STATISTICAL ANALYSIS OF THE SURFACE FATIGUE CRACK GROWTH IN WELDED JOINTS

Y.H. ZHANG

Department of Manufacturing Engineering,
Beijing University of Aeronautics and Astronautics,
Beijing 100083, P.R. China

L.J. WANG

Department of Mechanical Engineering, Tianjin University
Tianjin 300072, P.R. China

ABSTRACT

A method of analysis based on probabilistic theory is presented for statistical modelling of the surface fatigue crack growth(SFCG) at the weld toe. A procedure to estimate parameters in the model is illustrated, using the experimental data of the SFCG at the weld toe obtained from a replicate test program of A131 steel butt joint specimens under constant amplitude loadings. The statistical natures of the SFCG at the weld toe were investigated. The limit state of the surface fatigue crack growth at the weld toe was constructed, and the residual life distribution function for the surface fatigue crack growth at the weld toe is derived.

KEYWORDS

Welded joint, surface fatigue crack, statistical analysis

INTRODUCTION

Most fatigue failure in welded joint are related to the surface fatigue crack growth(SFCG) at the weld toe. Hence, the SFCG analysis is one of the major tasks in the fatigue design and life prediction of the welded structures. Several models based on the principles of fracture mechanics for prediction of fatigue crack growth(FCG) in components and structures under dynamic loads have been proposed, the best known being the Paris–Erdogan law:

$$\frac{dX}{dt} = C(\Delta K)^m \quad (1)$$

in which dX/dt is fatigue crack growth rate, t is time or cycle, X is crack size at t, ΔK is stress intensity factor range, C, m are constants.

The SFCG at the weld toe usually shows considerable statistical variability, because inevitable effects of the welded joint, such as the stress concentration , residual stress, microstructural and mechanical properties of HAZ, ets. , involve characteristically random. The nature of the SFCG has been considered to be a nonequilibrium irrevesible time-dependant kinetic process, that is a stochastic process. The deterministic method is unavailable for this kind of time–dependant fracture. As a result, probabilistic fracture mechanics approches to deel with the FCG have received considerable attention(Prown, 1987) and some probabilistic models have been proposed. Probabilistic models of FCG are random growth laws where C, m are considered to be random variables(Tanaka et al., 1981). Based on the probabilistic model of FCG, we shall present a new model which will for the first time give the statistical distribution of the SFCG at the weld toe.

EXPERIMENTAL DATA

To show the statistical variability of the SFCG at the weld toe, crack growth time histories are given in Fig. 2. These test results were obtained from a replicate test program of A131 steel butt joint subjected to constant amplitude loadings. The specimens used in each test were take from a single butt joint of A131 steel (Fig. 1.). Each test was conducted by the same operation on the same machine. The load range is 300kN and load ratio is 0. 1.

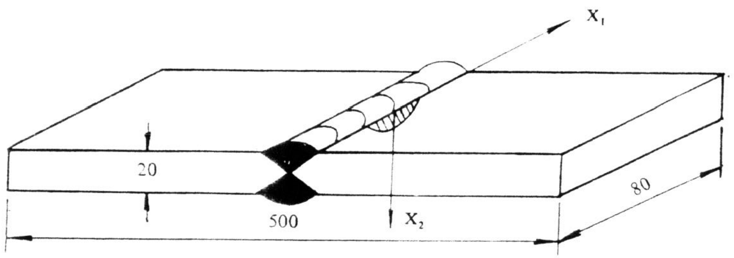
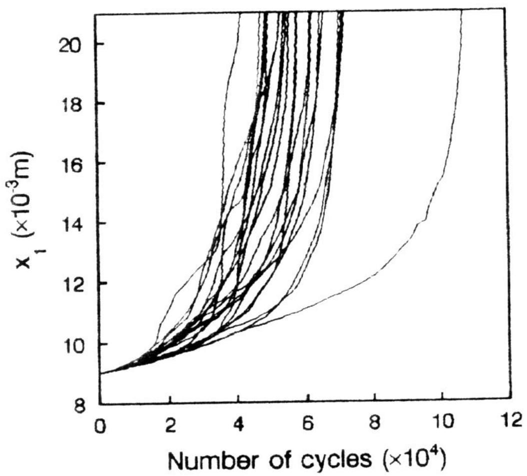
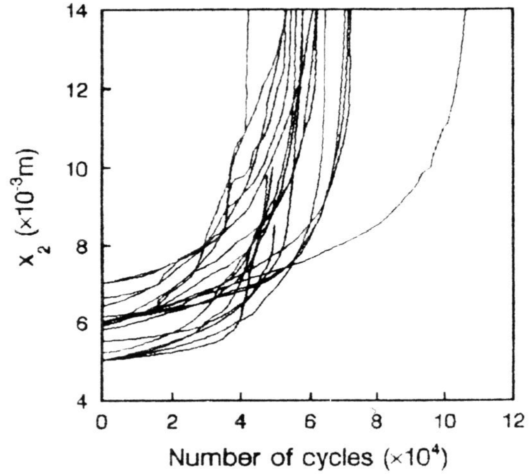


Fig. 1. Butt joint specimen with surface crack.



(a) crack length direction,



(b) crack depth direction.

Fig. 2. Surface fatigue crack growth time histories of Butt joint specimen.

From the fracture mechanics standpoint, the fatigue growth rate of the crack length X_l and depth X_2 is given by the Paris–Erdogan law:

$$\frac{dX_i}{dt} = C_i(\Delta K_i)^{m_i} \qquad (i=1,2) \tag{2}$$

where the stress intensity factor of the crack length and depth can be expressed as:

$$K_i = \alpha_i S\sqrt{\pi X_i} \quad , \tag{3}$$

where α_l is a geometrically related parameter, and S is applied stress. If the specimen size is given, then $\alpha_l = \alpha_l(X_l)$. If introducing Eq.(3) into Eq. (2) one obtains:

$$\frac{dX_i}{dt} = C_i(\Delta S\sqrt{\pi})^{m_i}\alpha_i^{m_i}(X_i)X_i^{m_i/2} \tag{4}$$

indicating that dX_l/dt is function of X_l. The data processing shows that each sample process dX_i/dt vs. X_l relationship can be expressed as follows:

$$\lg\frac{dX_i}{dt} = \lg D_i + n_i \lg X_i \tag{5}$$

or:

$$\frac{dX_i}{dt} = D_i X_i^{n_i} \tag{6}$$

where D_l, n_l are random variables. The SFCG rate data show that dX_2/dt is linealy related to dX_2/dt in given crack size range, i. e. :

$$\frac{dX_2}{dt} = k\frac{dX_1}{dt} \tag{7}$$

where k is a random variable.

STATISTICAL ANALYSIS

Experimental test results show that the SFCG data exhibits considerable statistical variability. Such a variability should be taken into account appropriately in the analysis and design of fatigue–critical welded joints. In particular, in the fatigue reliability analysis of welded structures, statistical is required for the SFCG at the weld toe. In this section, statistical analysis is carried out for parameters of the SFCG model. For each test specimen the parameter $\lg D_l$, n_l and k are estimated using the linear regression analysis. All data sets give a sample of 20 estimates of $\lg D_l$, n_l and k. The normality hypothesis applied to the three variables, $\lg D_l$, n_l and k, cannot be rejected on a 15% significance level by

Kolmogorov–Smirnor(K–S) tests for goodness–of–fit, indicating an excellent fit for the normal distribution.

The correlation between the $\lg D_l$ and n_l is linear, i.e.:

$$\lg D_1 = -6.7984 + 1.9231 n_1 \tag{8}$$

$$\lg D_2 = -6.7934 + 2.0620 n_2 \tag{9}$$

and $\lg D_l$ and $\lg D_2$ or n_l and n_2 are independent.

In order to recognize the basic statistical nature of the SFCG at the weld toe we take n_l as the ensemble average n and D_i as random variable $\tilde{D}_i$. K–S test show that $\lg\tilde{D}_i$ follow the normal distribution, $\tilde{D}_i$ becomes a lognormal variable. Eq.(6) is referred to as the lognormal random varialble model. The mean value and standard deviation for all the parameters are summarized in Table 1.

Table 1. Mean value and standard deviation of parameters.

Parameter	$\lg D_1$	n_1	$\lg D_2$	n_2	$\lg\tilde{D}_1$	$\lg\tilde{D}_2$	k
Mean	3.4265	5.3169	4.1330	5.2989	2.7990	3.4913	0.6338
S.D	1.1979	0.6211	1.4476	0.6896	0.0956	0.2518	0.1679

For the lognormal random variable model, the statistical distribution of the crack growth damage accumulation can be derived analytically as follows.

The distribution function of the lognormal random variable Z is given by:

$$F(z) = P[Z \leqslant z] = \Phi\left[\frac{\log z - E[\log z]}{\sqrt{var[\log z]}}\right] \tag{10}$$

where $Z = D_i$ .The distribution function of crack size $X_i(t)$ at eny service life t can be obtained from that of Z given by Eq.(10) though the integration of Eq.(6). The results are given as follows:

$$F(x_i) = P[X_i(t) \leqslant x_i]$$

$$= \Phi\left[\frac{\log[(x_{0i}^{-q} - x_i^{-q})/qt] - E[\log D_i]}{\sqrt{var[\log D_i]}}\right] \tag{11}$$

in which $q = n - 1$, X_{0i} is the initial crack size.

Let $T(X_c)$be a random variable denoting the time to reach any given crack size,then,the distribution function of $T(X_c)$ can be obtained. The results can be expressed as follows:

$$F(\tau) = P[T \leqslant \tau)]$$

$$= 1 - \Phi\left[\frac{\log((x_{0i}^{-q} - x_c^{-q})/q\tau) - E[\log D_i]}{\sqrt{var[\log D_i]}}\right] \tag{12}$$

RESIDUAL LIFE DISTRIBUTION

In the design based upon the damage telerance principle, it is most important to know the structural components life. If the crack growth process has some uncertain factors, we must treat the structural componts life as a stochastic variable. In the bivariate problem of surface fatigue crack growth, a failure criterion illustrated in Fig. 3. where the shaded area represents the region of failure. The unstable failure takes place when the vector process $X(t)$ grows to arrive at this shaded area for the first time(Tanaka et al., 1989a, b). Let $L(X_1,X_2)=0$ be the limit state function for the growth of the surface crack. If $L(X_1,X_2)<0$, then the component can stand against the general service, when the process arrives at the state $L(X_1,X_2)=0$ for the first time, the unstable failure takes place.

As for the boundary of failure region, we are concerned with the simple circular curve in Fig. 3. The radius of the circulur curve is given as follows:

$$L_c = \frac{x_{c1}^2 + x_{c2}^2}{2x_{c2}} \tag{13}$$

Therefore, the time of the following stochastic process:

$$L(t) = \frac{X_1^2(t) + X_2^2(t)}{2X_2(t)} \tag{14}$$

takes to grow up to L_c for the first time can be considered as the surface crack growth life of the component.

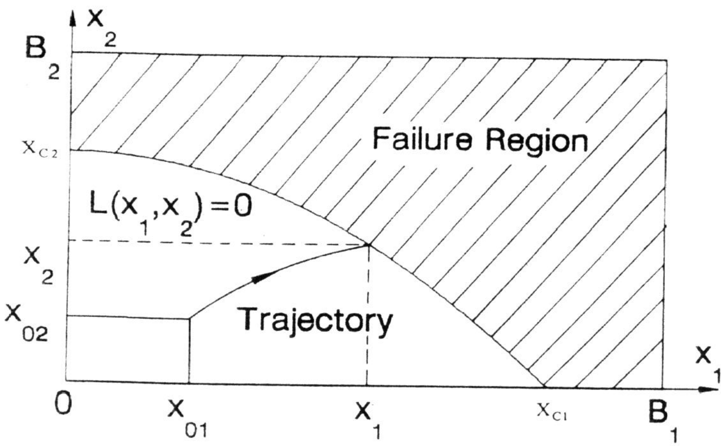


Fig. 3. limit state curve and failure region.

Integration of Eq.(7) yields:

$$X_2(t) = kX_1(t) \tag{15}$$

Then, Eq.(13) and Eq.(14) becomes:

$$L_c = \frac{(1+k^2)x_{c1}}{2a} \tag{16}$$

and:

$$L(t) = \frac{(1+k^2)X_1(t)}{2a} \tag{17}$$

Then, the problem is transformed to the first time of the process $X_1(t)$ growing up to L_{c1}. From the Eq.(12) we can obtain quantities which are of prime interest in fatigue life prediction. For instance, Probability ditribution function of the residual life as follows:

$$H_L(t|X_0,\ X_c) = 1 - \Phi\left[\frac{\log((x_{01}^{-q} - x_{c1}^{-q})/qt) - \mathrm{E}[\log D_1]}{\sqrt{\mathrm{var}[\log D_1]}}\right] \tag{18}$$

The residual life distribution function at different service time for several values of the initial aspect X_0 and limit state are presented in Fig. 4.

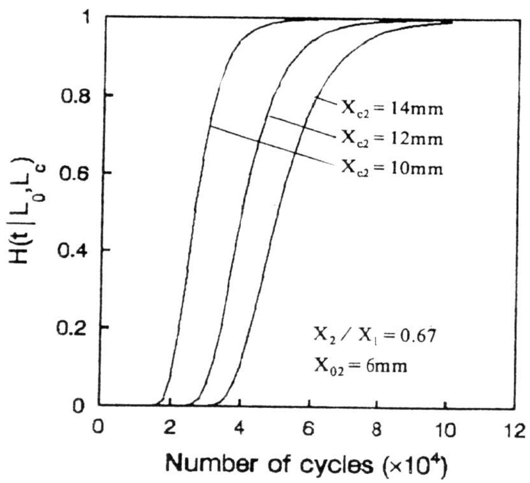


Fig. 4. Residual distribution function

CONCLUSIONS

In this paper, the probabilistic model of the SFCG at the weld toe was investigated based on the test results. The statistical analysis was carried out for parameters of the probabilistic model, using experimental data of the SCFG at the weld toe. The probabilistic model correlates well with the test results. It is very attractive for practical applications due to the reasons: (a) it is mathematically very simple for practical applications including analysis and design requirements, (b) it may reflect closely the SFCG behavior of real welded structures in service.

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