METHOD FOR DETERMINING CRACK RESISTANCE OF MATERIALS UNDER BIAXIAL LOADING

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ABSTRACT

The paper presents a basically new calculation and experimental method for studying various aspects of crack resistance of sheet structural materials under biaxial loading. A procedure is proposed and substantiated for processing test results for two types of specimens with cracks for the case of brittle and elasto-plastic fracture. Experimental results are presented which illustrate the application of the proposed method.

KEY WORDS

Crack growth resistance, biaxial loading, stress intensity factor, crack opening, sheet materials.

INTRODUCTION

The paper presents fundamentally new equipment and method for studying various aspects of sheet structural material crack resistance under biaxial static and cyclic loading. The idea of the method is in the establishment of the relation between the stress-strain state at the crack tip and the displacements on the boundary of the area surrounding it. This approach is the result of practical considerations and differs from the conventional ones (e.g. see Moyer and Liebowitz, 1984) when the stress-strain state at the crack tip is related to the stresses at infinity. Indeed, if a crack is initiated in a structure, it is unlikely that the area of the material uniform stress state could be found around it, which would correspond to the model of an infinite plane, due to the presence of various technological stress concentrators and other design factors. At the same time, it is practically always possible to measure displacements on the boundary of a comparatively small area surrounding the crack. And in this case to predict the crack behaviour in a structure one should determine the material crack resistance under biaxial loading by displacements adequate to those occurring in the structure. Thus the feasibility of the realization of the approach proposed is obvious and it predetermines the development of specific experimental devices, as well as respective mathematical models for the interpretation of the experimental data.

SPECIMENS AND TESTING DEVICE

The test method involves disc-shaped (Boiko and Karpenko, 1984) and plane specimens with central precracks. The loading scheme for a disc-shaped specimen is presented in Fig. 1. Specimen 1 is placed with its conical surface on round support 2

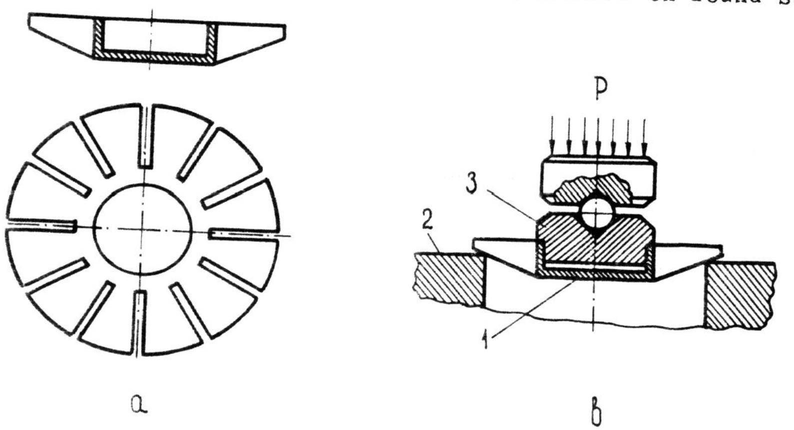

Fig. 1. Disc specimen (a) and the scheme of its loading (b).

and is uniformly loaded along the inner edge of the rim by means of punch 3. The rim gets buckled to thrust with its inner surface against the punch circular recess and the specimen working section is subjected to uniform biaxial tension. The principle of disc-shaped specimen loading can be used for testing as-received sheet materials with plane specimens. For this purpose one should separate the disc-specimen working section and its rim (Fig. 2). In this case the rim plays the role of reusable fixture 1 of plane circular specimen 2 fixed by means of pins 3 arranged round the perimeter of the fixture. In order to reduce the loads applied to the fixture, the latter is composed of separate sectors made of a high-strength material. During the specimen assembly the sectors are arranged symmetrically with respect to the punch axis owing to centring capability of the punch cylindric projection 4. Important experimental parameters in the specimen testing are radial displacements of its working section boundary which can be measured, for instance, with tensometers. The application of the above experimental characteristics to define the stress-strain state of the specimen

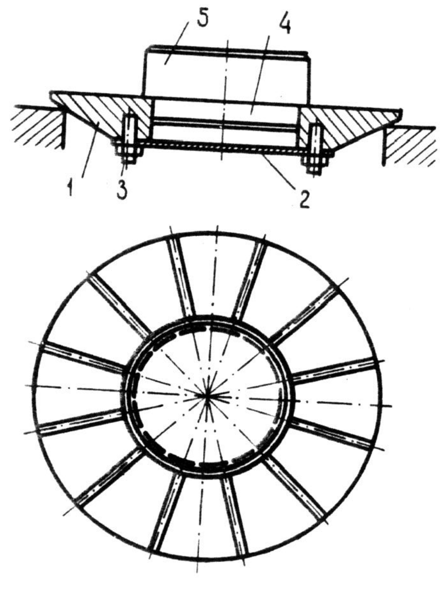

Fig. 2. A device for testing plane specimens in biaxial tension

working section having a crack will be discussed below.

Kinematic relation between the geometrical parameters of the punch, the support, the fixture section and the specimen during its loading is described by a transcendental algebraic equation, the analysis of which allows one to conclude that variation of the fixture and support contact arm size is an effective means to achieve the conditions of biaxial tension with the prescribed displacements in the specimen working section. A schematic of the device for the realization of the above approach is shown in Fig. 3. It comprises aligned punch 1 and support 2 with fixture 3 of specimen 4. The fixture is made as a trunkated cone consisting of separate sectors 5 with radial arrangement in respect to punch 1 whose tapered surface 6 can come into contact with supporting elements 7. Supporting elements 7 are made of equal height and positioned in guiding recesses 8 and can be moved in radial direction and clamped in position. Guiding recesses 8 for the installation of supporting elements 7 are made along the generating surface 9 of support 2. The quantity of recesses 8 corresponds to the number of sectors 5 in fixture 3. The working surfaces of the sectors and the support are made equidistant that provides the contact of each supporting element (of equal height) with the tapered surface of sectors 5 when the supporting elements are positioned in guiding recesses 8 of support 2 at a different distance from the axis of punct 1. The plane of the larger base 11 of fixture 3 is mating with punch 1 while the smaller base is intended for fastening specimen 4 with bolts 13.

CALCULATION SCHEMES

Parameters which characterize the loading of a disc or plane specimen (Figs 1, 2) are radial displacements of its working section boundary measured by means of tensometers. Considering the geometry of the specimen working section, it is feasible to formulate the problem of theoretical determination of the load carrying capacity for a disc or plane specimen with a crack as a problem of the elasticity or pla-

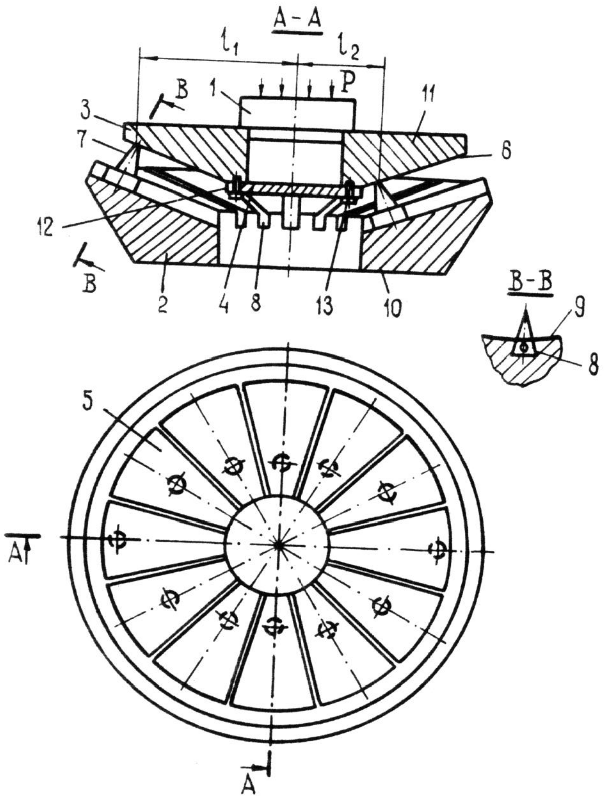

Fig. 3. A device for testing plane specimens in biaxial tension.

sticity theory for a crack in a round disc with specified radial displacements $w_p(\theta)$ of the circular boundary points whose distribution is described by the following law:

$$w_p(\theta) = w_1 \cos^2\theta + w_2 \sin^2\theta, \tag{1}$$

where θ is the polar angle, w_1 and w_2 are the radial displacements along the abscissa and ordinate axes, respectively, which intersect in the centre of the circle. At small values of w_1 and w_2 distribution (1) does not practically differ from elliptical.

The elasticity theory problem for a circle of radius R with a straight-line central crack of $2l$ length with prescribed radial displacements on its boundary (1) is reduced to the following singular integral equation to be solved for the

unknown function $g(t)$ proportional to the derivative of the displacement discontinuity on the crack faces (Boiko, 1991):

$$\frac{1}{\pi i}\int_{-l}^{l}\frac{g(t)}{t-t_0}\,dt + \frac{1}{2\pi i}\int_{-l}^{l} S(t_0,t)\,g(t)\,dt = G(t_0), \quad -l<t_0<l, \tag{2}$$

where

$$S(t_0,t) = \frac{4t}{\varkappa(\varkappa-1)R^2} + \frac{1}{\varkappa}\frac{(\varkappa^2+7)t-8t_0}{R^2-tt_0} - \frac{2}{\varkappa}\frac{t(t-t_0)(2t-3t_0)}{(R^2-tt_0)^2} - \frac{2}{\varkappa}\frac{t^2t_0(t-t_0)^2}{(R^2-tt_0)^3}, \tag{3}$$

$$G(t_0) = -\frac{2\mu}{R}\,\frac{w_1+w_2}{2}\left(\frac{2}{\varkappa-1} - \frac{b}{2} - \frac{3b}{2\varkappa} + \frac{6b}{\varkappa R^2}t_0^2\right), \tag{4}$$

$$b = (w_1-w_2)/(w_1+w_2), \tag{5}$$

μ is the shear modulus, $\varkappa = 3-4\nu$ for plane strain and $\varkappa = (3-\nu)(1+\nu)$ for a generalized plane stress state, ν is the Poisson ratio.

The numerical and approximate analytical solutions of eq. (2) have been obtained. On the basis of the latter, the expression for a dimensionless stress intensity factor at the crack tip is written in the form

$$\frac{k_1\sqrt{R}(\varkappa-1)}{2\mu(w_1+w_2)} = \frac{\sqrt{\pi\lambda}}{1+\eta\lambda^2}\left[1 - b\,\frac{\varkappa-1}{4\varkappa}(\varkappa+3-8\lambda^2)\right], \tag{6}$$

where $\lambda = l/R$,

$$\eta = (\varkappa^3-\varkappa^2+7\varkappa-3)/[4\varkappa(\varkappa-1)]. \tag{7}$$

Comparison of the numerical and approximate analytical calculations reveals that eq. (6) provides a fairly high accuracy of the calculation at $\lambda \leqslant 0.5$.

The elasto-plastic problem for a round plate of elastic-ideally plastic material with a central Dugdale's crack (Dugdale, 1960) of length $2l$ with specified radial displacements on the plate boundary (1) is reduced to the following singular integral equation (Boiko, 1990):

$$\frac{1}{\pi i}\int_{-l_1}^{l_1}\frac{g(t)}{t-t_0}\,dt + \frac{1}{2\pi i}\int_{-l_1}^{l_1} S(t_0,t)\,g(t)\,dt = F(t_0), \quad -l_1<t_0<l_1, \tag{8}$$

where

$$F(t_0) = \begin{cases} \sigma_p + G(t_0), & l<|t_0|<l_1, \\ G(t_0) & |t_0|<l. \end{cases}$$

Functions $S(t_0,t)$ and $G(t_0)$ are defined by eqs (3) and (4), $l_1 = l + a$, a is the plastic zone size at the crack tip determined from the solution of the problem; σ_p is the material yield strength, $\varkappa$ is the elastic constant.

The numerical and approximate analytical solutions of eq. (8) have been obtained. On the basis of the latter an expression for the plastic zone size is written in the form:

$$\arccos(s) + \lambda^2 \eta s \sqrt{1-s^2} = \frac{\pi}{2}\left[1 - \delta \frac{\varkappa - 1}{4\varkappa}(\varkappa + 3 - \delta\lambda^2)\right]\frac{p}{\sigma_p}, \quad (9)$$

where

$$p = 2\mu(w_1 + w_2)/[(\varkappa-1)R], \quad \lambda = \ell_1/R,$$

expressions for δ and η are determined in accordance with (5) and (7), $s = \ell/\ell_1 = 1 - a/\ell_1$. The crack tip opening displacement related to ℓ_1 is defined from the formula

$$\delta(s) = \frac{(1+\varkappa)\sigma_p}{2\mu}\left[-b\lambda^2 \frac{\varkappa-1}{\varkappa}\frac{p}{\sigma_p}(1-s^2)^{\frac{3}{2}} + \frac{2}{\pi} s \ln\frac{1}{s}\right]. \quad (10)$$

Comparison of the numerical and approximate analytical calculations reveals that eqs (8) and (10) ensure an appreciably high accuracy of the calculations at $\lambda < 0.5$.

Of certain interest for the analysis and interpretation of the experimental results is the distribution of local stresses σ_x acting along the crack in the region adjacent to the crack tip. As is known (Alpa et al., 1979) the crack model used is valid at stresses σ_x which in the plastic zone at the crack tip satisfy the condition

$$\sigma_x / \sigma_y \leq 1,$$

in which we shall use the stresses at the crack tip $\sigma_x = \sigma_x(s)$.

EXPERIMENTAL RESULTS

Let us consider some experiments which illustrate the application of the above method. Disc specimens were used successfully to study the ultimate state of elastic plates weakened by the cracks of complex shape in uniform biaxial tension. Those results are presented in the paper by Boiko and Karpenko (1984). Elastoplastic deformation of plates with cracks in biaxial tension including nonuniform one was studied using plane 1.5 mm thick specimens (Fig. 2) of aluminium alloy. Central linear cuts of lengths 10, 20 and 30 mm oriented along one of the main axes of the specimen deformation were made by electric-spark technique that made it possible to get the slits with the curvature radius at the tips of 0.03 mm. Mechanical properties of the aluminium alloy were determined using a standard method and were: E=71500 MPa, ν =0.32, σ_p =320 MPa. The anisotropy of the material mechanical properties did not exceed 1%. The experiments were performed with the prescribed displacements on the boundary of the specimen working section in accordance with eq. (1) for different w_1 and w_2 ratios along the main axes of the specimen

deformation. The limiting w_1^* and w_2^* values of displacements corresponding to the crack growth onset moment were measured on the basis of 2R=50 mm.

The procedure of the experimental results processing is as follows. First, the dependences of the relative crack tip opening displacement $\Delta = 2\mu\delta(s)/[(1+\varkappa)\sigma_p]$ on the parameter $\psi(w_1 + w_2)/R$ (where ψ is the dimensionless constant $2\mu/[(\varkappa-1)\sigma_p]$) are determined by computation. Then, using the results obtained we define Δ_* which corresponds to the crack growth onset for displacements w_1^* and w_2^* determined from the experiment. This makes it possible to find the dependence of the critical value of the absolute crack tip opening displacement δ_A^* on the w_1/w_2 parameter (Fig. 4 a). These

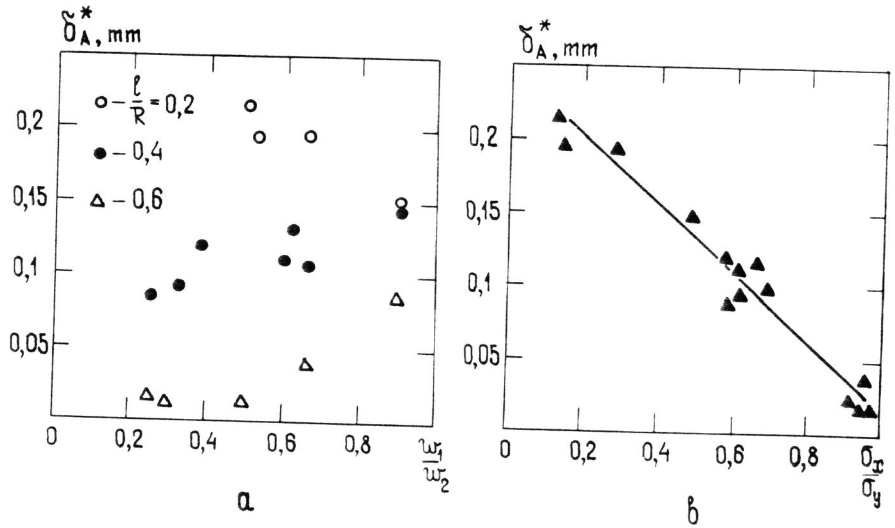

Fig. 4. Dependence of the critical crack tip opening displacement on the parameter (a) and on the normal stress ratio at the crack tip (b).

data are useful for the evaluation of the structural element ultimate state within the framework of the conception formulated in the Introduction. The following step is the construction of the σ_x/σ_y (the local stress ratio at the crack tip) dependence on the parameter $\psi(w_1+w_2)/R$. Using these results and the experimental w_1^* and w_2^* values, we find the relationship between the critical value of the absolute crack tip opening displacement δ_A^* and the σ_x/σ_y parameter (Fig. 4 b). The above dependence characterizes the crack growth resistance of the tested thin-sheet material of the given thickness.

CONCLUSION

The proposed method based on the establishment of relationship between the stress-strain state at the crack tip and the displacements on the boundaries of the area surrounding it appears to be promising from the standpoint of the maximum approach (as to the type of stress state) of the laboratory test conditions to real service conditions of a structural material. In addition, the advantage of the equipment developed over the known one is in the possibility of biaxial loading of specimens using the simplest types of testing machines: presses for static loads and pulsators for cyclic loads. This simplifies the experiment appreciably and makes it cheaper, allows the experiment to be performed in practically any laboratory engaged in mechanical testing of materials.

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