# THEORETICAL STUDY ON PECULIARITIES OF SMALL ELLIPSOIDAL DEFECTS DETECTING AND EVALUATION IN LAYERED MEDIAS

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ABSTRACT

On the basis of the equivalent sources method the small defect with its maximal dimension more less than electromagnetic wave length in the material under evaluation are substituded by electric and magnetic dipoles. Equivalent dipole moments are presented as a product of a polarizability tensor and field intensity vector at the defect coordinate. Such representation allows the analysis of the defect shape and orientation influence upon scattered field to reduce to the polarizability tensor investigation. Numerical analysis of the dependence of the void spheroidal defect shape and orientation upon equivalent dipole moment has been carried out.

KEYWORDS

Equivalent sources method, equivalent electric dipole, layered media, ellipsoidal defect, nondestructive testing.

## THEORETICAL BACKGROUND

Dipole Model. For the case of small defects while $a \ll \lambda$, in which $a$ is the greatest defect dimension and $\lambda$ is a wavelength in the material with defect, scattered field with sufficient for the practical purposes accuracy is described by a dipole model. Such mathematical model results from the equivalent sources method in which small defect is substituted by sum of the electric and magnetic dipoles with orientation depending on defect shape and orientation and primary electromagnetic field electric and magnetic intensity vectors direction. Besides dipoles moment value depends upon defect and media electric and magnetic permeability correlations (Senior,1976). Then the problem is reduced to three independent subproblems:

a) field determination at defect coordinate by given primary field sources in defectless material;
b) polarizability tensor calculation as a function of the defect electrical and magnetic parameters with regard of that of the material and defect shape (electric and magnetic dipole moments equal to the product of the electric and magnetic polarizability tensors upon electric and magnetic field intensity vectors);
c) derived equivalent dipole field calculation at points under consideration as a defect scattered field.

Presentation Advantages. By such treatment of the problem one can independently consider and analyze the influence of the defect dimensions, shape and orientation, its depth, and the geometric or physic material parameters. It is convenient for eddy current probes design too allowing apart consideration and choise of the primary and the secondary probe coils.

Solution Method. Knowing the electric and magnetic field intensities at the point of the small defect location is necessary to calculate equivalent electric and magnetic dipole moments. In general one can represent such a field as a sum of the primary electric and magnetic field intensities $\vec{E}^{inc}$, $\vec{H}^{inc}$ in the material with no defect and components $\vec{E}^r$, $\vec{H}^r$ to taking into account secondary sources (i.e. equivalent dipoles) interaction with piecewise homogeneous media boundaries. Components $\vec{E}^r$ and $\vec{H}^r$ values are determined having found a solution of the problem of the electric and magnetic dipole radiation in the layered media (Stoyer,1977). In order to obtain the analytical expressions for the components $E^r_\alpha$, $H^r_\alpha$ were derived the limit values of the reflected from the boundaries dipole field components by $z$ and $r$ tend to zero (coordinate onset and defect are at the same point and $z$ is perpendicular to boundaries):

$$E_\alpha(M_\beta, r=0, z=0) = \lim_{\substack{r\to 0\\ z\to 0}} E_\alpha(M_\beta) = M_\beta e_\alpha^{M_\beta},$$

$$H_\alpha(M_\beta, r=0, z=0) = \lim_{\substack{r\to 0\\ z\to 0}} H_\alpha(M_\beta) = M_\beta h_\alpha^{M_\beta}, \qquad (1)$$

$$\alpha,\beta \in \{1,2,3\} = \{x,y,z\}, \qquad M_\beta \in \{p_\beta,\ m_\beta\},$$

in which $p_\beta$, $m_\beta$ are electric and magnetic dipole moment components, and $r = (x^2+y^2)^{1/2}$.
Obtained due to the limit transition (1) reflected from planar layered media boundaries dipole field structure is shown in Table 1.



Table 1. Electric and magnetic intensities field components at the defect centre due to the defect field interaction with plain boundaries.

| Dipole components | Field components $E_1$ | $E_2$ | $E_3$ | $H_1$ | $H_2$ | $H_3$ |
|---|---|---|---|---|---|---|
| $p_1$ | $e_1^{p_1}$ | 0 | 0 | 0 | $h_2^{p_1}$ | 0 |
| $p_2$ | 0 | $e_2^{p_2}$ | 0 | $h_1^{p_2}$ | 0 | 0 |
| $p_3$ | 0 | 0 | $e_3^{p_3}$ | 0 | 0 | 0 |
| $m_1$ | 0 | $e_2^{m_1}$ | 0 | $h_1^{m_1}$ | 0 | 0 |
| $m_2$ | $e_1^{m_2}$ | 0 | 0 | 0 | $h_2^{m_2}$ | 0 |
| $m_3$ | 0 | 0 | 0 | 0 | 0 | $h_3^{m_3}$ |

Taking into account abovementioned correlations the algebraic equation system to calculate the electromagnetic field intensities components at the point of the defect location has been obtained:

$$E_\alpha = e_\alpha^{p_\beta} T_{\beta\gamma}(\varepsilon)E_\gamma + e_\alpha^{m_\beta} T_{\beta\gamma}(\mu)H_\gamma + E_\alpha^{inc},$$

$$H_\alpha = h_\alpha^{p_\beta} T_{\beta\gamma}(\varepsilon)E_\gamma + h_\alpha^{m_\beta} T_{\beta\gamma}(\mu)H_\gamma + H_\alpha^{inc}, \qquad (2)$$

$$\alpha,\beta,\gamma \in \{1,2,3\},$$

in which $T_{\beta\gamma}$ is electric ($\varepsilon$) or magnetic ($\mu$) polarizability tensor. Here the electric and magnetic dipole moments are written as

$$p_\beta = T_{\beta\gamma}(\varepsilon)E_\gamma, \quad m_\beta = T_{\beta\gamma}(\mu)H_\gamma. \qquad (3)$$

Polarizability tensor dependence upon the defect orientation with reference to plain layer media boundaries is next:

$$T_{11} = N_{11}\vartheta_{11}^2 + N_{22}\vartheta_{21}^2 + N_{33}\vartheta_{31}^2,$$

$$T_{12} = T_{21} = N_{11}\vartheta_{12}\vartheta_{11} + N_{22}\vartheta_{22}\vartheta_{21} + N_{33}\vartheta_{32}\vartheta_{31},$$

$$T_{13} = T_{31} = N_{11}\vartheta_{13}\vartheta_{11} + N_{22}\vartheta_{23}\vartheta_{21} + N_{33}\vartheta_{33}\vartheta_{31}, \qquad (4)$$

$$T_{22} = N_{11}\vartheta_{12}^2 + N_{22}\vartheta_{22}^2 + N_{33}\vartheta_{32}^2,$$

$$T_{32} = T_{23} = N_{11}\vartheta_{13}\vartheta_{12} + N_{22}\vartheta_{23}\vartheta_{22} + N_{33}\vartheta_{33}\vartheta_{32},$$

$$T_{33} = N_{11}\vartheta_{13}^2 + N_{22}\vartheta_{23}^2 + N_{33}\vartheta_{33}^2.$$

Here $\vartheta_{\alpha,\beta} = \cos(x'_{\alpha} \hat{,} x_{\beta})$, in which $(x'_{\alpha} \hat{,} x_{\beta})$ is the angle of the local frame of axes with the shape tensor $N_{\alpha\beta}$ to be calculated, with respect to the common one with the dipole radiation to be determined. Shape tensor for ellipsoid is described analitically (Dodd et.al.,1971; Kolodiy et.al.,1985; Orlowsky,1982; Rudakov et.al.,1984), and by the primary field time dependence $\exp(-i\omega t)$ it equals

$$N_{\alpha\beta} = -\frac{4}{3}\, i\omega\pi \prod_{j=1}^{3} a_j \, \frac{\xi_0(\xi_2-\xi_1)\delta_{\alpha\beta}}{\xi_1+(\xi_2-\xi_1)n_{\alpha\beta}}, \quad \xi\in(\varepsilon,\mu), \qquad (5)$$

in which $a_j$ are the ellipsoid semi-axes, $\delta_{\alpha\beta}$ is the Kronecker symbol,

$$n_{\beta} = \frac{a_1 a_2 a_3}{2} \int_0^{\infty} (\varkappa+a_{\beta})^{-2} \prod_{j=1}^{3} (\varkappa+a_j)^{-1/2} d\varkappa, \qquad (6)$$

$\varepsilon,\mu$ are electric and magnetic permittivities, and indices $0,1,2$ identify free space, material media and defect correspondingly.
Conductors have complex electric permittivity $\underline{\varepsilon} = \varepsilon + i\sigma/\omega$, in which $\sigma$ is the material conductivity.

Conductive Media. To investigate defect shape and orientation relationship it is worth while to consider the nondestructive eddy current testing commonest case with nonmagnetic media and defect and media material to be distinct only by its electric parameters. For good conductors while $|\underline{\varepsilon}_1| \gg |\varepsilon_2|$ formula (5) accurate to the terms order $O(\varepsilon_2/(\underline{\varepsilon}_1(1-n_{\beta})))$ can be written in the form $N_{\alpha\alpha} = \frac{4}{3} i\omega\pi a_1 a_2 a_3 \; \varepsilon_0(1-n_{\beta})$. (7) In such a case polarizability tensors don't depend upon media material conductivity.

Spheroidal Defect Investigaion. To study shape and orientation influence upon the defect scattered field a void in a form of the ellipsoid of revolution ($a_2=a_3$) was chosen. For the analysis clarity such a defect was located fairly far of boundaries to neglect their influence. There were two cases of axes change under investigation a) $a_1 = a_{max}$ = const, $0 < a_2 = a_{min} < a_1$, b) $0 < a_1 = a_{min} < a_2$, $a_2 = a_{max}$ = const. In the first case defect changes its shape from sphere ($a_2=a_1$) to needle ($a_2 \approx 0$). In the second case shape varies from sphere ($a_1=a_2$) to disc ($a_1 \approx 0$).



NUMERICAL ANALYSIS.

The ratio of the equivalent dipole moment of the ellipsoidal void to that of the circumscribing spherical one $p_c$ was taken as a figure of merit in numerical study of void shape influence upon the value of equivalent dipole (Fig.1).

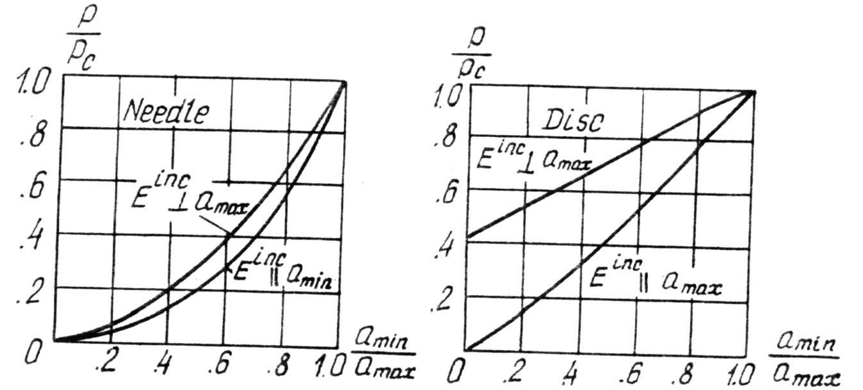


Fig.1. Dipole moment dependence upon the ellipsoid semi-axes ratio.

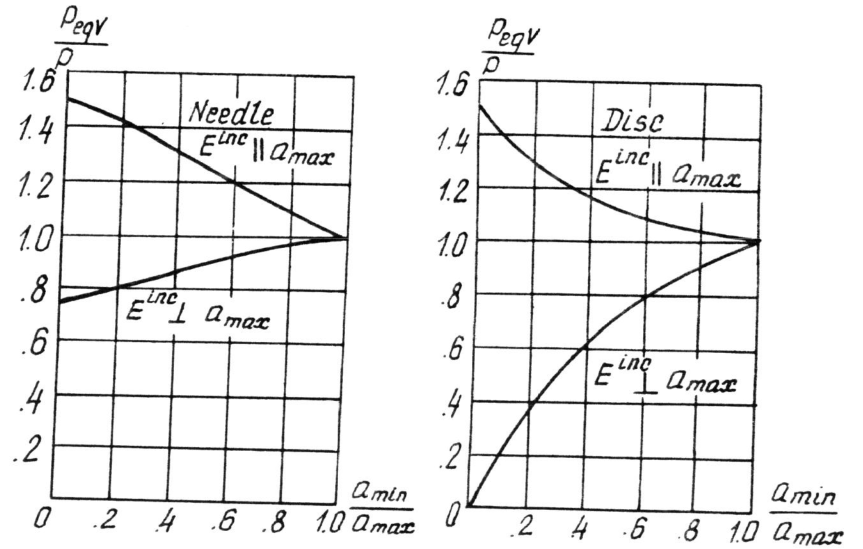


Fig.2. Equivolume spherical void dipole moment reduced to that of the ellipsoidal one.

Curves on Fig.2. represent the ratio of the equivolume spherical void dipole moment to that of the ellipsoidal one as a function of its semi-axes ratio.

For the analysis of the orientation influence upon the defect scattered field the ellipsoidal void was rotated around its axis $a_3$. This axis was chosen to be perpendicular to the plain boundary surfaces of the material under evaluation, and all the axes were perpendicular to each other as well. The results of calculations are presented on Fig.3.

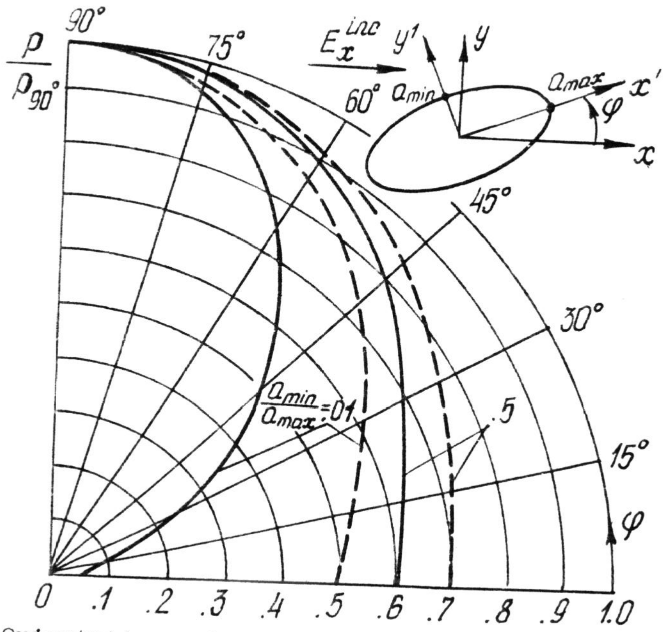


Fig.3. Orientation relationship for disc (—) and needle (--).

CONCLUSIONS

1) The investigation of the local defect shape and orientation influence upon the scattered electromagnetic field value is reduced to the equivalent dipole moment change analysis and, in particular, to the study of the polarizability tensor by means of the dipole moment is presented.

2) The greatest disturbance of electromagnetic field is caused by perpendicular defect orientation with reference to primary field intensity direction and the smallest one - by parallel orientation.



3) Disclike defect locating perpendicular with reference to primary field direction causes disturbance less than twice by that of curcumscribing spherical defect, and by parallel location its dusturbance approaches to zero.

4)The most hardly detectable defect shape is a needle. By perpendicular to primary field orientation its field disturbance is twice greater than that due to such defect parallel orientation.

5)Field disturbance due to disclike as well as needle like defect by it's parallel location is one and half times less than that due to equivolume spherical defect.

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