

NEW METHODS OF MICRO- AND MACROCRACK RETARDATION

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ABSTRACT

A method is proposed to determine the chemical composition of a material with given retardation characteristics of the micro- and macrocrack growth which is based on the Geoffriono-optimum and Pareto optimum solutions of multi-criteria problems. Some methods are proposed which provide the optimization of the shapes of three-dimensional bodies featured by a maximum resistance to the initiation and growth of fatigue cracks and which use a specific strain energy equalization algorithm and the Pontryagin maximum principle. An efficient strengthening of structural components is proposed which prevents the crack initiation from stress concentrators (hole and large cutout edges, riveted and bolted junctions, lugs, welded spots and joints, stringer and stiffening rib interruptions). Some methods of designing fail-safe optimum structures are proposed.

KEYWORDS

Material optimization, crack growth retardation, strengthening methods, shape optimization, structure optimization.

METHOD OF THE DETERMINATION OF CHEMICAL COMPOSITION AND MANUFACTURING PROCESS OF MATERIALS HAVING SPECIFIED OPTIMUM MICRO- AND MACROCRACK RETARDATION CHARACTERISTICS

Let chemical compositions and manufacturing processes of material-prototypes and V_i ($i=1,2,\dots,m$) of quality criteria for each material be known. The quality criteria may be most diverse and be dependent on such operational requirements for the materials being considered as: yield strength, fracture toughness, volume microcrack initiation energy density, cold-brittleness (for rail steels and steels used for north-construction bridges, S-N curve parameters, surface flaking (for bearing steels), wear resistance, hardness (for cutting

and boring tools), resistance to radioactive radiation and thermocycling (for steels used for nuclear reactors), corrosion properties (for steels used in petroleum chemistry), crack resistance, creep resistance (for steels used for turbine blades), microcrack initiation resistance (for ceramics used in engine manufacturing), superconductivity temperature (for superconducting materials). Let us consider the problem of determining the chemical composition and manufacturing process of a new (T+1)-th material that consists of components having the chemical compositions of the prototypes and that is manufactured using some fragments of the prototype manufacturing processes. This material is featured by a given property combination, and it is proposed to estimate how close is the material being designed to the above property combination using the Geoffriono-optimum criterion L_1 (Lagutin, 1991, b):

$$L_1 = \left\{ \sum_{i=1}^m (1 - V_i / V_{s,i})^2 \right\}^{1/2} = \min \quad (1)$$

where $V_{s,i}$ is the value of the i -th criterion which is desirable to have in the i -th material being designed; V_i is the current value of the i -th quality criterion in the course of the optimum material selection. It is possible to show that V can be expressed as a function of the generalized parameter B that depends on the chemical composition of the prototype materials in question and their manufacturing processes:

$$V_i = V_i(B) \quad (2)$$

Substituting (2) into (1), after the optimization, we shall determine a sought optimum value of $B = B_{opt}$, an optimum chemical composition and an optimum manufacturing process of the (T+1)-th material corresponding to this value. We shall show how it is possible to find V_i , provided that V_i is a parameter of the S-N curve. Assume that there is a critical structural component that takes up loads of γ levels during its service. The application of n_j cycles at the stress σ_j corresponds to each j -th level of the γ load levels. Let be $n_1 > n_2 > \dots > n_j > \dots > n_\gamma$ at $\sigma_1 < \sigma_2 < \dots < \sigma_j < \dots < \sigma_\gamma$. In accordance with the hypothesis of the linear damage summation we have

$$\xi = \sum_{j=1}^{\gamma} n_j / N_j \quad (3)$$

where ξ is the measure of damage accumulation; N_j is the limiting number of the cycles of the application of σ_j resulting in the structure component failure. The development of a material featured by the most advantageous S-N curve means the determination of those values of N_j , that minimize (3), at which the area under the S-N curve remains constant

$$\sum_{j=1}^{\gamma} \Delta_j N_j = \text{const} \quad (4)$$

where $\Delta_j = \sigma_j - \sigma_{j-1}$ at $2 \leq j \leq \gamma$; $\Delta_j = \sigma_j$ at $j=1$. The above problem is solved as follows

$$N_j = \sqrt{n_j / \Delta_j} \left(\text{const} / \sum_{j=1}^{\gamma} \sqrt{\Delta_j n_j} \right) \quad (5)$$

Instead of the criterion L_1 (Lagutin, 1986, b) it is possible to use the following Pareto-optimum criterion L_2 :

$$L_2 = \prod_{i=1}^m V_i(B) = \max$$

for which it is not obligatory to know the values of $V_{s,i}$. Note that at a finite value of m the optimum versions obtained by using L_1 and L_2 coincide with each other.

METHOD OF STRUCTURAL COMPONENT STRENGTHENING FOR RETARDATION OF MICROCRACK NEAR STRESS CONCENTRATORS

The method of strengthening in question was proposed in order to increase fatigue resistance of hole edges (Lagutin, 1982, b, 1989, a, b) and stringer interruption zones (Lagutin, 1982, a). But it turned out that this method can also be modified for an efficient strengthening of other stress concentrators (see Table 1, where σ is the specimen's one-axial tension stress, * is the operational load, and \rightarrow means the failure absence in test sections).

Table 1. Experimental verification of proposed strengthening methods

N Stress concentrator type	Strengthening effect as compared to non-strengthened specimens		
	Fatigue resistance increase (number of times)	Static strength increase (%)	Time increase up to creep failure (number of times)
1. Drain hole in integral aircraft solid aluminium-alloy panel stringer	10.75	at $\sigma = 207$ MPa	
2. Hole with cone cavity in aluminium-alloy specimen	5.15	at $\sigma = 294$ MPa	
3. Repair version of hole edge strengthening (with cutting of microcrack-accumulated layer) in aluminium-alloy specimen	5.0	at $\sigma = 200$ MPa	
4. Large hole (≈ 0.08 M) with a nonsymmetric collar in aluminium-alloy spar wall of maneuverable aircraft	12.0	at *	
5. Large oval cutout in aluminium-alloy spar wall of passenger aircraft	13.5	at *	
6. Welded butt connection of aluminium-lithium alloy specimen	11.0	at $\sigma = 200$ MPa	

7. Welded spot in specimens of steel KVK-32		6.22	
8. Lug in aluminium-lithium alloy specimen	4.0 at $\sigma=120$ MPa		
9. Lug in VT-20 titanium alloy specimen	6.4 at $\sigma=560$ MPa		
10. VT-18 titanium alloy specimen surface		8.0 at $\sigma=600$ MPa T=500° C	
11. VT-19 titanium alloy specimen surface		6.2 at $\sigma=800$ MPa T=450° C	

Note that the efficiency of the proposed modifications of strengthening methods N 1 and N 4 exceeded the American strengthening method, consisting in size-reduction of the hole by tube punches, by the factors of 6.7 and 5.2 respectively. The application of the strengthening methods, listed in Table 1, to conventional structural alloys does not result in a structural component mass increase and it is equivalent, in its strengthening effect, to the development of new alloys featured by an increased crack resistance. This is attributed mainly not only to favorable residual stresses but also to sinergetic structures in the stress concentration zone which are formed in the course of material strengthening. It is especially characteristic of the modifications N 10 and N 11 (see Table 1), where sheet structural components are reinforced by increased-strength zones being formed as a result of local strengthening (Lagutin et al, 1991, d). The modification N 1 was also used for strengthening stringer drain holes, riveted airliner wing panels (the application of the American method turned out to be impossible due to the presence of a fillet transition from the stringer attachment flange to its wall wich prevented the indentation of tube punches in the vicinity of drain holes). After the application of the proposed strengthening method there was no failure of a specimen in its test section (that is, starting from strenghtened edges of the drain hole). Therefore, the calculated residual stresses resulted from strengthening (Lagutin, 1989, b) were used to estimate an optimum relative volume of the material pressed out during strengthening $W=W_1/W_2$ (where W_1 is the volume of the punches pressed into the stringer, W_2 is the volume of the drain hole). The effect of the growth of two initial typical through cracks of the length of $l=0.1 a$ (where a is the drain hole radius), which go from drain hole edges, on the residual stress variation was simulated by cutting cylindrical metal layers from the srenghened hole surface. It turned out that if the detected crack length is equal to 0.8 a, the ratio $W=0.1$ is the most advantageous and this value was recommended to implement the proposed strengthening method in the aircraft construction.

OPTIMIZATION METHODS OF BODY SHAPES FEATURED BY A MAXIMUM RESISTANCE TO MICROCRACK INITIATION AND INITIAL TYPICAL THREE-DIMENSIONAL CRACK GROWTH

The first method provides the detemination of such a shape of a three-dimensional body when the specific strain energy becomes the same for the whole body volume, and therefore the conditions of a retarded microcrack initiation are provided in a volume unit. In this case, the following formulae of the iterative redistribution of the load-carrying material (Lagutin, 1980, 1986, a) are used:

$$h_k = S_k^{-1} \sum_{i=1}^k [R_{N,N,i} (V - \sum_{l=1}^{i-1} V_{N,l}) / \sum_{l=1}^i R_{N,N,l}]$$

where h_k is the varying height in the i-th prism, into which the body is divided; S_k is the base area of the i-th prism; V is the body volume; $\sum_{l=1}^i V_{N,l}$ is the volume of prisms whose heights are not varied because of the necessity of satisfying the boundary conditions imposed on the body; $R_{N,N,l}$ is the convolution

$$R_{N,N,l} = 2(A+B+C)/E$$

$$A = \sum_{j=1}^3 (U_{N,N,i,j} / \epsilon_{N,N,i,j})$$

$$B = -2M [U_{N,N,i,1} U_{N,N,i,2} / (\epsilon_{N,N,i,1} \epsilon_{N,N,i,2}) + U_{N,N,i,2} U_{N,N,i,3} / (\epsilon_{N,N,i,2} \epsilon_{N,N,i,3}) + U_{N,N,i,3} U_{N,N,i,1} / (\epsilon_{N,N,i,3} \epsilon_{N,N,i,1})]$$

$$C = 2(1+\mu) \sum_{j=1}^3 (U_{N,N,i,j}^2 / \epsilon_{N,N,i,j}^2)$$

where E, μ is the modulus of elasticity and the Poisson's ratio. The proposed method is aimed at the application of existing finite-element programs intended to determine strains and stresses in three-dimensional bodies and it is of interest for the selection of optimum shapes of high-rise constructions, crack-resistant dams, mines and cavities, stiffening rib ports and interruptions. This method can also be used to determine optimum locations and diameters of fasteners in a joint unit of a maximum rigidity and to prove the convergence of the optimization process using the Cauchy inequality (Lagutin, 1986, a). Note that the locations of the fasteners in the unit and their diameters are quite different if the fault accumulation rate is used as an objective function being minimized.

The second method provides the selection of such a shape of a three-dimensional body (e.g., thickness and width of a central through-hole structural component under cyclic tension) when the dissipation of the energy spent for the growth of an initial typical corner crack (as a 1/4 portion of an ellipse) at the hole exit on the structural component surface during n loading cycles is minimum

$$U = \int_0^{n_1} U_n dn = \min \tag{6}$$

The minimization (6) at known differential equations for crack growth rates at two points that fully determine its shape, $dx_1/dn=f_1$; $dx_2/dn=f_2$; is equivalent to the maximization of the Hamiltonian functions:

$$H = \psi_0 U_n + \psi_1 f_1 + \psi_2 f_2 = \max$$

when the Hamiltonian canonical equations are satisfied:

$$d\psi/dn = -CH/\sigma x \quad (i=0,1,2)$$

$$dx_i/dn = CH/\sigma \psi_i; \quad dx_2/dn = CH/\sigma \psi_2$$

where auxiliary variables ψ_0, ψ_1, ψ_2 are nonzero continuous functions that, in the last loading cycle, assume the following values: $\psi_0(n) = -1; \psi_1(n) = 0; \psi_2(n) = 0$.

The proposed approach, under using the Pontryagin maximum principle, reduces this problem to a time-dependent problem of the theory of optimum control with a fixed left and a free right curve ends for a fixed time interval when a particular case of the piece-constant control, is used. The Pontryagin maximum principle used in the optimization of the body shape makes it possible to avoid a tedious numerical determination of the three-dimensional crack failure front location in each loading cycle. If some constraints in the form of inequalities are imposed on the phase coordinates x and x it is necessary to use a more complex Pontryagin maximum principle. It is possible to show that the optimum control change-overs, typical for the maximum principle, suggest an efficient solution of the urgent problem of the crack resistance increase of the welded butt connection of two pipelines made of different-modulus materials used in atomic power plants.

DESIGNING METHODS OF FAIL-SAFE OPTIMUM STRUCTURES

The first method is referred to the problem of the synthesis of optimum structures. The problem of the optimization of pressure shell cross-section shapes of stiffener locations and sizes (stringers, frames, crack stoppers) in the presence of about two dozens of varying design parameters) and about a hundred of constraints in the forms of inequalities which include the requirements for a fail-safe service in the presence of typical crack and holes, the requirements for static strength and buckling of structural components (flight loading and emergency water landing), the requirements for pressure shell rigidity and cross-section dimensions, the requirements for natural mode frequencies of the pressure structure skin sections that are subject to acoustic exposures of operating engines, consists in the selection of the method of a nonlinear mathematical programming that is capable to solve the above problem of the synthesizing of an optimum structure. To solve this problem, it is proposed (Lagutin, 1990) to apply a combination of the method of internal logarithmic penalty functions with the method of a coordinate descent, where the variation step of variables is found by using the Fibonacci numbers, which is their new application for optimization problems in the presence of many variables. The required coefficients of tension and compression of the variation step for each variable which were calculated by using the Fibonacci numbers turned out to be equal to 1.236 and 0.764, respectively (Lagutin, 1990). The numerical investigations confirmed that these values provided the most decrease in the objective function, i.e. the structure mass, as compared both to other tension-compression coefficients and other methods of nonlinear programming, i.e., the method of deformable polyhedron and the method of steepest descent. The above approach is implemented in the structure

optimization program "SPURT" (Lagutin, 1990).

It is suggested to estimate the effect of stress biaxiality σ_y, σ_z on stress intensity coefficient K_I at the 2 l - long crack being under tension σ_y in the pressure structure skin using the following formula (Lagutin et al., 1991, a):

$$K_I(\sigma_x, \sigma_y) = \sigma_y \left(\pi \left[1 + 1/2 \left(\sigma_y/\sigma_m + \sigma_z/\sigma_m \right)^2 \right] \right)^{1/2}$$

where σ_m is the skin material yield limit, σ_c is the stress acting along the crack sides.

It is suggested to estimate the stress σ_L at the tips of the Paris's crack using a modified formula of Kolosov-Inglis (Lagutin, 1991, e):

$$\sigma_L = \sigma_y \left[1 + 2(GV/\Delta l)^{1/2} \right]$$

where G, V are the parameters depending on the ordinate axis crack position and the slope angle of the second section of the kinetic fatigue diagram; l and Δl are the length and the length increment of the crack, and σ_y is the stress.

The second method is applied to the selection, at a fixed mass M of the load-carrying material, of a maximum-rigidity structure consisting of membrane finite elements, in each of them a typical crack, whose length does not exceed 1/7 of the diameter of the circumference inscribed into a finite element, can initiate. In this case, it is proposed to combine the problem of the selection of optimum thicknesses of each finite element with the problem of the selection of optimum (from the standpoint of failure mechanics, static strength, buckling and rigidity) moduli of elasticity of each finite element among deformable aluminium alloys (m=1), titanium alloys (m=2) and structural steels (m=3), since there is a linear dependence between the modulus of elasticity $E_{N+1}^{k,m}$ and the density $\rho_{N+1}^{k,m}$ with an accuracy of 5%

$$\rho_{N+1}^{k,m} = a E_{N+1}^{k,m}$$

where N is the iteration number; m the order number in the k-th material family which makes it possible to introduce a new variable $t_{N+1}^{k,m} = \sigma_{N+1}^{k,m} E_{N+1}^{k,m} / l$.

In this case, the problem is reduced to the minimization of the strain energy

$$\sum_{i=1}^n (R_{N+1}^{k,m})^2 S_i / 2 t_{N+1}^{k,m} = \min$$

in variables $t_{N+1}^{k,m}$ under the isoperimeter condition

$$\sum_{i=1}^n t_{N+1}^{k,m} S_i = M/a \cdot l$$

and constraints in the form of inequalities:

$$t_{N+1}^{k,m} \geq E_{N+1}^{k,m}$$

where

$$\begin{aligned} t_{N+1}^{1,m} &= \min && \text{from } \Delta E_{N+1}^{1,m} : \Delta E_{N+1}^{2,m} : \Delta E_{N+1}^{3,m} \\ \Delta_{N+1}^{1,m} &= \min && \text{from } \max \sigma_{N+1}^{1,m} : \sigma_{N+1}^{2,m} : \sigma_{N+1}^{3,m} \\ \Delta_{N+1}^{2,m} &= \min && \text{from } \max \sigma_{N+1}^{1,m} : \sigma_{N+1}^{2,m} : \sigma_{N+1}^{3,m} \\ \Delta_{N+1}^{3,m} &= \min && \text{from } \max \sigma_{N+1}^{1,m} : \sigma_{N+1}^{2,m} : \sigma_{N+1}^{3,m} \end{aligned}$$

Similarly, it is also possible to use the new variable $t_{N+1}^{k,m}$ in the problem of the selection of a structure having a minimum mass. It is possible to show that the problem of the selection of the most rigid truss allowing safe failure of any one truss rod (in this case, the truss will continue to carry the cyclic

load, and the stress of all undamaged rods will not exceed allowable tension or compression stresses) is reduced to the above problem. This problem is of interest for designing fail-safe truss structures of railway bridges, power transmission line supports, helicopter tail booms.

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