ON SEQUENCE EFFECTS IN RANDOM-LOAD FATIGUE

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ABSTRACT

Fatigue behaviour of cyclic hardening aluminium alloy under two-level strain control is investigated. The load-history-sensitive model of cyclic curve is suggested. The model characterizes cyclic instability of material as well as the influence of loading history. Computer-based algorithm for generation of random load sequence is developed for the given form and parameters of amplitudes distribution and autocorrelation properties of a process. A numerical example of fatigue damage summation based on the load-history-sensitive cyclic curve model is presented.

KEYWORDS

Random loading, fatigue damage accumulation, load-history-sensitive cyclic curve, cyclic hardening material.

The accuracy of structures' fatigue life predictions depends on adequacy of mathematical and physical models of fatigue damage accumulation under irregular loading. The well known linear damage hypothesis use leads to the considerable scatter of the results comparing to the experimental data (Swanson,1968). One of the sources of discrepancies is that the effects of loading history are not being taken into account in the analyses.

To improve the fatigue model a new approach to fatigue damage analyses is proposed. The principal means of it are seen in load-history-sensitive cyclic curve introduction. To obtain its parameters the experimental results of cyclic and program load tests were used.

The hourglass specimens with 6 mm diameter of cyclically hardened aluminium alloy were tested under strain control in constant and program conditions. The loading program consisted of two-level blocks with lo-hi succession within every of blocks. Mechanical properties of the alloy under tensile conditions are as follows: ultimate strength = 401.8 MPa; yield stress = 294.0 MPa; modulus of elasticity = 71000 MPa; elongation = 11 %. At cyclic and programmed tests transverse strains at the gage part of the specimen along with axial loads were recorded periodically according to cyclic hardening law and changes in load amplitudes. These recordings were used to calculate the plastic hysteresis loops in axial stress-strain format. The aim of the experiments was to study the stage of fatigue crack initiation, therefore the 20 % tensile load drop was recognised as the material failure.

The response of the material has been analysed on the load changes in low-to-high and high-to-low sequences. The results of program tests revealed the particular character of strain hardening. The cyclic hardening process at higher level of the program was less pronounced than at the same strain level in cyclic tests. At the lower level of the program the cyclic hardening was more active as a result of prestrains. This result can be explained by the influence of residual microstresses induced at changes of load conditions and by the process of its relaxation. It represents macroscopically what is usually known as the load history effects. It should be noted that qualitatively similar cyclic response of the materials of other cyclic properties, softened and stable, has been displayed by Manson (1965), Topper et al. (1968) and others.

The experimental data analysis was used to develop the model of load-history-sensitive cyclic curve. The model is based on a set of transient stress-strain curves to describe the cyclic response of the material considering the hardening and the load history dependence. It implies that the strain range is an unique function of the current stress range and the transient cyclic state. The model is presented in a typical form, i.e. as a sum of elastic and plastic strain ranges, the later in a current load reversal depends on the cyclic hardening parameters and the load history:

$$\Delta e_k = \Delta e_{ek} + \Delta e_{pk} =$$

$$= (\Delta s_k / E)(1 + M m_1 (\Delta s_k / \Delta s_c - 1)^{m_2}), \qquad (1)$$

where, k = number of cycle reversals; e_k and s_k are strain and stress ranges in current load reversal "k" ; E = Young's modulus; M = plastic strain correction factor considering over- and underload sensibility of the material; m = an average correction for relation between plastic and elastic components of the strain range in load reversal; Δs_c = stress range corresponding to cyclic proportionality limit at "k"-th reversal; m_2 = loop form parameter. As follows from analysis of tests data the parameter m_2 may be assumed as independent on the loading type: cyclic or program loading. The influence of the cyclic hardening on the inelastic strain in the model is covered by Δs_c which increases with the number of reversals.

Correction M is a ratio of plastic strain range at program test to plastic strain range during the cyclic test of the same load amplitude level. At constant amplitude loading $M = 1$. The resulting form of correction factor for the material discussed:

$$M = k^m \qquad (2)$$

where, m is the function of the ratio $\Delta e_k / \Delta e_{k-1}$.

The mathematical formulation of cyclic curve under the program loading (1) is suitable for a computer based simulation of strain response of a material. It may be introduced into the common procedure of fatigue evaluation based on the linear hypothesis. Involving the proposed corrections to cyclic curve considering the transient cyclic state and the dependence on load history this procedure may be represented as:

- generation of random load sequence;
- editing of load sequence to evolve of its "damaging" part;
- strain calculations in each load reversal according to cyclic curve model;
- determination of cycles number to failure using the strain versus life curves, for example, Coffin criterion $e_p = C N^{-\alpha}$;
- calculation of damage in a halfcycle defined as the ratio $d_k = 1 / N$,
- damage summation, $D = \Sigma d_k$.

According to this scheme the damage of the above material for loading sequence used was estimated. The results were compared with the experimental data and with the cumulative damage values which were calculated under the suggestion that the fatigue damage could be measured in terms of inelastic strain energy. The identical kinetics of calculated cumulative damage is seen either obtained using the suggested model or inelastic strain energy summation. Close agreement was found between the estimated cumulative damage and experimental data (Yermolayeva, 1990).

As illustrative example to the discussed above the fatigue of the material under random loading has been predicted in case of load sequence modeled by blocks of stationary random process typical to ship-hull wave loads. Its amplitudes are distributed according to Rayleigh law. The corresponding density probability function of load amplitudes is:

$$p(\zeta) = \zeta \exp(-\zeta^2 / 2) , \qquad (3)$$

where, $\zeta = s/s_s$ = standardized stress amplitude; s_s = stress deviation standard. The correlation properties of the process are presented by the standard autocorrelation function (Price and Bishop, 1974):

$$\rho(\tau) = \exp(-\alpha |\tau|) \cos \beta\tau , \qquad (4)$$

where, τ = time interval, sec ; α, β = parameters which depend on sea dynamics. Approximately, α = 0.383 1/sec and β = 1.0 1/sec.

On the basis of autocorrelation function the random load sequence has been developed using the procedures of inversion and superposition of statistical modelling (Bendat and Piersol, 1971; Shalygin and Palaghin, 1986). The autocorrelation function in form (4) corresponds to the density probability

$$f(x) = \sum_{i=1}^{n} p_i f_i(x) , \qquad (5)$$

where, $n = 2$; $p_i = 0.5$; $\sum_{i=1}^{n} p_i = 1$; p_i = probability of discrete value "i" appearence. Density probability function consists of two Cauchy distributions,

$$f(x) = \alpha / \pi (\alpha^2 + (x - b)^2) , \qquad (6)$$

which differs one of the other by the shift parameters $b = \pm\beta$. One may use the superposition technique for modelling of random sequence meeting the probability distribution function in form (5). The modelling is carried out in two steps. At first, the discrete random instantiation, where the quantities are equal to the number of the event 1,...,n according to probability p_k , is performed. As the result, the value of "k" is obtained. Afterwards the random quantity corresponding to probability distribution function $f_k(x)$ is computed. This value is accepted as "ζ" in the inversion statistical procedure. On the basis of random instantiation corresponding to arbitrary distribution function it allows to generate the random sequence with desired distribution law

$$F(x) = \int_0^x f(x)\, dx .$$

If the standard independent random quantities distributed uniformly at interval [0,1] - γ = rnd (), rnd () = firmware random values generator, then it is necessary to apply the nonlinear transformation:

$$\zeta = F^{-1}(\gamma) . \qquad (7)$$

It means the solving of equation:

$$F(\zeta) = \gamma . \qquad (8)$$

From (4) - (8) follows the model algorithm:

$$\zeta = \alpha\, tg(\pi(\gamma - 0.5)) + \beta æ , \qquad (9)$$

where, α, β = approximation coefficients of standard autocorrelation function (4); γ = rnd () (see eq.(7)); $æ = \pm 1$, with equal probability.

To verify the equivalence hypothesis of density probability function of random sequence generated according to the model algorithm (9) and theoretical Rayleigh distribution the Pirson χ^2 - criterion was used. The loading sequence computed according to the algorithm (9) is given in Fig.1. Its amplitudes' distribution meets the Rayleigh law with the level of equivalence of 0.1 . It was assumed that the stresses below the cyclic proportionality limit do not contribute in the damage accumulation.

Two $\Delta e - N$ equations have been used in the illustrative example. One of it

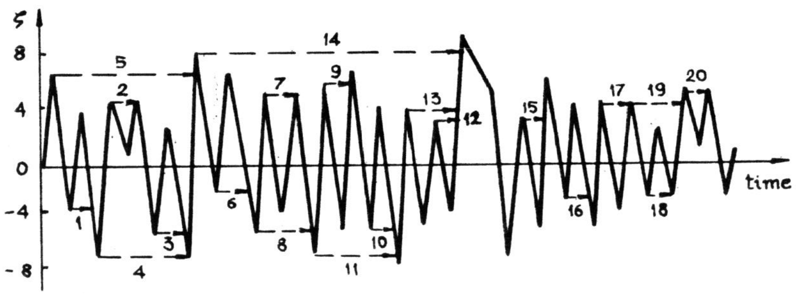

Fig.1. Loading sequence. Dashed lines show schematicaly "rainflow" method of closed loops counting.

in low-cycle range is developed in accordance with regression equation which has been derived from above constant amplitude cyclic tests. For high-cycle range the reference data for aluminium alloy of the same chemical composition and mechanical properties is used (Troshchenko and Sosnovskij, 1987).

The fatigue damage was calculated in terms of cycle-by-cycle technique on the basis of linear damage rule and the load-history sensitive curve using Δe - N equations. It represents the model of damage accumulation under the random loading sequence generated according to (9), (Fig.2). As seen from Fig.2 the damage accumulation is nonlinear which means that application of developed model permits to consider the real character of damage accumulation within the linear rule.

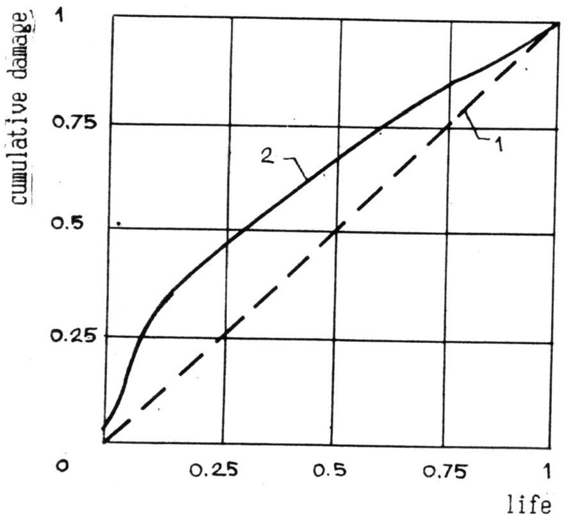

Fig.2. The rate of accumulation: 1 - conventional linear rule; 2 - proposed method.

It is reasonable to compare the above calculated value to number of loadings to failure of the material in conventional form:

$$N = 1 \Big/ \int_{s_{min}}^{s_{max}} (p(s) / N(s))\, ds \; ; \qquad (10)$$

where, s_{min} = minimum of $\Delta s/2$ according to the results of program tests; $s_{max} = \infty$; p(s) = the density probability function of stress amplitudes distribution (Rayleigh law); N(s) = cycles number to failure from the strain criteria. In this case a generalized cyclic curve corresponding to 40 % of life from constant amplitude tests is used. Predicted value of fatigue life of the material according to eq.(10) is N = 49000, while the suggested in this report model (1) which takes into account history effects gives number of reversals to failure equal to 21737.

To conclude, the fatigue life estimations based on fatigue behaviour of a material under the irregular loadings might be more precise if the load-history sensitive curve is used. For the purposes of engineering analysis the linear damage rule including proposed in this paper corrections remains as useful and reasonable technique.

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