THE WAVE METHOD FOR ANALYSIS OF STRUCTURAL MEMBERS INJURY UNDER IMPACT

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ABSTRACT

The problem connected with the determination of striking object measures and density, its initial velocity of impact, as well as the moment and coordinates of the location of impact is considered by way of the example of a cylindrical body impact upon a thin transversely isotropic composite plate. As a method of solution, the ray method is used, which allows one to construct the solution behind the wave fronts arising during impact up to the boundary of the contact region. This method in conjunction with indications of five seismic sensors (or other analogous instruments measuring velocities of displacements) causes to the system of five algebraic equations with respect to the enumerated values to be found. From comparison of the obtained initial velocity of impact V_0 with the ultimate velocity of shock failure, one can predict the emergence of cracks during impact.

KEYWORDS

Impact, ballistic velocity of punching, target, plate, composite, ray method

PROBLEM DEFINITION AND DETERMINING EQUATIONS

Let a thin elastic bar, whose axis coincide with the x_3-axis, move along this axis with the velocity V_0 towards an elastic transversely isotropic plate. The plane of isotropy at every point of the plate is parallel to the middle plane. At t=0 the normal impact of the cylindrical bar upon the plate occurs, the centers of the cylinder face and the plate being coincident at the moment of impact (Fig.1).

The plate dynamic behavior is described by the system of equations (Uflyand, 1948; Mindlin, 1961)

$$M_{1,1} + M_{12,2} - Q_1 = -1/12\rho h^3 \dot{\Phi}_1$$
$$M_{12,1} + M_{2,2} - Q_2 = -1/12\rho h^3 \dot{\Phi}_2 \quad ,$$
$$Q_{1,1} + Q_{2,2} = \rho h \dot{W} \quad ; \qquad (1)$$

$$\dot{M}_1 = -D(\Phi_{1,1} + \sigma\Phi_{2,2}) \ , \quad \dot{M}_2 = -D(\Phi_{2,2} + \sigma\Phi_{1,1}),$$
$$\dot{M}_{12} = -D_1(\Phi_{1,2} + \Phi_{2,1}) \ ,$$
$$\dot{Q}_1 = k\mu' h(W_{,1} - \Phi_1) \ , \quad \dot{Q}_2 = k\mu' h(W_{,2} - \Phi_2) \quad , \qquad (2)$$

where M_i and Q_i $(i=1,2)$ are the bending moments and transverse forces, respectively; M_{12} is the torsional moment; $D=Eh^3/12(1-\sigma^2)$, E is the modulus of elasticity for the directions in the plane of isotropy, σ is the Poisson's ratio characterising contraction in the plane of isotropy under tension in the same plane, h is the plate thickness; $W=\dot{w}$ is the velocity of the deflection; Φ_1 and Φ_2 are the angular speeds of the normal to the plate in the x_1- and x_3-directions, respectively; ρ is the plate density; $D_1 = \mu h^3/12$, $\mu=1/2E(1+\sigma)^{-1}$ is the shear modulus; μ' is the shear modulus in the plane which is perpendicular to the plane of isotropy; $k=\pi^2/12$; an overdot denotes a derivative with respect to time, an index after a comma indicates a derivative with respect to the corresponding coordinate.

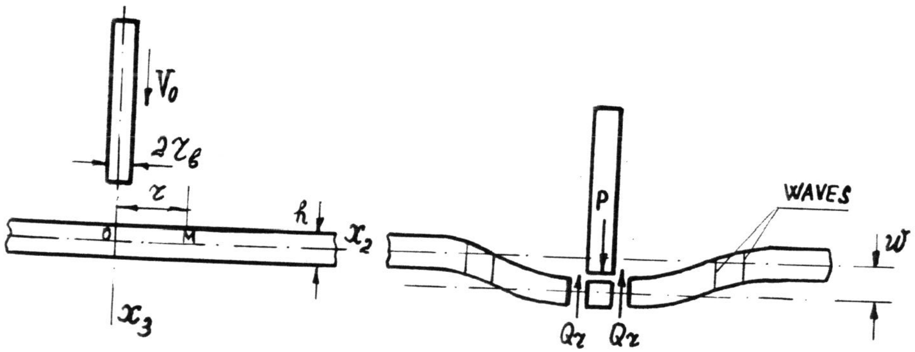

Fig.1 Scheme of an elastic bar shock interaction with an Uflyand-Mindlin plate

As a result of the dynamic action on the plate, the cylindrical waves Σ_α of a strong or weak discontinuity propagate in the plate, whose generators are parallel to the x_3-axis but guides locating in the middle plane are circumferences extending with the normal velocities $G_{(\alpha)}$.

At the same time, a longitudinal wave propagates along the thin bar with the velocity $G_b=(E_b/\rho_b)^{1/2}$ (E_b and ρ_b are the Young's modulus and volume density of the striking bar, respectively), representing itself a plane of the strong discontinuity. In virtue of the fact, that the jumps of kth derivatives of the bar's particles displacement velocities with respect to time are constant values, on this plane the dynamic condition of compatibility is satisfied

$$\sigma' = \rho_b G_b (V_o - W) = P(\pi r_b^2)^{-1} \ , \qquad (3)$$

where $\sigma' = \sigma|_{x_3=0}$ is the contact stress, $W = V|_{x_3=0}$ is the displacement velocity of the beam's contacting part which is considered as a rigid, P is the contact force.
With due account of (3), the equation of motion of the beam part, which is in contact with the bar, may be written as

$$-\rho\pi r_b^2 h\dot{W} + \pi r_b^2 \rho_b G_b (V_o - W) + 2\pi r_b Q_r = 0 \ , \qquad (4)$$

where Q_r is the transverse force affecting on the area perpendicular to the polar radius $r = (x_1^2 + x_2^2)^{1/2}$.
It is necessary to add the initial condition

$$W|_{t=0} = 0, \qquad (5)$$

as well as the relation

$$\partial W/\partial r|_{r=r_b} = 0 \qquad (6)$$

to Eq.(1).

METHOD OF SOLUTION

Behind the front of the each wave surface Σ, a certain desired function $Z(x_\alpha, t)$ is represented by a series in terms of the powers $t-(r-r_b)G^{-1} \geq 0$ (Achenbach and Reddy, 1967)

$$Z(x_\alpha, t) = \sum_{k=0}^{\infty} \frac{1}{k!}[Z,_{(k)}]\{t-(r-r_b)G^{-1}\}^k H\{t-(r-r_b)G^{-1}\}, \qquad (7)$$

where the values $[Z,_{(k)}]$ are calculated at $t=(r-r_b)G^{-1}$, $H(t)$ is the unit Heaviside function, a sign $[\ ]$ denotes the jump of the corresponding value on the wave surface. An index indicating an ordinal number of the wave is omitted for simplicity.

As the axially symmetric problem is solved hereafter, then we assume that the desired values are independent of the angle coordinate φ.

To determine coefficients of the ray series (7) for the desired functions Q_r and W, we differentiate Eqs.(1) and (2) k times with respect to time t, take their difference on the different sides of the wave surface Σ, and apply the condition of compatibility (Thomas, 1961)

$$G[\partial Z_{,(k)}/\partial x_\alpha] = -[Z_{,(k+1)}]\nu_\alpha + \nu_\alpha d[Z_{,(k)}]/dt \quad , \qquad (8)$$

where ν_α ($\nu_1=\cos\varphi$, $\nu_2=\sin\varphi$) are the components of the normal vector to the wave surface.

As a result we obtain

$$(1-\rho h^3G^2/12D)\omega_{(k+1)} = 2d\omega_{(k)}/dt + Gr^{-1}\omega_{(k)}$$

$$+ k\mu' hGD^{-1}X_{(k)} + F_{1(k-1)} \ , \qquad (9)$$

$$(1-\rho G^2/k\mu')X_{(k+1)} = 2dX_{(k)}/dt + Gr^{-1}X_{(k)}$$

$$- G\omega_{(k)} + F_{2(k-1)} \ . \qquad (10)$$

Here

$$\omega_{(k)} = [\Phi_{2,(k)}]\nu_\alpha \ , \quad X_{(k)} = [W_{,(k)}] \ ,$$

$$F_{1(k-1)} = d^2\omega_{(k-1)}/dt^2 - Gr^{-1}d\omega_{(k-1)}/dt$$

$$+G^2r^{-2}\omega_{(k-1)} - k\mu' hGD^{-1}dX_{(k-1)}/dt - k\mu' hG^2D^{-1}\omega_{(k-1)} ,$$

$$F_{2(k-1)} = Gd\omega_{(k-1)}/dt - d^2X_{(k-1)}/dt^2 + Gr^{-1}dX_{(k-1)}/dt$$

$$+ G^2r^{-1}\omega_{(k-1)} \ .$$

Setting $k=-1,0,1,2,\ldots$ sequentially in Eqs.(9) and (10), one can obtain the jumps of any order for the desired functions up to arbitrary constants $c^{(\alpha)}_{(k)}$ ($k=0,1,2,\ldots$, $\alpha=1,2$). Substituting the known jumps into Eqs.(4)-(6) and equating coefficients at like powers of t, we find all arbitrary constants $c^{(\alpha)}_{(k)}$. The relations obtained allow one to calculate the velocity of the cylindrical striking bar displacement, contact force, as well as

to construct the fields of generalized forces and displacements behind the wave fronts up to the boundary of the contact area, in other words, to solve the primal problem of the impact interaction.

As an illustration corroborating the efficiency of the above method, we consider the transverse impact of a circular steel bar of length 100 mm and radius r_b=5 mm upon a composite target with the following parameters: E=51.1 GPa, μ=19.5 GPa, μ' =4.14 GPa, σ=0.31, E' =11.9 GPa, and σ' =0.06. Calculations are performed with due account of the five terms of the ray series for the desired functions. The time functions of the dynamic strains ε_r and ε_φ at the place of contact are presented in Fig.2 (solid lines). The results obtained are in close agreement with the results of Mochihara and Tanaka (1989) (light dots).

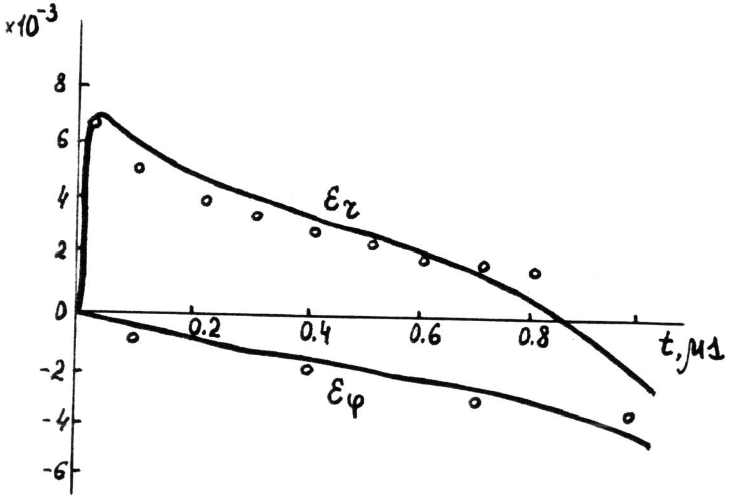

Fig.2. Shock strain dependences from time

INVERSE PROBLEM

Assume that a long body of the cylindrical form at the moment T bumps against a plate with some initial velocity, the radius of the striking object, its density (mass), the moment of the impact, and the location of collision, as well as the shock initial velocity being unknown values.

Using indications of five seismic sensors C5C (or other analogous instruments measuring velocity of displacement) placed at different points on the plate, it is necessary to determine all enumerated unknown values and, moreover, to establish the convenience of plate's material failure in the area of the impact by means of the analysis of the shock initial velocity value.

Figure 3 schematically shows the location of the impact and position of the sensors.

It is seen from Fig.3 that with the known relative position of the sensors, the only unknown value characterizing the striking object location with respect to the sensors is the distance from the striker to the nearest sensor (the fifth sensor is the nearest in Fig.3). Denoting this distance by r_x , we have

$$r_i = r_{i-5} + r_x \qquad (i=1,2,3,4) , \qquad (11)$$

where r_{i-5} is the difference between the radii of the two circles with the center in the middle of the striker, the both circles running along the ith and fifth sensors.

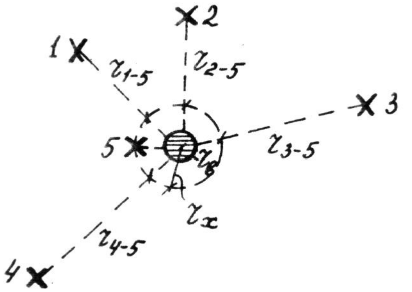

Fig.3. Scheme of the sensors position

By means of the ray series for the displacement velocities, one can obtain the expressions for the displacement velocities W_i of the plate points where sensors locate

$$W_i = \sum_{k=0}^{\infty} \frac{1}{k!} (T + r_0 G^{-1})^k [W_{,(k)}] \Big|_{t=(r_i - r_0)/G} \qquad (12)$$

$$(i=1,2,\ldots,5),$$

where G is the velocity of the fast wave.

From the system of five Eqs.(12), one can determine the five desired values: T, V_0, r_b, ρ_b, and r_x .

Comparing the obtained magnitude V_0 with the ultimate ballistic velocity V_l (Zukas et.al., 1982), one can answer the question: whether this collision cause the plate injury or not.

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