ON STATISTICAL DISTRIBUTION OF FATIGUE CRACK GROWTH AT WELD TOES

L.J. WANG and J.D. KANG

Department of Mechanical Engineering, Tianjin University, Tianjin-300072, P.R. China

Y.H. ZHANG

Department of Mechanical Engineering, Qinghua University, Beijing-100084, P.R. China

ABSTRACT

This paper provides and studies the N versus a curve family which describes statistically the fatigue crack growth at weld toes. A probability compatible condition is given, which relates the distribution of the fatigue life for a given crack size to the distribution of crack size for a given fatigue life and the random initial condition of the fatigue crack propagation. Furthermore, it is confirmed that the depth size a of fatigue cracks at weld toes follows the distribution $\Phi\ ((\mu(a)-\lg N)\ /\ \sigma(a))$ for a given N.

KEYWORDS

Fatigue crack growth, weld toe, statistical analysis, probabilistic distribution, reliability.

INTRODUCTION

The weld toes, generally speaking, act as one of the potential cracking sources in a welded structure. Because of the effect of fatigue loading, some tiny surface cracks will form at this place, and grow in the direction of thickness of plates until penetrating at first the Heat–Affected–Zone (HAZ) and then the base metal. For the past few years, some researchers have discovered that the geometrical parameters of a weld are practically the stochastic variables (Ushirokawa, 1983), and the variation of them influences directly the value of stress intensity factors to a certain extent (Niu and Glinka, 1989). Based on what have been obtained above, it can be deduced that even if there is no change in service conditions, the fatigue cracks which form at the different position along the toe of a weld will practically propagate under the control of different driving forces, especially in the early stage of propagation. No doubt, the uncertainty of this sort in driving forces would make the scatter of fatigue crack growth at weld toes more marked than that of a weldless structure. On the other hand, the complicated mecroscope status and various micro–defects, as a matter of fact, might be considered as the random initial condition of the fatigue crack growth at weld toes. From the point of view suggested above, it is obviously suitable to say that the study of statistical property of the fatigue crack growth at weld toes is one of the branches which could not be neglected in fatigue researches of the welded structures. In this paper, some statistical characteristics of the fatigue crack growth at weld toes are presented with emphasis on the distribution of depth size of fatigue cracks for a given life, which has to be provided in durability or reliability analysis of structures.

STATISTICAL FATIGUE TEST

Specimen. The 25 as–welded specimens tested are made of the A3 steel by use of the manual arc welding. The processing of them and their size are shown schematically in Fig.1. Otherwise, no notch is deliberately prepared before testing at the weld toes of these specimens.

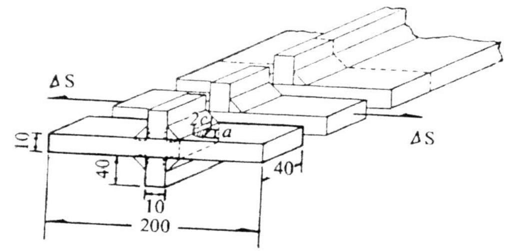

Fig.1. The size and processing of fatigue specimens.

Testing Detail. All the specimens are cyclically tensioned as shown in Fig.1, with the aid of a Hongshan high–frequency fatigue testing machine. The testing conditions used are as follows: Loading frequency $f \approx 140$Hz; Constant stress range $\triangle S = 170$MPa; Cyclic ratio $r \approx 0$; Laboratory temperatuce $T \approx 25$℃.

Testing Results. In testing, the fatigue failure occurs along a certain weld toe of a specimen. The appearance of a fracture surface is presented by a macroscope photograph in Fig.2. According to the fractographic data of crack growth, as indicated by Fig.2 for instance, a series of the depth size a of the fatigue crack growth at weld toes (Fig.1) are secured, and then related respectively to the cyclic number N at that time to obtain discrete points (N_i, a_i) of the crack growth. Afterwards, the McCartney relationship recommended by Duggen and Byrne (1977) is selected to fit the (N_i, a_i) for each specimen , and results of the 25 Specimens are given in Fig.3, which visualy illustrate the statistical scatter of the fatigue crack growth at weld toes in the direction of depth a.

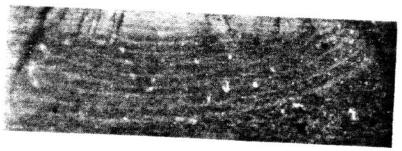

Fig.2. The fracture surface of a specimen

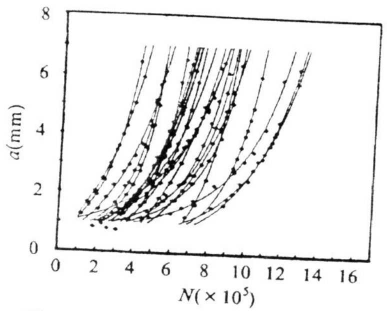

Fig.3. Experimental N vs. a curves

PROBABILITY COMPATIBLE CONDITION

The conditional distribution $F(a|N)$ and $F(N|a)$ involved in Fig.3 are basic statistical information for fatigue reliability design and structure durability analysis. Based on the concept of cycle loading and unloading in the fatigue crack growth, Fu(1988) has proved, with the aid of aggregate method, that the cumulative probability of N for a given a equals the cumulative exceedance probability of a for a give N. In order to reach the conclusion with a more general sense, the following discussion will be carried out on a monotonic increasing curve family, the boundary shape of which is illustrated in Fig.4 in which both N and a are considered as common variables, and the a_u is referred to as the upper bound of a as $N=0$. Through the point ($N^* > 0$, $a^* > 0$) selected arbitrarily, two straight lines, i.e. $N=N^*$ and $a=a^*$, are made to intersect the N vs. a curves respectively. For a certain intersection point on $N=N^*$, if its ordinate satisfies the condition of $a < a^*$, it is called that the random event A occurs. Similarly, if its abscissa satisfies the conditon of $N > N^*$ for a certain intersection point on $a=a^*$, it is called that the random event B occurs (see Fig.4). Now, Let us suppose that one of the sample functions in this curve family is $a=\varphi_j(N)$. If $\varphi_j(N^*) < a^*$ occurs, it is known, according to the property of monotonic increasing of this curve family, that the φ_j^{-1} exists and $\varphi_j^{-1}(a^*) \geqslant N^*$ also occurs certainly. That is to say the occurrence of A induces that of B, namely

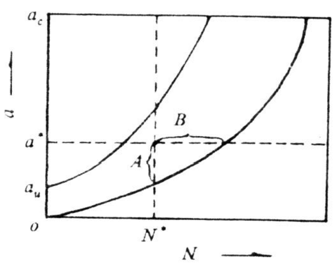

Fig.4. The N vs. a curve family

$$P(A) \leqslant P(B) \tag{1}$$

On the other hand, we can postulate that one of the sample functions is $N=\psi_j(a)$. If $\psi_j(a^*) > N^*$ is true, it is certain that $\psi_j^{-1}(N^*) \leqslant a^*$ is also true according to the same principle. So the occurrence of B results in that of A, namely

$$P(B) \leqslant P(A) \tag{2}$$

From inequality (1) and (2), we obtain

$$P(A) = P(B) \tag{3}$$

Next, substituting the $P(A)$ and $P(B)$ in equation (3) by their corresponding conditional distributions , and meanwhile noticing the arbitrariness of point (N^*, a^*), we set up a probability compatible condition for the curve family in Fig.4 as follows:

$$F(N|a) = \begin{cases} 1 - F(a|N), & a \geqslant a_u \\ F(a|o) - F(a|N), & a < a_u \end{cases} \tag{4}$$

Generally speaking, the eqn (4) is suitable to any random process whose change is similar to that shown in Fig.4. Moreover, it may be known, compared with the testing results shown in Fig.3, that a_c in Fig.4 represents the critical crack depth or thickness of plates, and a_u is correspondent to the upper bound of depth of initial cracks or micro–defects at weld toes. The $F(a|o)$ in eqn(4) can be considered as the random initial condition of the fatigue crack growth.

EXAMPLE

In the durability analysis methodology of aircraft structures (Manning et al., 1987), a definite differential equation with a random initial condition is used to describe the fatigue crack growth in the small crack size region, i.e.

$$da(t)/dt = Q\,[a(t)]^{b},\quad t \geqslant 0;\ a(0) = a_0 \tag{5}$$

where Q and b are parameters, a_0 is a random variable. For the situation of $b = 1$, which is recommended for durability analysis, it is easy to obtain the solution of eqn (5):

$$a(t) = a_0 exp(Qt) \tag{6}$$

Now, it is assumed further that a_0 obeys the exponential distribution being cutted off at its upper bound point a_u, as it may be at weld toes,

$$f(a_0|a_0 \leqslant a_u) = \frac{exp(-\lambda a_0)}{1 - exp(-\lambda a_u)},\quad 0 < a_0 \leqslant a_u, \tag{7}$$

the conditional distribution $F\,[a(t)|t]$ can be obtained as follows:

$$F\,[a(t)|t] = \begin{cases} \dfrac{1 - exp\,[-\lambda a(t) exp(-Qt)]}{1 - exp(-\lambda a_u)}, & 0 < a(t) \leqslant a_u exp(Qt), \\ 1.0, & a(t) > a_u exp(Qt), \end{cases} \tag{8}$$

for $a(t)$ is an increasing function of a_0 for a given time t. Likewise, the $F\,[t|a(t) > a_u]$ will be in the form of

$$F\,[t|a(t) \geqslant a_u] = \begin{cases} 0, & t < [ln(a(t)/a_u)]\,/\,Q, \\ \dfrac{exp\,[-\lambda a(t) exp(-Qt)] - exp(-\lambda a_u)}{1 - exp(-\lambda a_u)}, & t \geqslant \dfrac{1}{Q} ln\dfrac{a(t)}{a_u}, \end{cases} \tag{9}$$

for t decreases monotonously with the increasing of a_0 for a given $a(t) > a_u$. Obviously, the eqn(4) is satisfied.

DISTRIBUTION OF N FOR A GIVEN CRACK SIZE

According to the results of K–S testing, we conclude, for the significance level $\alpha = 0.05$, that the $F(N|a)$ of the fatigue crack growth at weld toes can be roughly expressed by a two–parameter log–normal distribution,

$$F(N|a) = \begin{cases} \Phi\left(\dfrac{lgN - \mu(a)}{\sigma(a)}\right), & N > 0 \\ 0, & N \leqslant 0 \end{cases} \tag{10}$$

where $\Phi(\cdot)$ denotes the standard normal distribution. The samples of N corresponding to several different crack sizes are shown on a two–parameter log–normal probability paper (Fig.5). As we have seen from Fig.5, the sample points of N for each given a approximate respectively a straight line. The performance of $\mu(a)$ and

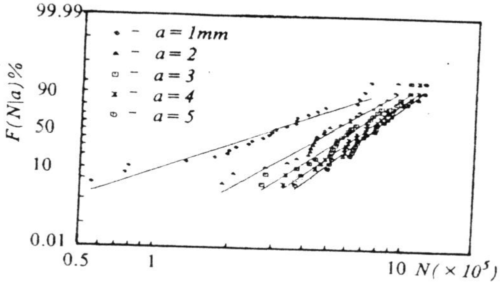

Fig.5. Two–parameter log–normal probability paper

$\sigma(a)$, as the function of a, are shown in Fig.6.

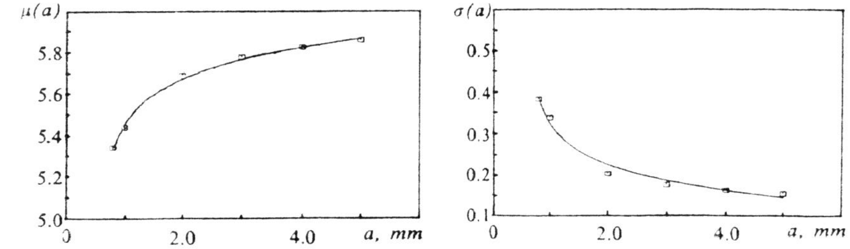

Fig.6. The performance of distribution parameter $\mu(a)$ and $\sigma(a)$

DISTRIBUTION OF A FOR A GIVEN CYCLIC NUMBER

Substituting eqn(10) into eqn(4), we obtain immediately

$$F(a|N) = \Phi((\mu(a) - lgN)\,/\,\sigma(a)),\quad N > 0;\ a \geqslant a_u \tag{11}$$

Hence, it is known that when $F(N|a > a_u)$ is in the form of normal or log–normal distribution, $F(a|N > 0)$ can also be represented by a standard normal distribution. For a parallel straight line family on the plane of N vs. a, $\sigma(a)$ is a constant. At that time, if $\mu(a)$ is the linear function of a, $\log a$ or $\log(a-\eta)$, where η is a constant, the corresponding $F(a|N)$ will be in the form of normal, two–parameter or three–parameter log–normal distribution respectively. However, since the $\sigma(a)$ in Fig.6 is a nonlinear function of a, what we have discussed above is not true to the growth of fatigue crack at weld toes. Otherwise, it is learned from eqn(11) that the $F(a|N)$ describes a straight line on a probability paper where the $(\mu(a) - \lg N)\,/\,\sigma(a)$ is selected as the abscissa and the $\Phi(\cdot)$ as the ordinate. So, in order to examine the expression of eqn(11), let $N = 3.5 \times 10^5$ and $N = 4.5 \times 10^5$ cycles respectively. It is found that if no extrapolation is made for experimental curves in Fig.3, two straight lines $N = 3.5 \times 10^5$ and $N = 4.5 \times 10^5$ can not intersect with some of those curves. As a matter of fact, it is impossible to obtain all the 25 sample points of a for a given $N > 0$ in Fig.3. Consequently, the examination for eqn(11) can only be done on the partial curves in Fig.3. Ten curves in Fig.3 are chosen, whose ordinates of the intersection points with the straight lines $N = 3.5 \times 10^5$ and $N = 4.5 \times 10^5$ satisfy the condition of $a > 1$mm. For the ten curves, we obtain

$$\begin{cases} \hat{\mu}(a) = 0.3741 lg(a - 0.880) + 5.5120 \\ \hat{\sigma}(a) = -0.014 lg(a - 0.999) + 0.1110 \end{cases} \tag{12}$$

The $F(a|N)$, in which the $\mu(a)$ and $\sigma(a)$ are substituted respectively by the $\hat{\mu}(a)$ and $\hat{\sigma}(a)$ in eqn(12), and ten sample points of a for the given $N = 3.5 \times 10^5$ and 4.5×10^5 are depicted on their corresponding probability paper as shown in Fig.7. It is obvious, according to the results in Fig.7, that the depth size a of the fatigue crack growth at weld toes for a given N follows the distribution $\Phi((\mu(a) - \lg N)\,/\,\sigma(a))$, which is also supported by the results of K–S testing for the significance level $\alpha = 0.05$.

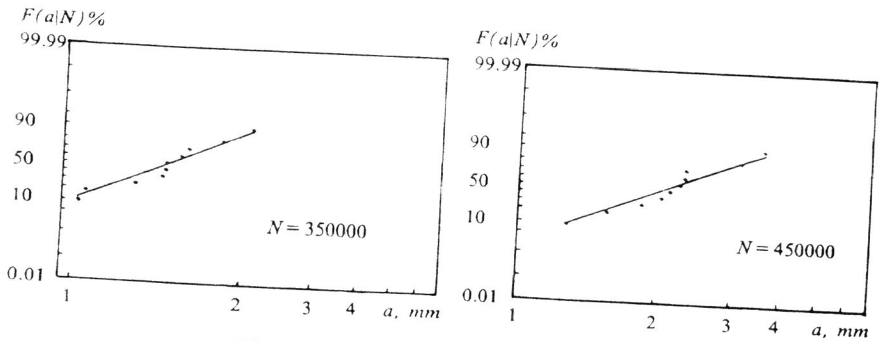

Fig.7. $\Phi((\mu(a)-\lg N)\,/\,\sigma(a))$ probability paper

CONCLUSIONS

A probability compatible condition (4) has been set up for the statistical fatigue crack growth, which relates the distribution of fatigue life for a given crack size to the distribution of crack size for a given fatigue life and the random initial condition of the fatigue crack growth.

It is confirmed that the depth size a of the fatigue crack growth at weld toes for a given cyclic life N follows the distribution $\Phi((\mu(a)-\lg N)\,/\,\sigma(a))$, in which both $\mu(a)$ and $\sigma(a)$ are the nonlinear function of a.

REFERENCES

Duggen T.V. and Byrne J.(1977), Fatigue as a Design Criterion , the Gresham Press, London.

Fu H.M. and Gao Z. T. (1988), Distribution of Crack Sizes at a Given Life, Acta Aeronautica et Astronautica Sinica, 9(3), A113–A118.

Manning S.D. , Yang J. N. and Rudd J. L. (1987), Durability of Aircraft Structures, in Probability Fracture Mechanics and Reliability (Edited by Provan J. W.), 213–267, Martinus Nijhoff Publishers, Dordrecht, The Netherlands.

Niu X. and Glinka G. (1989), Stress–Intensity Factors for Semi–Elliptical Surface Cracks in Welded Joints, Int. J. Fract. , 40, 255–270.

Ushirokawa O. (1983), Stress Concentration Factor at Welded Joints –2nd Report Probabilistic Approach–,Ishikawajima–Harima Engineering Review,24(2), 98–103.