ANALYSIS OF ONE-SIDED BONDED REPAIR TO A CRACKED THICK PLATE

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ABSTRACT

One-sided bonded composite repair to a centrally cracked thick plate has been analysed. The stress intensity factors of the repaired crack were calculated by using the slice synthesis technique in which the stress distributions in the crack location for the corresponding uncracked patched plate were obtained by a semi-analytical method. Efforts were focused on examination of the bending effects on the maximum stress intensity factors in the plate due to the neutral axis offset caused by the one-sided repair. A large number of patches consisting of various materials and geometries were examined. It appears that the maximum reduction of the stress intensity factor occurred when the length of the patch is approximately 6 times the thickness of the plate. Numerical results showed that reduction of the stress intensity factor is enhanced by having thicker and stiffer patch with fiber oriented perpendicular to the crack.

KEYWORDS

Bonded repair, Composite patch, Stress intensity factor, Crack, Thick plate.

INTRODUCTION

Adhesive bonded patches of advanced fiber composites can provide highly efficient and cost-effective repairs for cracked metallic aircraft components and structures. This method has been successfully used to repair RAAF aircraft in Australia (Baker and Jones, 1988). Hitherto, however, intensive investigation of patches has been aimed at the repairs of thin section and most successful applications to repair cracked aircraft components have been for thin sections. This is because the repair of a thick section is less effective than the repair of a thin section. A typical case is one-sided repair to a thick section with a through-thickness crack. In this case, a high bending stress occurs on the non-patched side due to the neutral axis offset and hence increases the stress intensity factor at the unpatched side. Thus, how to reduce this bending effect is a major issue in the design of the patch.

In the past twenty years, the majority of the computations of stress intensity factors in patched cracked plates were carried out by the three-dimensional finite element method. However, this method is time consuming and costly because a refined mesh is always required in the cracked region. In this paper, a cost-effective technique, the slice synthesis technique developed by Saff and Sanger (1984) for calculating stress intensity factors in unrepaired cracked plate, was employed to determine the stress intensity factors for a through-thickness crack in a thick plate with one-sided repair. In this application of the technique, the required stress distributions in the crack location for the corresponding uncracked patched plate were provided by the semi-analytical method developed by Zhu et al. (1996) and Zhu (1996).

For all the models analysed in this paper, the ends of the plates are free to rotate. Since in practical structures the ends are generally clamped, the stress intensity factors calculated in this paper are the maximum values for the individual cases. In other words, an upper limit of the stress intensity factor in a repaired structure can be determined using the method presented in this paper and the conclusions drawn based on this investigation are valuable for practical design of the patches.

SLICE SYNTHESIS TECHNIQUE

The slice synthesis technique deals with three dimensional crack problems through two dimensional analysis. Fig. 1 illustrates the idea of the technique, in which the cracked body is divided into two series of orthogonal slices of infinitesimal thickness. These slice are coupled through the introduction of pressure distribution $P(x,z)$ acting on the face of the crack to re-established the original three dimensional problem.

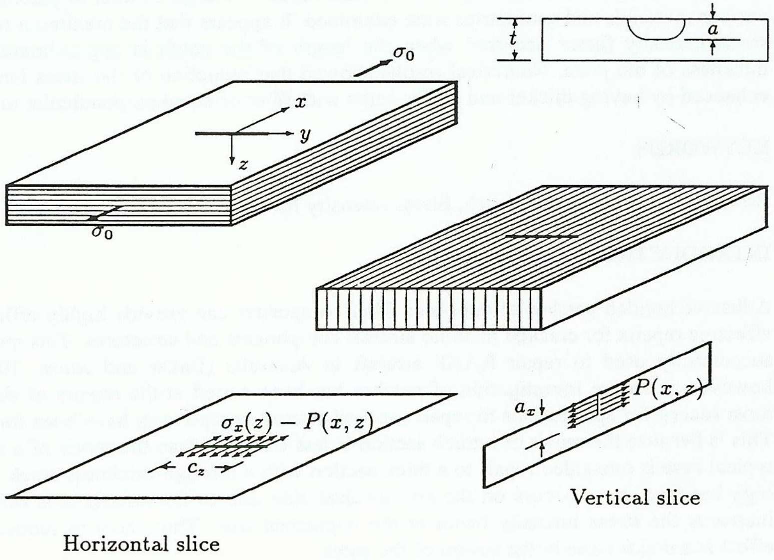

Fig. 1. Illustration of slice synthesis technique for surface crack problem

In this way, the original three-dimensional model is simplified into two two-dimensional models, one subjected to $\sigma_x(z) - P(x,z)$ and another only subjected to $P(x,z)$ on the crack surfaces. The pressure $P(x,z)$ is determined by the requirements for displacement continuity over the crack face, namely $d_a(\sigma,p) = d_c(P)$ where d_a and d_c are the associated displacements for the vertical and horizontal slices respectively. $\sigma_x(z)$ is the stress distribution induced by the applied load at the crack location in the original uncracked body. According to weight function theory, the stress intensity factors for the vertical and horizontal slices can be written as follows

$$K_a(a_x) = \int_0^{a_x} P(x,z)g(z,a_x)dz \tag{1}$$

$$K_c(c_z) = \int_0^{c_z} [\sigma_x(z) - P(x,z)]g(x,c_z)dx \tag{2}$$

where $g(z,a_x)$ and $g(x,c_z)$ are the weight functions for the vertical slice and horizontal slice respectively. It can be seen that in the formulations, two sets of quantities which must be known are stress distribution $\sigma_x(z)$ and the weight functions for the two sets of two dimensional models. In other words, this technique is applicable provided that the two sets of quantities are available.

SLICE SYNTHESIS TECHNIQUE FOR REPAIRED THROUGH-THICKNESS CRACKS

Since weight function is a local quantity related to crack geometry only, it can be argued that when the weight function of a crack with a bonded composite patch is calculated, the material difference between the plate and the patch can be ignored. In other words, those weight functions derived for unrepaired cracks can be employed for the analysis of cracks with bonded composite repairs as used by Lam et al. (1995). However, the stress distribution in an uncracked plate with bonded composite patches becomes much more complicated than that in an uncracked isotropic plate. The stress distribution in a patched uncracked plate is a function of multivariables related to the materials and geometries of the plate and patches as well as the loads applied on the plate. For a cracked metallic plate with bonded unidirectional/cross-ply patch, the stress distribution $\sigma_x(z)$ in equation (2) can be economically determined by the analytical formulae developed by Zhu et al. (1993). In this investigation, the stress distribution is determined by using the semi-analytical method developed by Zhu et al. (1996) and Zhu (1996). In the semi-analytical method, Rayleigh-Ritz solutions are developed based on a modified potential energy functional and an assumed displacement field. The assumed displacement field has been constructed to account for the continuity of transverse stresses explicitly. As a result, the stresses can be calculated directly and analytically through the constitutive equations.

VALIDATION OF THE METHOD

A one-sided repaired component consisting of a 10 mm thick aluminium plate with a centre crack, a 0.2 mm thick adhesive layer and a 2.55 mm thick boron/epoxy patch was used in validating the method presented above. Other geometries of the component and the coordinate system used for the current analysis are shown in Fig.2.

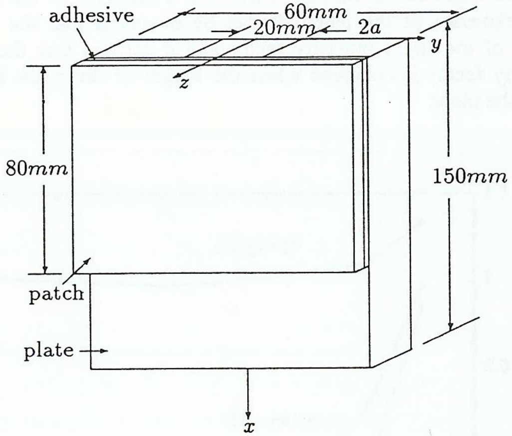

Fig. 2. A cross-section showing the plane containing the crack for a centrally cracked plate with one-sided bonded patch.

The loading was a tensile stress of 1 MPa in the x-direction applied to the end of the plate and the loaded end was free to rotate. The material of the plate was aluminium with $E = 72GPa$ and ν=0.3 The compliances of the boron/epoxy patch were

$$S_{xx} = 4.807 \times 10^{-6} MPa^{-1}$$
$$S_{yy} = S_{zz} = 3.932 \times 10^{-5} MPa^{-1}$$
$$S_{xz} = S_{yx} = S_{zy} = 1.381 \times 10^{-4} MPa^{-1}$$

The properties of the adhesive were taken as $E = 1.89GPa$ and $\nu = 0.35$. The stress intensity factors for the patched crack with the above loading and restrain were calculated using a full three-dimensional finite element analysis. The stress intensity factors at the four points distributed along the crack front in the thickness direction of the plate are obtained from both the finite element analysis and the method presented in this paper and are listed in Table 1 where h_0 and K_0 are the thickness of the plate and the stress intensity factor for the unpatched crack.

It can be seen from Table 1 that the results from both methods are in good agreement. This indicates that the proposed approach for calculating stress intensity factors for one-sided patched crack is valid and accurate.

Table 1. Nondimensionalized stress intensity factors for patched crack

z / h_0 (y=10, x=0)	K_I / K_0 (FEA)	K_I / K_0 (Proposed approach)	Difference (%)
0.	4.37	4.59	4.8
0.333	3.31	3.45	4.1
0.667	2.31	2.34	1.3
1.0	1.20	1.21	0.8

PARAMETER ANALYSIS

The parameters examined in this paper are the fiber angle of the patch, the material properties of the patch, and the length and thickness of the patch. Thus, a large number of patches need to be analysed.

Symmetric Angle-ply Patches

Fiber orientation of the patch for the one-sided repair can greatly affect the maximum stress intensity factor at the unpatched crack end. This can be observed from the following examples in which 19 patches with different fiber angles taken from 0 to 90 degree are analysed, where a fiber angle θ is defined as the angle between the x-axis and the fiber (Note: for every layer of $+\theta$ orientation, there is an identical layer of $-\theta$ orientation.). In these examples, a 10 mm thick aluminium plate with $E = 72GPa$ and $\nu = 0.3$ is considered. All patches have the same thickness of 10 mm and the same material properties as used above. Other geometries are the same as shown in Fig.2 and all loadings used are 1 MPa. The variation of the maximum stress intensity factor as a function of the fiber angle is shown in Fig.3. This figure shows that 0^o unidirectional patch produce the lowest value of the maximum stress intensity factors, while the patch with fiber angle of 45^o gives the largest value of the stress intensity factor. Between 0^o to 45^o, $K_{\max}$ increases rapidly with an increase in ply angle. Therefore, when all fibers in the patch are perpendicular to the crack face, the repair is the most efficient as expected.

Patches with Various Elastic Moduli

Elastic modulus of the patch is another parameter which affects the efficiency of the repair. In order to examine the effect of elastic modulus of the patch on the efficiency of the repair, 19 different patches with different isotropic materials are used in this investigation. The geometries of the plate and the patches as well as the material properties of the plate are the same as in the analysis of angle-ply patches. The variation of the maximum stress intensity factor as a function of the elastic modulus is shown in Fig. 4. It is indicated that the higher of the elastic modulus of the patch, the lower value is the value of the maximum stress intensity factor. This suggests that the patch with the highest tensile elastic modulus provides the best one-sided repair. It also indicated that for low modulus patch, an increase in the stress intensity factor above that of the unpatched cracked plate could occur.

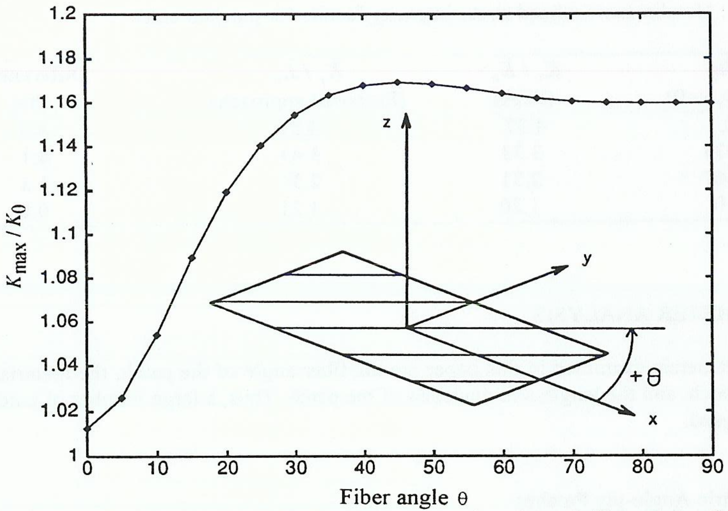

Fig. 3. Variation of maximum stress intensity factor as a function of fiber angle of the patch.

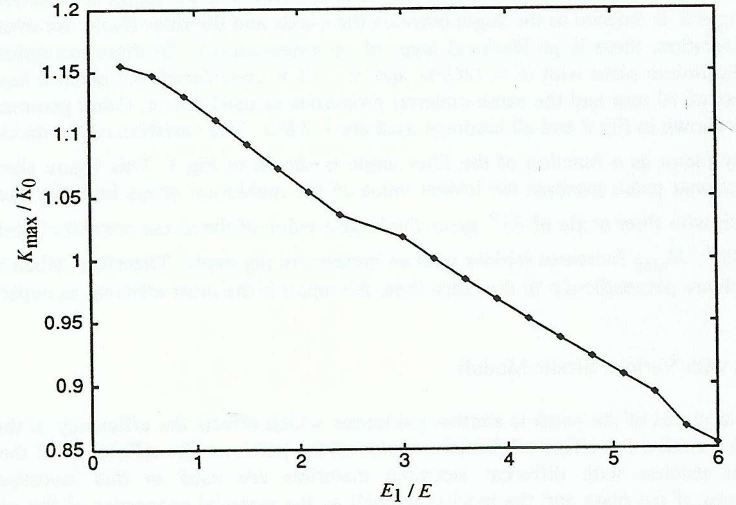

Fig. 4 Variation of maximum stress intensity factor as a function of elastic modulus E_1 of the patch.

Patches with Various Lengths and Thicknesses

In this analysis, 36 patches corresponding to 3 thicknesses and 12 lengths are considered. The plate is the same as used in the previous section. The materials of the patches are assumed to be isotropic with elastic modulus of 3 times that of the plate and Poisson ratio of 0.3. Fig. 5 shows the maximum stress intensity factors as a function of the length of the patch for three different thicknesses of the patch. It can be observed that the thickest patch produces the lowest value of the stress intensity factor and it appears that the maximum reduction of the stress intensity factor is achieved when the length of the patch is approximately 6 times the thickness of the plate.

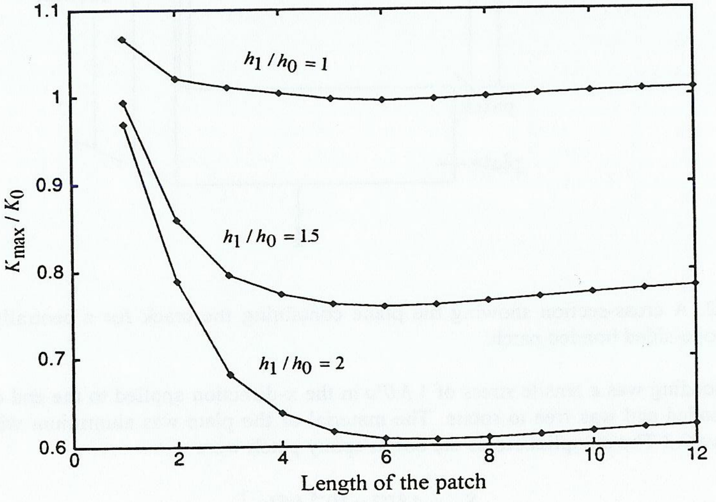

Fig. 5. Variation of the maximum stress intensity factor as a function of the length of the patch (h_1 and h_0 are the thicknesses of the patch and plate respectively).

CONCLUSIONS

Numerical analyses have been carried out for one-sided bonded repair to a thick plate with a through-thickness crack. The slice synthesis technique developed for the analysis of unpatched three-dimensional cracks has been demonstrated to be suitable for the calculation of stress intensity factor for one-sided repair of cracked plate. The application of this technique has been validated by comparisons of the results with those obtained from three-dimensional finite element analysis. Variations of the maximum stress intensity factors as functions of four parameters, fiber angle of angle-ply patch, elastic modulus of isotropic patch, and the length and thickness of the patch, have been obtained based. Reduction of the maximum stress intensity factor is achieved by having 0^o ply angle, thicker and stiffer patch. In addition, it appears that maximum reduction of the stress intensity factor occurred when the length of the patch is approximately 6 times the thickness of the plate.

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