THE INFLUENCE OF OPEN PROFILES WARPINGS UPON KINETICS OF CRACKS DEVELOPMENT IN THEM

T.I. RYBAK

Ternopil Instrument-Making Institute,
56, Ruska St., 282001 Ternopil, Ukraine

For thinwalled bar constructions consisting of open profile elements of different channels, bar , T-beam cross-sections types, the main fractures are those which appear under tensile stresses. The main calculative characteristic is the stress intensity factor (*SIF*) of normal break K_I. In spite of is of great importance of this problem nowadays, the methods of determining of *SIF* in the above-mentioned standard profiles are not available in modern literature. However, these profiles are very frequently used in designing new and developing existing basic carry-systems, particularly for the mobile agricultural machines. In the paper Rybak, (1985) the first attempt to determine on the basis of approximate approach the calculative dependences for *SIF* in the case of pure bending of channel beam with a edge crack was made. The paper deal with the general ideas of this problem. The main task there is to determin *SIF*, taking into account the influence of additional stresses from the warpings of elements of an open profile.

Task and Solution. Lets take thinwalled bar with channel cross-section, which is characterized by the following dimensions (Fig.1): H-channel's height; b-shelf's width; d-wall's thickness; t-average thickness of a shelf. Let channel has a crack with length $L=b\times\varepsilon$ which starts on a shelf's edge. Let's determine *SIF* at the crack's tip if the bar is loaded by the bend torsion bimoment B_ω.

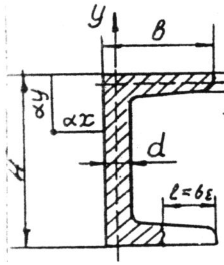


Fig.1. Standard beam's cross-section of channel profiles with an edge [illegible]

Stress [illegible] plane, can be aproximately modelled, if we examine the plate of the same thickness t and width b with the edge crack with the same force load. As stresses in the shelf under the influence of bimoments are distributed according to the linear law, they can be represented as a combination of tension and bend Fig.2.).By analogy with the existing formulas we find the proper solutions of *SIF* equation.

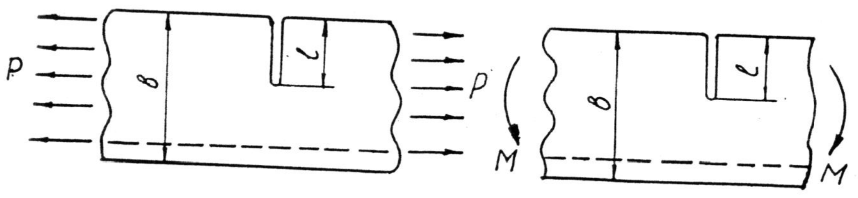
-in the case of tension:

$$K_I^{(t)}=p\times\sqrt{\pi\times L}\times\sqrt{2\times tg(\pi\times\varepsilon/2)/(\pi\times\varepsilon)}\times\left[0,572+2,02\times\varepsilon+0.37\times(1-Sin(\pi\times\varepsilon/2)^3\right]/Cos(\pi\times\varepsilon/2) \qquad (1)$$

-in the case of bend:

$$K_I^{(b)}=\sigma_b\cdot\sqrt{\pi\times L}\times\sqrt{2\times tg(\pi\times\varepsilon/2)/(\pi\times\varepsilon)}\times\left[0,923+0,199\times(1-Sin(\pi\times\varepsilon/2)^4\right]/Cos(\pi\times\varepsilon/2) \qquad (2)$$

where $\varepsilon=L/b$; $\sigma_b=6\times M/b^2\times t$ -maximum stresses.



a) b)
Fig.2.Diagram of shelves load under channel warping
a-tension b-bend

In the relations (1) and (2) letit take the nominal tensions which act directly in the neck cross-section,weakened by the crack.As in tension

$$\sigma_{nom}^{(t)}=p\cdot b/(b-L)=p/(1-\varepsilon)$$

and in bend (3)

$$\sigma_{nom}^{(b)}=6\times M/(b-L)^2\times t=\sigma_b/(1-\varepsilon)^3 \qquad (4)$$

Then the total quantity of *SIF* in linear distributed stresses and in accordance with (1) and (2) will have the value:

$$K_I=K_I^{(t)}+K_I^{(b)}=\sqrt{\pi\times L}\times\sqrt{2\times tg(\pi\times\varepsilon/2)/(\pi\times\varepsilon)}\times\Big\{\sigma_{nom}^{(t)}\times(1-\varepsilon)\times\left[0,752+2,02\times\varepsilon+0,37\times(1-Sin(\pi\times\varepsilon/2)^3\right]+\sigma_{nom}^{(b)}\times(1-\varepsilon)^3\times\left[0,923+0,199\times(1-Sin(\pi\times\varepsilon/2))^4\right]\Big\}/Cos(\pi\times\varepsilon/2) \qquad (5)$$

Equation (5) is the first eguation for calculating *SIF* in the crack tip point of the channel shelf.It is nessesary only to 1 choose force parameters $\sigma_{nom}^{(t)}$ and $\sigma_{nom}^{(b)}$ in such away that they

completely correspond to the real picture of nominal stresses in the supporting cross-section of the crackted shelf. In its turn the task is to determination the stress-strained state in the net cross section of the bar (hatched part in fig.1),which is asymetrical channel with the lower shelf length $b_1=b-L=b\times(1-\varepsilon)$.

Analyses of Redistribution of Nominal Stresses in Cross-Sections Weakend by Crack.Nominal stresses in the thinwalled bars while being loaded by bend-torsion bimoments,are changed according to sector-area law and are described by the dependence

$$\sigma_\omega=B_\omega\times\omega/I_\omega \qquad (6)$$

where $I\omega$ - sector moment of cross-section inertia,ω - sector coordinate of definite point in placing the pole in the centre of the bend,and the starting point (beginning of count)-in the main sector point.Thus,detection of normal stress distribution,appearing in the channel netto cross-sections is based, first of all, on the determination of sector geometrical characteristics of this cross-section.To solve this problem let's use arbitary integration epure method.For this purpose,draw arbitary epures of linear and sector coordinates of given cross-section in such a way,as to simplify and decrease to the maximum the quantity of calculations while being integrated.

Let's calculate integrals on all the area of cross-section (Fig.1) from the squares of epures and their derivatives, taken in pairs:

$$\int_F x^2dF=b^3\times t/3+b_1^3\times t/3 \;;\; \int_F y^2dF=H^3\times d/3+H^2\times b_1\times t$$

$$\int_F z^2dF=b\times t+H\times d+b_1\times t \;;\; \int_F x\times ydF=-b_1^2\times H\times t/2$$

$$\int_F x\times zdF=b^2\times t/2+b_1^2\times t/2;\; \int_F y\times zdF=-H^2\times d/2-H\times b_1\times t \qquad (7)$$

$$\int_F y\times\omega_0dF=-H^2\times b_1^2\times t/2 \;;\; \int_F x\times\omega_0dF=b_1^3\times H\times t/3$$

$$\int_F z\times\omega_0dF=H\times b_1^2\times t/2 \;;\; \int_F \omega_0^2dF=H^2\times b_1^3\times t/3$$

Let's mark through α_x and α_y the coordinates of the bending's centre in the selected system of coordinates and also take the value

$$\beta=\alpha_x\times y_0-\alpha_y\times x_0 \qquad (8)$$

where (x_0,y_0)-the co-ordinates of the main sector point.

To calculate the necessary parametres α_x,α_y,β ,taking into account (7) and after the transformations, we'll get the system of algebraic equations

$$
\begin{aligned}
&2\times b\times(1+\xi^3)\times\alpha_y+3\times H\times\xi^2\times\alpha_x+3\times(1+\xi^2)\times\beta=-2\times b\times H\times\xi^3 \\
&-3\times b^2\times t\times\xi^2\times\alpha_y-2\times H\times(H\times d+3\times b\times t\times\xi)\times\alpha_x-(3\times H\times d+6\times t\times b\times\xi)\times\beta= \\
&\qquad=3\times b^2\times H\times t\times\xi^2 \\
&3\times b^2\times t\times\alpha_y+H\times d\times\alpha_x+(3\times H\times d+6\times b\times t)\times\beta=0
\end{aligned}
\qquad (9)
$$

where $\xi=1-\varepsilon$.

Solving the system of equations (9), we'll determine

$$\alpha_y=A_1\times H\times\xi^2/A \quad ; \quad \alpha_x=A_2\times b\times\xi^2/A \quad ; \quad \beta=b\times H\times A_3\times\xi^3/A \qquad (10)$$

where A, A_1, A_2, A_3 the determinants of the 3d order. Opening and simplifying these formulas, we'll get:

$$
\begin{aligned}
&A=-6\times S_2\times(S_1+S_2)-6\times S_1\times(4\times S_2+3\times S_1)\times\xi-18\times S_1\times S_2\times\xi^2-6\times S_2\times \\
&\quad\times(S_2+4\times S_1)\times\xi^3-6\times S_1\times(S_2+3\times S_1)\times\xi^4 \ ; \\
&A_1=9\times S_1\times S_2+6\times S_2\times(S_2+4\times S_1)\times\xi+6\times S_1\times(S_2+3\times S_1)\times\xi^2 \ ; \\
&A_2=9\times S_1\times(S_1+2\times S_2)+18\times S_1\times S_2\times\xi+9\times S_1^2\times\xi^2 \ ; \\
&A_3=-6\times S_1\times S_2-12\times S_1\times S_2\times\xi-9\times S_1^2\times\xi^2 \ ;
\end{aligned}
\qquad (11)
$$

where $S_1=t\times b$; $S_2=H\times d$ – the squares of the channel's shelf and wall.

The relations (10) and (11) determine the co-ordinates of the bending's centre and zero sectorial point for the channel's netto cross-section. It's necessary admit, that if $\xi=1$ (that is $\varepsilon=0$, there is no crack) it follows:

$$\alpha_y=-H/2 \quad ; \quad \alpha_x=-3\times S_1\times b/(S_2+6\times S_1); \quad \beta=-H\times\alpha_x/2 \qquad (12)$$

that completely coincide with the corresponding dependences for an equal shelved channel Bychkov, 1962.

To determine the the sectorial moment of channel's inertia we use the formula:

$$I_\omega=\alpha_y\times\int_F x\times\omega_0 dF-\alpha_x\times\int_F y\times\omega_0 dF+\beta\int_F z\times\omega_0 dF+\int_F\omega_0^2 dF \qquad (13)$$

By substituting into (13) the above-mentioned values of integrals and parameters α_x, α_y, β we'll get:

$$I_\omega=-b^3\times H^2\times t\times\xi^3\times\left[12\times S_2\times(S_1+S_2)+3\times S_1\times(3\times S_1+4\times S_2)\times\xi\right]/6\times A \qquad (14)$$

Epure of the main sector co-ordinates is described by the relation:

$$\omega=\alpha_y\times x-\alpha_x\times y+\beta+\omega_0 \qquad (15)$$

that within the limits of shelf netto cross-section of the channel ($y=-H$; $0\leqslant x\leqslant b\times\xi$) gives:

$$\omega=\left[x\times H\times(A_1\times\xi^2+A)+6\times H\times\xi^2\times(A_2+A_3)\right]/A \qquad (16)$$

By substituting (14) and (16) into (6) we'll determine the distribution of normal stress in the netto cross-section which is under consideration:

$$
\begin{aligned}
\sigma_\omega=&-6\times B_\omega\times\left[x\times(A_1\times\xi^2+A)+6\times\xi^2\times(A_2+A_3)\right]/b^3\times H\times t\times\xi^3\times \\
&\times\left[12\times S_2\times(S_1+S_2)+3\times S_1\times(3\times S_1+4\times S_2)\times\xi\right]
\end{aligned}
\qquad (17)
$$

Let's distribute this linear epure of stresses into constituents which correspond to the tension and pure bending. For this purpose we determine the nominal stresses the crack tip (point 1; $x=b_1=b\times\xi$) and in the angle traverse point of a wall and a shelf of the channel (point 2; $x=0$). After the calculations we get:

$$
\begin{aligned}
&\sigma_1=6\times B_\omega\times\left[2\times S_2\times(S_1+S_2)+S_1\times(4\times S_2+3\times S_1)\times\xi+S_1\times S_2\times\xi^2\right]/ \\
&\quad/6\times H\times S_1\times\xi^2\times\left[4\times S_2\times(S_1+S_2)+S_1\times(4\times S_2+3\times S_1)\times\xi\right] \\
&\sigma_2=-6\times B_\omega\times\left[S_1\times(3\times S_1+4\times S_2)+2\times S_1\times S_2\times\xi\right]/b\times H\times S_1\times\xi\times \\
&\quad\times\left[4\times S_1\times(S_1+S_2)+S_1\times(3\times S_1+4\times S_2)\times\xi\right]
\end{aligned}
\qquad (18)
$$

It's obvious, that the nominal stresses from pure bending $\sigma^{(b)}_{nom}$ and tension $\sigma^{(t)}_{nom}$ are expressed through the received values by the relation relation:

$$\sigma^{(b)}_{nom}=(\sigma_1-\sigma_2)/2 \quad ; \quad \sigma^{(t)}_{nom}=(\sigma_1+\sigma_2)/2 \qquad (19)$$

So,

$$
\begin{aligned}
&\sigma^{(b)}_{nom}=6\times B_\omega\times\left[2\times S_2\times(S_1+S_2)+2\times S_1\times(4\times S_2+3\times S_1)\times\xi+3\times S_1\times S_2\times\xi^2\right]/ \\
&\quad/2\times b\times H\times S_1\times\xi^2\times\left[4\times S_2\times(S_1+S_2)+S_1\times(4\times S_2+3\times S_1)\times\xi\right] \\
&\sigma^{(t)}_{nom}=6\times B_\omega\times\left[2\times S_2\times(S_1+S_2)-S_1\times S_2\times\xi^2\right]/2\times b\times H\times S_1\times\xi^2\times \\
&\quad\times\left[4\times S_2\times(S_1+S_2)+S_1\times(4\times S_2+3\times S_1)\times\xi\right]
\end{aligned}
\qquad (20)
$$

Taking into account that for undefective channel the sectorial moment of resistance acuires value

$$W = b\times H\times S \times(S +3\times S)/6\times(3\times S +2\times S)$$

and transfering from geometrical parameter $\xi=1-\varepsilon$ to $\varepsilon=l/b$ let's represent the dependence(20) in the form:

$$\sigma^{(b)}_{nom} =B_{\omega}\times(3+2\times\lambda)\times\left[(6+\lambda)\times(1+2\times\lambda)-2\times(3+7\times\lambda)\times\varepsilon-3\times\lambda\times\varepsilon^{2}\right]/W_{\omega}\times$$
$$\times 2\times(1-\varepsilon)^{2}\times(3+\lambda)\times\left[(1+2\times\lambda)\times(3+2\times\lambda)-(3+4\times\lambda)\times\varepsilon\right]$$
$$\sigma^{(t)}_{nom}=B_{\omega}\times(3+2\times\lambda)\times\left[\lambda\times(1+2\times\lambda)+2\times\lambda\times\varepsilon-\lambda\times\varepsilon^{2}\right]/W_{\omega}\times 2\times(1-\varepsilon)^{2}\times$$
$$\times(3+\lambda)\times\left[(1+2\times\lambda)\times(3+2\times\lambda)-(3+4\times\lambda)\times\varepsilon\right] \qquad (21)$$

where $\lambda=S_{2}/S_{1}$.

<u>Determination of Calculating Dependence for *SIF*.</u> Above-mentioned results give us approximate solution of the problem for *SIF* determination in the case of channal warping of the edged crack, originating from shelf tip. This solution is represented by:

$$K_{I}=B_{\omega}\times\sqrt{b}\times F(\varepsilon,\lambda)/W_{\omega} \qquad (22)$$

where B_{ω}/W_{ω} - maximum normal stresses from warping; B_{ω} - bimoment, which is transfered through cross-section of cracked channel with crack; W_{ω} -sector moment of the channel resistance; $F(\varepsilon,\lambda)$-correction function, which takes into account the influence of crack sizes and cross-section geometry on *SIF* ; $\lambda=S_{2}/S_{1}=H\times d/b\times t$ - geometrical parameter of channel form; $\varepsilon=l/b$-dimensionless crack length.

Taking into account relations (5) and (21) we'llget for the function $F(\varepsilon,\lambda)$ following expression:

$$F(\varepsilon,\lambda)=\sqrt{2\times tg(\pi\times\varepsilon/2)}\times(3+2\times\lambda)\times\left\{\left[\lambda\times(1+2\times\lambda)+2\times\lambda\times\varepsilon-\lambda\times\varepsilon^{2}\right]\times\left[0,75\times\right.\right.$$
$$\left.\times\lambda+2,02\times\varepsilon+0,37\times(1-Sin(\pi\times\varepsilon/2))^{3}\right]+(1-\varepsilon)^{2}\times\left[(6+\lambda)\times(1+2\times\lambda)-\right.$$
$$\left.\left.-2\times(3+7\times\lambda)\times\varepsilon+3\times\lambda\times\varepsilon^{2}\right]\times\left[0,923+0,199\times(1-Sin(\pi\times\varepsilon/2))^{4}\right]\right\}/ \qquad (23)$$
$$/\left[Cos(\pi\times\varepsilon/2)\times 2\times(3+\lambda)\times(1-\varepsilon)\times(1+2\times\lambda)\times(3+2\times\lambda)-(3+4\times\lambda)\times\varepsilon\right]$$

For standard rolling channel N10 ГОСТ 8240-56(H=100мм, d=4,5мм; b=46мм; t=7,6мм) parameter λ equals value $\lambda=1,28$ and the relation (23) takes the form:

$$F(\varepsilon)=\sqrt{2\times tg(\pi\times\varepsilon/2)}\times\left\{0,149\times(1+0,564\times\varepsilon-0,282\times\varepsilon^{2})\times\left[0,752+2,02\times\varepsilon+\right.\right.$$

$$\left.+0,37\times(1-Sin(\pi\times\varepsilon/2)^{3}\right]/(1-\varepsilon)\times(1-0,412\times\varepsilon)+0,851\times(1-\varepsilon)\times$$
$$\times(1-0,924\times\varepsilon+0,148\times\varepsilon^{2})\times\left[0,923+(1-Sin(\pi\times\varepsilon/2))^{4}\right]/ \qquad (24)$$
$$\left./(1-0,412\times\varepsilon)\right\}/Cos(\pi\times\varepsilon/2)$$

In this case the function $F(\varepsilon)$,mapping the dependence of dimensionless of *SIF* on the crack length, is shown graphically in fig.3.

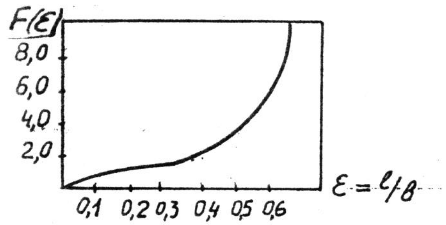


Fig.3.Value of the correction function under the determination of channel's *SIF* with a crack while being loaded B_{ω}.

<u>Experimental Investigation.</u>The idea of the given method is corroborated by special specimens of standard channels, which are loaded according to the scheme of eccentrical bending.

Each specimen is welded into the frame-work, thus its ends are held rigidly.Two horizontal plates,to which testing machine's loadis are being applied,are welded to the middle part of the specimens.These two plates enable us to regulate the magnitude of force eccentricity h(distance between the channel's bending centre and the point of the force application) and desired ratio of torque and bending moment values.The notch,which promotes appearance and gradual growth of fatigue crack, is made in the working part of the specimen,between two plates,at the free edge of channel's lower shelf.The surface of channel's shelf was grinded;measure mark were drawn on it, everything in order to observe crack growth and to check its length visually.

Tests were carried out on the pulsator under the regular loads at frequency of of 300 cycles per minute and asymmetry is 0.1.Conditions of fatigue crack origion with respect to the notch,and also loading conditions where chosen according to the standard method .During the test, the value of applied load and its eccentricity were constant,thus sudden transitions from one loading level to another were eliminated,and these could influence unnatural growth of propagating crack.Crack length l_{i} and the number of loadings N_{i} were consecutively measured in fixed interval of time during the experiment.Values $l_{i}$and N_{i} and load P were used us original data for drawing figures of crack growth and defining the speed of its growth (Fig.4).

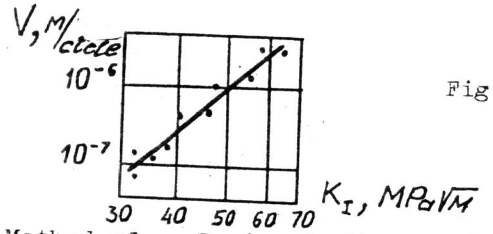


Fig.4. Kinetic diagram of specimen fatigue fracture under given loading conditions.

<u>Method of Designed Structures Analysis with Account of Peculiarities of Warping</u>. Modified method of potential energy minimum in the most loaded structural section of machines, with consideration of actual dynamic loads, having obtained got under natural the conditions, was developed for determining bimoment B_ω real values. Proposed method of dynamic design of supporting system is based on combination of manipulated Langrange's principle and Kastiliano's theorem. Designed structural scheme is based on the principle of final elements. The author introduced the definition and the function of potential energy of warping U_ω, that is the basis and efficiency of the investigation:

$$U_\omega=\Sigma\int B_\omega dS/2\times E\times I_\omega \quad ; \quad \partial U_\omega/\partial B_{\omega i}=0 \qquad (25)$$

Software *"DEPLAN-DINAMIK"* (Rybak et al., 1990), was designed for calculations of differential equation system and obtaining the results in engineering design.

REFERENCES

1. Bychkov D.V., (1962), Structural Mechanics of Thin-walled Bar Structures, [in Russian], Gosstroiizdat, Moscow.
2. Rybak T.I., (1985), Methods of Machine Supporting Capasity and Durability Estimation for Chemical Protection in Plant Groving, [in Russian], Naukova Dumka, Kiev.
3. Rybak T.I., Suhorolski M.A. and Sychca Y.B., (1990), Methodical and Programming Provision of Carrier Systems Dynamic Calculation by Variational Method *"DEPLAN-DINAMIK"*, [in Russian], 20, LPI, Lvov.