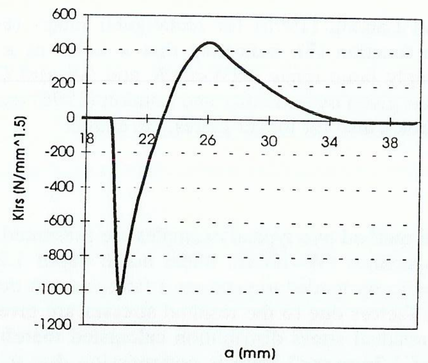


Fig 5: Stress intensity factor in a fatigued Charpy specimen (a_0=1.2mm)

Fig. 6: Residual normal stresses in the crack plane of the fatigued Charpy specimen

As a second example the described method was applied to a precracked plate made of mild steel (yield stress ca. 300 MPa, ultimate strength about 520MPa) of a width W=40 mm that contains a crack of initial length a_0=20mm. Prior to the measurement it was bent such that general yielding occurred, then unloaded. Regarding the magnitude and the gradients of the residual stresses this system and load history is a rather extreme one. Fig. 7 and 8 show the experimental results. Qualitatively, regarding the compressive stress peak adjacent to the crack tip and their general distribution on the ligament, the residual stresses (Fig. 8) correspond to what one expects on theoretical grounds. Quantitatively, however, there are some peculiarities that need further explanation: Instead of the expected plateau-like residual stress distribution in the vicinity of the crack tip (see next section), a sharp compressive peak is exhibited that exceeds the yield stress considerably. This surprising behaviour is analysed and discussed by means of a simple mechanical model in the next section.



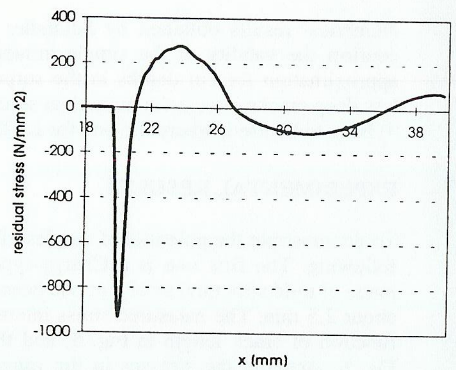


Fig. 7: Stress intensity factor as a function of crack length in the ligament of a pre-loaded cracked beam

Fig. 8: Residual normal stress distribution calculated from Fig. 7

THEORETICAL CONSIDERATIONS AND DISCUSSION

In the following we consider the residual stress that is left in the vicinity of the crack tip of a pre-cracked plate (see Fig. 2) after unloading from the plastic limit load. The bending moment required to cause general yielding of the ligament under plane stress conditions is

$$M_f = \frac{\sigma_f \cdot B \cdot b_0^2}{4} \tag{8}$$

where σ_f is the flow stress of the material, which can be assumed to be a value near (actually somewhat less than) the ultimate tensile strength, $b_0=W-a_0$ the ligament width and B its thickness. Assuming that in the pre-loaded state the elastic singularity is completely wiped out by plastic yielding, and that the unloading process is essentially linear, a negative stress intensity factor of

$$K_I^{unload} = -\frac{4M_f}{B \cdot b_0^{3/2}} = -\sigma_f \cdot \sqrt{b_0} \tag{9}$$

is produced by unloading, where first relation follows from the corresponding SIF solution given by (Tada et al. 1973), the second by inserting (8). By using a Dugdale-type strip yield model (see e.g. Anderson 1991) and assuming that the flow stress in compression is approximately the same as in tension, one finds the length of the compressive plastic zone, l_p^c, to be

$$l_p^c = \frac{\pi}{8} \cdot \left(\frac{K_I^{unload}}{2\sigma_f}\right)^2 = \frac{\pi \cdot b_0}{32} \cong 0.1 \cdot b_0 \tag{10}$$

In this zone, i.e. in the first 10% of the ligament adjacent to the crack tip, the residual stress is expected to reach σ_f, exhibiting a distribution as shown in Fig. 9.

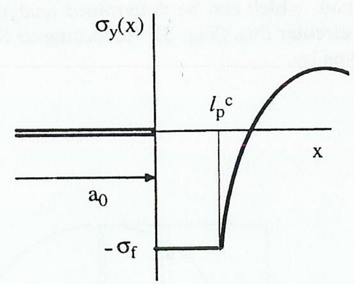
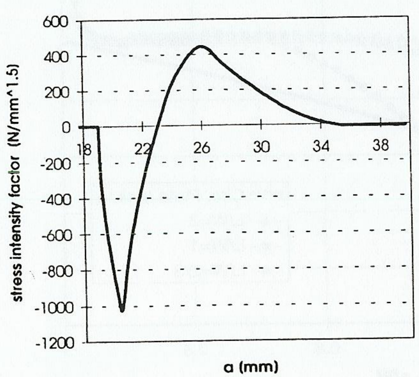


Fig. 9: Distribution of residual stresses in the vicinity of a pre- and unloaded crack tip (schematic)

Consider now the case where a short cut is progressively introduced from the original crack tip into the compressive yield zone. The released residual stresses give rise to an additional stress intensity factor at the cut tip, which can be measured by the strain gage as described above. From the singular term that dominates the weight functions in the vicinity of the crack tip, one readily finds that the SIF produced by cutting in the vicinity of the original crack tip (for $a-a_0<<a_0$) is

$$K_{Irs}(\Delta a) = -2\sqrt{\frac{2}{\pi}} \cdot \sigma_f \cdot \sqrt{a-a_0} \tag{11}$$

The measured SIF in the vicinity of the crack tip (Fig. 7) rises obviously much more steeply than theoretically predicted according to (11), This behaviour, which is, thus, physically obviously not possible, can be explained as follows: The sharp cut that is introduced into the compressive plastic zone gives rise to a SIF as given by (11), and therewith, to additional local plastic yielding. Since the surrounding material already is in a state of high loading, there is - so to say - no reserve left for an additional plastic zone. Thus, local plastic yielding and the corresponding stress rearrangement takes place to a much higher extent than in a less loaded region. Therefore the strain change measured at M is no longer only due to the stresses released directly by cutting, but also to the ones released by the accompanying local plastic yielding. This is the reason why eq. (2) leads to an overestimation of the SIF and of the residual stresses in the very first phase of cutting in the compressive plastic zone.



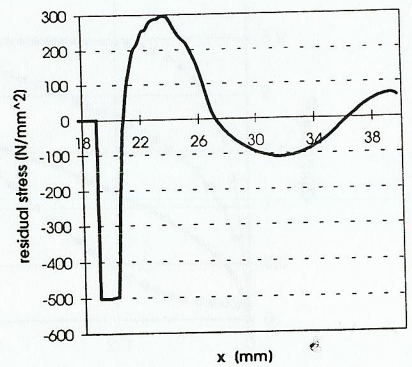


Fig. 10: SIF as a function of crack depth, modified as described in the text

Fig. 11: Residual stress distribution calculated from Fig. 10

To correct approximately for this effect, the calculated $K_{Irs}(a)$-curve (Fig. 7) can be modified such that its initial (negative) rising part follows the function given by (11), assuming the flow stress to be 500N/mm² (Fig. 10). Since the same peak value of K_{Irs} is to be reached, this modification is accompanied by a small virtual shift (of about 1 mm in the present case) of the crack-tip to the left. Disregarding this physically somewhat unsatisfactory side-effect, the residual stresses calculated from the modified curve appear to be reasonable (Fig. 11), indicating that this simple procedure is well suited to be used as a first measure to account approximately for local yielding.

CONCLUSIONS

The residual stresses in the ligament of pre-cracked components can be determined quite easily and accurately by the crack compliance method. Besides, this methods also delivers the stress intensity factors that arises when the crack extends. However, because of the high residual stresses near the crack tip, effects of local plastic yielding can occur and affect the results significantly. A simple, approximate way to correct the measured data for this effect is suggested. However, further research will be necessary to develop a more general and more accurate correction procedure.

Regarding their magnitudes and gradients the residual stresses in the ligament of pre-cracked and pre-loaded parts are rather extreme cases. The theoretical analysis and the preliminary experimental results indicate that the CC-method is well suited to be applied even to such difficult situations of residual stress measurements. The method appears to be a promising new tool to analyse the driving forces of subcritical crack growth, particularly retardation mechanisms, crack closure or shielding effects .

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