FATIGUE LIFE PREDICTION METHOD BASED ON ENERGY CRITERION

P.A. FOMICHEV, V.V. ZVYAGINTSEV and I.Y. TRUBCHANIN
Kharkov Aviation Institute, Kharkov, Ukraine

ABSTRACT

After an analysis of experimental data on number of steels, aluminium and titanium alloys cyclic inelastic deformation investigation in transient and proper high-cycle fatigue region models to evaluate kinetics of cyclic residual strain during constant and programmable varied load tests were built up. To calculate fatigue crack initiation life new energy criterion is used. This criterion determines fatigue failure relative energy as a function of total dissipated energy depending on cyclic residual strain and provides constant value of relative energy under regular or irregular loading with present or absent mean stress.

KEYWORDS

Fatigue life, hysteresis loop, residual strain, stress concentration, energy criterion.

INTRODUCTION

Investigation of damage accumulation before crack initiation is an important step in analysis of fatigue failure. There are different methods of fatigue damage summation based on phenomenological approach to the process of fatigue. In those methods both criterion of failure and damage accumulation law are defined by their authors' hypotheses and assumptions. Because term "fatigue damage" is rather indefinite it is difficult to realize direct experimental verification of those hypotheses. Advance in summation method has to be achieved in the field of criterion that is proved experimentally and has phisically clear substantiation. Failure of structural members practically always begins at the notch root or stress concentration zones. Today there are two kinds of approach to calculate fatigue life of such members - using nominal stress or local strain. Past is more universal but it requires sophisticated investigation of material cyclic deformation under constant and variable amplitude loading. To develop fatigue life prediction method it is neccessary to establish fatigue failure criterion, mechanism of damage accumulation and parameter that determines material fatigue life under uniform and non-uniform stress state. Energy approach may be prefered because it permits phisically clear summation of dissipated energy under constant and variable amplitude loading.

EXPERIMENTAL RESULTS AND THEORETICAL ANALYSIS

Experimental investigations were made with means of installations created on the basis of general purpose electromechanical test mashins. Cyclic hysteresis loop method was realized. Resolution of strain measurement was about $5*10^{-6}$ mm/mm. To measure strain at the notch root and smooth specimens extensometers with gage length 2 mm and 15 mm were used. Amplitude of both total and residual strain was measured. Residual strain amplitude (denoted further as RSA) is strain attained to moment when external load is equal to its mean value, for symmetrical nominal cycle this is a moment when external load is equal to zero. Specimens were made of materials having contrasting cyclic deformation properties. They were D16AT (stiff Al-Cu-Mg alloy), AMzM (soft welded Al-Mn alloy) and 30HGSA (Cr-Mn-Si alloy steel). Specimens of steels 45, 40H, 12HN3A and other were also tested. As an result of numerous experimental investigations it was suggested to define equation of kinetic cyclic diagram (in plots RSA vs cycle ratio) in the form:

$$\varepsilon_{ar} = \varepsilon^{*}_{ar} f(\sigma_a, \sigma_m, x) \tag{1}$$

where ε_{ar} and ε^{*}_{ar} - current and average value of RSA; X=n/N -cycle ratio that is equal to current number of cycle divided on number of cycles to failure (or specimen fatigue life); $f(\sigma_a, \sigma_m, X)$ - material softening (hardening) function. For cyclically stable metals this function is equal to unity.

Average value of RSA can be computed with numerical integration of experimental kinetic cyclic diagram. Softening (hardening) function has to correspond to the following condition

$$\int_0^1 f(\sigma_a, \sigma_m, X)\,dX = 1 \tag{2}$$

Definition of principal cyclic stress-strain curve (CSSC) is used. It is a dependence between average value of RSA and stress amplitude. It was shown experimentally that this dependence may be presented as straight or broken straight line in double log plots for both symmetrical and asymmetrical cycle. Then

$$\varepsilon^{*}_{ar} = (\sigma_a / K_m)^{1/m} \tag{3}$$

for most tested metals

$$K_m = K[1-(\sigma_m/\sigma_b)^{\nu}] \tag{4}$$

where K and m - principal CSSC parameters of symmetrical cycle; ν - coefficient; σ_b - material ultimate tensile stress.

Concerning the problem of stress state evaluation with the reversal - by - reversal technique it is necessary to take into account a real cyclic stress-strain curve that is hysteresis loop naturally. After numerical analysis of experimental real CSSC it was shown that equation of real CSSC can be represented as sum of linear and exponential items. This equation is to be writen in plots of unloading e-S those are set in stress and strain reversal point

$$e = S/E + 2\varepsilon_{ap}(S/2\sigma_a)^{\gamma} \tag{5}$$

Correlation between RSA and plastic strain amplitude is

$$\varepsilon_{ap} = \varepsilon_{ar}(1+t) \tag{6}$$

where $t = 1/(2^{\gamma-1}-1)$
Exponent γ may be evaluated with hysteresis loop relative parameters. Following equation is suitable

$$\gamma = 2 + h\,\varepsilon_{ar}/\varepsilon_{at} \tag{7}$$

Variety range of h was from 7 to 13. If experimental data are absent it is allowed to use h =10.

Model to calculate RSA kinetics under programmable loading was developed. In accordance with this model correlation between values of RSA when stress amplitude changes from σ_{ai} to σ_{ai+1} is

$$\mathring{\varepsilon}_{ari+1} = (\sigma_{ai+1}/\sigma_{ai})^{1/c}\,\varepsilon_{ari} \tag{8}$$

where $\mathring{\varepsilon}_{ari+1}$ - initial RSA on σ_{ai+1}; c - parameter of CSSC after overloading.

If RSA after transfer does not exceed maximum value attained under regular loading with same actual stress amplitude then kinetics of RSA occurs. In alternate case RSA does not change and remains equal to $\mathring{\varepsilon}_{ari+1}$. After transition from one stress amplitude to another softening (hardening) process obeys an Eq. 1, where cycle ratio changes from x_{eqi+1} to $x_{eqi+1}+x_{i+1}$ corresponding to $\varepsilon_{ar\,i+1}$. Equivalent cycle ratio needed to achieve $\varepsilon_{ar\,i+1}$ under regular loading can be found as value of function that is inverse of softening (hardening) one

$$x_{eq\,i+1} = \Psi(\varepsilon^{*}_{ar\,i+1}, \sigma_{a\,i+1}, \sigma_{m\,i+1}) \tag{9}$$

then

$$\varepsilon_{ar\,i+1} = \varepsilon^{*}_{ar\,i+1} f(\sigma_{a\,i+1}, \sigma_{m\,i+1}, x_{eq\,i+1} + x_{i+1}) \tag{10}$$

Very important feature of metal cyclic deformation under block loading is following. After overloading on step of maximum stress amplitude value of RSA on other steps exceeds corresponding value attained under regular loading with same stress amplitude and cycle ratio. If number of blocks to failure is rather big then it can be assumed without considerable error that RSA kinetics occurs on overload step of block only.

Total energy dissipated in elemental material volume is ussually evaluated as square of hysteresis loop

$$W_c' = K_f \sigma_a \varepsilon_{ar}, \tag{11}$$

where K_f - hysteresis loop shape coefficient. With regard for equation of real CSSC K_f can be computed as follow

$$K_f = 4[(\gamma - 1)/(\gamma + 1)]/(1 + t) \tag{12}$$

Values of K_f varies in rather narrow range from 2.67 for high cycle fatigue to 3.2 for low cycle fatigue. In practical calculations mean value $K_f = 3$ may be adopted.

Energy criterion of fatigue failure is suggested. This criterion is based on separation of total dissipated energy on safe and dangerous components concerning fatigue. According to this criterion differential equation of relative dangerous energy accumulation is of the form

$$d\bar{W}/dx = N R_m (W_c'^{\alpha} - W_{-1}'^{\alpha}) \tag{13}$$

or

$$d\bar{W}/dx = N R_m W_c'^{\alpha} \tag{14}$$

where R_m and α - equation parameters; $\bar{W}$ - relative dangerous energy, $0 \leq \bar{W} \leq 1$; W_{-1}' - total energy dissipated on fatigue limit under symmetrical cycle.

Eq. 13 is suitable for metals having phisical fatigue limit. Considerable difference between results of calculation on Eqs. 13 and 14 takes place only for stress level close to

fatigue limit. Dependence of R_m on mean stress is found

$$R_m = R(1 + r\sigma_m/\sigma_b) \tag{15}$$

where R - value of parameter R_m under symmetrical cycle;
r - material constant.
Values of parameters α, R and r may be evaluated from plot log W_c^* - log N , where $W_c^* = K_f \sigma_a \varepsilon^*_{ar}$

Energy criterion and model of material cyclic deformation allow to construct method to calculate fatigue life under irregular loading. Because there are different softening and hardening functions and different kinetic diagrams for different metals it is impossible to establish some unique formula of fatigue life evaluation. However, dependences suitable for every specific condition and material may be obtained proceeding from common correlation. Differential equation of relative dangerous energy accumulation is presented as

$$d\bar{W}/dx = NG(x) \tag{16}$$

where function G(X) depends on amplitude and mean stress according to Eqs. 14 and 15.

Fatigue curve equation may be presented as

$$N \int_0^1 G(x)\,dx = 1, \tag{17}$$

and condition of failure under programmable loading of unstable materials as

$$\sum_{i=1}^{j} \Delta \bar{W}_i = 1 \tag{18}$$

where j - number of load steps to failure; $\Delta\bar{W}_i$ - relative energy increment at i-th load step.

If RSA kinetics takes place only at highest block step under block loading then failure condition has form

$$\int_0^{\lambda_p} \sum_{i=1}^{a} G_i(\lambda x_{max}) n_{\delta i}\, d\lambda = 1 \tag{19}$$

where λ_p - number of load blocks to failure; $n_{\delta i}$ - cycle ratio of highest step in block.

If moderate conservative estimation of calculated fatigue life is acceptable then average RSA value may be used

$$N_c = N_{max} / \sum_{i=1}^{a} (\sigma_{ai}/\sigma_{max})^{\alpha(1+d/c)} V_i \tag{20}$$

where V_i - cycle ratio of i-th block step; N_{max} fatigue life under regular loading with maximum stress amplitude of block. In the case of block loading with continued stress amplitude distribution and probability density $\varphi(\sigma_a)$ equation to calculate number of blocks to crack initiation may be presented as

$$n_\delta \int_{\sigma_y(l)}^{\sigma_{a\,max}} \int_0^{l_p} G(l, x_{max}) \varphi(\sigma_a) \, d\sigma_a \, dl = 1 \tag{21}$$

where σ_{max} - maximum stress amplitude taken into fatigue life account; n_δ - number of cycles in load block; $\sigma_y(l)$ - current value of fatigue limit that may be found from Eq. 14 and condition $d\bar{W}/dn = 0$.

Initial value of fatigue limit under asymmetrical loading with mean stress σ_y^o corresponds to equality of total energy dissipated under symmetrical and asymmetrical load cycles

$$\sigma_y^o = (K_m/K)^{1/m} \sigma_{-1}^o \tag{22}$$

Taking into account Eq. 4 for steels it may be obtained

$$\sigma_y^o = [1 - (\sigma_m/\sigma_e)^k]^{1/(k+m)} \sigma_{-1}^o$$

Two problems should be separated to calculate fatigue life of notched members. First is connected with accurate determination of local stress-strain state. Second problem is to investigate parameter that fatigue life under non-uniform stress state condition depends on. A whole complex of experimental exploration of local strain at notch root of different metals was fulfilled with means of small gage length extensometers. Experimental data were compared with number of approximate methods. It was recognized what in most cases the best compliance of calculated and experimental results may be ensured using Neuber's rule with correction function of Makhutov (see Ref. (4)). To solve second problem fatigue lives of smooth and notched specimens as function of some local stress-strain state parameters were compared. It was certainly shown that fatigue curves of smooth and notched specimens cover each other only in plots RSA vs fatigue life or dissipated energy Wr vs fatigue life . Here Wr that we call residual energy is define as follow

$$W_r' = K_f \sigma_a \varepsilon_{ar} \tag{23}$$

where σ_a - local stress amplitude; ε_{ar} - local RSA at the notch root.

Both plots are equal in accuracy but residual energy was prefered because it made possible to do simple summation of dissipated energy under irregular loading.

Equation to calculate local RSA takes form

$$\varepsilon_{ar} = \varepsilon_{at} - e_r \tag{24}$$

where ε_{at} - local total strain amplitude; e_r - strain in plots of unloading at notch root that may be evaluated using equation of real CSSC.

To take into account stress and strain gradients well-known hypothesis was applied. It assumes that notched member fatigue life to crack initiation is determined by stress parameters on some distance d from notch tip. Fomichev (1989) suggested functions to calculate stress and strain gradients. They are based on supposition of validity of Neuber's rule in vicinity of maximum stress concentration point. In same manner function to calculate residual energy gradient is derived. Because size d is rather small average value of dissipated energy on distance d from notch tip is equal to

$$W_{rd}^* = W_r^* - G_W d \tag{25}$$

After an analysis of extensive experimental and calculated data it was establish what mean value of d is close to 0.1 mm for aluminium and titanium alloys and 0.05 mm for steels.

W_{rd}^* may be evaluated with reasonable accuracy if to calculate local stress and strain Neuber's rule and elastic stress concentration factor on distance d from notch tip are used. Fomichev (1991) carried out evaluation of local stress-strain state kinetics at notch root. It was noted that auxiliary function ought to be obtained using modelling of dissipated energy kinetics at notch root on overload block step. This function is to comply with condition

$$W'_{r\,max} = W^*_{r\,max} f_W(\sigma_{H\,max}, x_{max})$$

Also it is neccessary to determine dependence between dissipated energy on i-th load block step and $W'_{r\,max}$ and represent function $G(X, \sigma_a, \sigma_m)$ as right part of Eqs. 14 or 15. If dissipated energy kinetics on overload step is not considerable then according to Eqs. 14 and 15 correlations to calculate fatigue life to crack initiation at notch root take form

$$l_p \sum_{i=1}^{a} R_{mi} (W_{ri}^{\prime\alpha} - W_{-1}^{*\alpha}) n_{\delta i} = 1 \tag{26}$$

or

$$l_p \sum_{i=1}^{a} R_{mi} W_{ri}^{\prime\alpha} n_{\delta i} = 1 \tag{27}$$

Parameter R_{mi} in Eqs. 26 and 27 should be calculated taking into account local mean stress.

So to calculate notched member fatigue life it is enough to have material cyclic stress-strain and fatigue properties obtained from smooth specimens tests.

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