ESTIMATION OF THE RESIDUAL STRESSES INFLUENCE UPON PLANE WELDED JOINTS STRENGTH

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ABSTRACT

Distribution of residual stresses in welded joints is approximated by functional analytical dependences and are determined by SIF at the crack tip. The influence of the residual stresses field on crack propagation and crack retardation was studied.

KEYWORDS

Residual stresses, stress intensity factor, welding, edge and central crack, critical crack size.

INTRODUCTION

Brittle fracture of welded joints takes place by propagation of cracks, which initiate during welding or some time after it. In this case residual stresses arising due to uneven heating or cooling of metal during welding as well as by inhomogeneous plastic deformation caused by various strengthening treatments substantially influence crack propagation. The aim of the work is to study possible types of residual stresses influence on brittle fracture development in the area of their distribution using the elementary model in plane formulation.

<u>Residual Atresses Approximation.</u> Normal residual stresses change in transversal direction (along the Ox axis) by different laws, depending on the type of welding, physico-mechanical properties of the welds material. Typical distribution of residual stresses (Fig. 1) can be approximately approximated by dependences:

$$\sigma_x(x) = \sigma_p[(1+\sigma_c/\sigma_p)/(1+(x/L)^2\sigma_c/\sigma_p)-\sigma_c/\sigma_p] \qquad (1)$$

$$\sigma_y(x) = \sigma_p[(1+\sigma_c/\sigma_p)(1+\sigma_c/\sigma_p)^{-(x/L)^2} - \sigma_c/\sigma_p] \qquad (2)$$

at $x \ll B$, where $2B$ is a specimen width. In this case σ_p and σ_c are maximum tensile and compressive stresses; L characterizes the length of the tensile stresses area. Expressions (1) and (2) satisfy conditions $\sigma(0)=\sigma_p$, $\sigma(\pm L)=0$, $\sigma(x>L)\approx-\sigma_c$ at $L \ll B$.

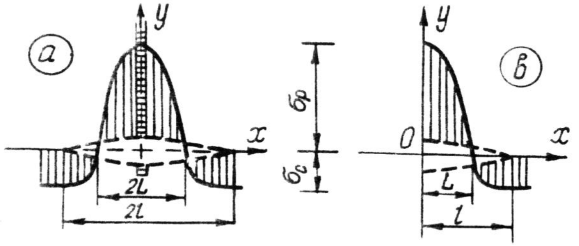

Fig. 1. Distribution of residual stresses in the area of (CC) central (a) and (EC) edge (b) cracks.

Complex distribution of residual stresses in the crack plane may be approximated with high accuracy by the analytical dependences in the form of trigonometric polynomial (Trush I.I. et al., 1987).

$$\sigma(x,0) = a_0 + \sum_{k=1}^{N} a_k \cos\frac{k\pi}{T}x + b \sin\frac{k\pi}{T}x \qquad (3)$$

Analysis of the elasticity problem solution for infinite plane weakened by CC shows that of the sell-balancing stresses $\sigma(x)$ and crack edges displacement $v(x)$ will be presented as:

$$\sigma(x) = \sum_{s=1}^{N} a_s U_{s-1}(x/l);$$

$$v(x) = \frac{\text{æ}+1}{4\mu}\sqrt{l^2-x^2}\sum_{s=1}^{N} b_s U_{s-1}(x/l), \qquad (|x|\leqslant l); \qquad (4)$$

the boundary conditions on the crack line ($|x|\leqslant l$) are satisfied at $a_s=-b_s$. Here, a_0, a_k, b_k, a_s, b_s are coefficients, evaluated by the known formulas, which in general case depend on the physico-mechanical properties of material, technological, service characteristics of a structure, shear μ-modulus, $\text{æ} = 3-4\mu$ for plain strain and $\text{æ} = (3-\nu)/(1+\nu)$ for a plane generalized stressed state, ν is a Poisson's coefficient.

<u>Stress Intensity Factor.</u> We suppose that crack edges do not come in contact, i.e. distance between defect faces is larger than the elastic displacements. Basing on fundamental solutions (Panasyuk V.V. et al., 1976) we obtain SIF value for CC that corresponds to distribution (1)

$$K_I = \sigma_p\sqrt{\pi l}[(1+\sigma_c/\sigma_p)(1+(l/L)^2\sigma_c/\sigma_p)^{-1/2} - \sigma_c/\sigma_p], \qquad (5)$$

and during distribution (2)

$$K_I = \sigma_p\sqrt{\pi l}[(1+\sigma_c/\sigma_p)e^{-p}I_0(p) - \sigma_c/\sigma_p], \qquad (6)$$

where $p=1/2(lT)^2\ln(1+\sigma_c/\sigma_p)$; $I_0(p)$ is a modified Bessel function of the zeroth order. For EC at distribution (2) we obtain:

$$K_I = \sigma_p\sqrt{2l/c}\left\{\left(1+\frac{\sigma_c}{\sigma_p}\right)\sum_{k=0}^{\infty}(-1)^k\frac{(2p)^k}{k!}\frac{\Gamma\left(\frac{2k+1}{c}\right)}{\Gamma\left(\frac{2k+1}{c}+\frac{1}{2}\right)} - \frac{\sigma_c}{\sigma_p}\frac{\Gamma\left(\frac{1}{c}\right)}{\Gamma\left(\frac{1}{c}+\frac{1}{2}\right)}\right\}; \qquad (7)$$

with $\Gamma(z)$ to denote gamma-function; $c=2\pi^2/(\pi^2-4)$. If the residual stresses distribution is described by dependence (3) the SIF value can be written as:

$$K_I^{\pm} = a_0 K_I^{(0)} + \sum_{k=1}^{N} a_k K_I^{(c)}(k) + b_k K_I^{(s)}(k) \qquad (8)$$

where $K_I^{(0)}$, $K_I^{(c)}$, $K_I^{(s)}$ are SIF values, when load $\sigma=1$, $\sigma=\cos k\pi x/T$ and $\sigma=\sin k\pi x/T$ respectively, is applied to the crack edges. These values are determined by the following relations:

In case of CC:

$$K_I^{(0)} = \sqrt{\pi l}; \quad K_I^{(c)} = \sqrt{\pi l}J_0(k\pi l/T),$$

$$K_I^{(s)} = \pm\sqrt{\pi l}J_1(k\pi l/T). \qquad (9)$$

In case of EC:

$$K_I^{(0)} = \sqrt{\frac{2l}{\pi c}}\, B\left(\frac{1}{c},\frac{1}{2}\right),$$

$$K_I^{(c)} = \sqrt{\frac{2l}{\pi c}} \sum_{n=0}^{\infty}(-1)^n \left(\frac{k\pi}{T}l\right)^{2n} \frac{1}{(2n)!} B\left(\frac{2n+1}{c},\frac{1}{2}\right), \qquad (10)$$

$$K_I^{(s)} = \sqrt{\frac{2l}{\pi c}} \sum_{n=0}^{\infty}(-1)^n \left(\frac{k\pi}{T}l\right)^{2n+1} \frac{1}{(2n+1)!} B\left(\frac{2n+2}{c},\frac{1}{2}\right).$$

Here $J_0(z)$, $J_1(z)$ are Bessel functions of the first order, $B(a,b)$ is a beta-function $c=2\pi^2/(\pi^2-4)$ All values of SIF for CC with upper sign (+) correspond to the right crack tip ($x=l$), and with lower sign (-) - to the left one ($x=-l$). We obtain rather simple relationships for CC if stresses $\sigma(x)$ and displacements $v(x)$ are approximated as (Panasyuk V.V. et al., 1976) :

$$K_I^{\pm} = \sqrt{\pi l}\sum_{s=1}^{N}(\pm 1)^s s b_s \quad \text{or} \quad K_I^{\pm} = \mp\sqrt{\pi l}\sum_{s=1}^{N}(\pm 1)^s a_s \qquad (11)$$

Crack Edges Displacement. Basing on results of study (Panasyuk V.V.,1968) crack edges displacement of the non-contacting CC when stresses (3) are applied to its edges can be written (Trush I.I. et al., 1987), as:

$$v(x,0) = a_0 v^{(0)} + \sum_{k=1}^{N} a_k v_k^{(c)} + b_k v_k^{(s)}; \qquad (12)$$

$$v^0(x,0) = \frac{\varkappa+1}{4\mu}\sqrt{l^2-x^2},$$

$$v_k^{(c)}(x,0) = \frac{\varkappa+1}{2\mu}\frac{T}{k\pi l}\sqrt{l^2-x^2}\sum_{s=0}^{\infty}(-1)^s J_{2s+1}\left(\frac{k\pi}{T}l\right)U_{2s}\left(\frac{x}{l}\right), \qquad (13)$$

$$v_k^{(s)}(x,0) = \frac{\varkappa+1}{2\mu}\frac{T}{k\pi l}\sqrt{l^2-x^2}\sum_{s=1}^{\infty}(-1)^{s+1} J_{2s}\left(\frac{k\pi}{T}l\right)U_{2s-1}\left(\frac{x}{l}\right).$$

When the cut edges displacement under normal forces effect $\sigma(x)$ are known can evaluate SIF values (Panasyuk V.V. et al., 1976). In our case, for displacements (12) and (4) SIF values are determined using expressions (8) and (11) respectively.

The Obtained Data Analysis. From numerical analysis of equations (5)-(7) we get that at $l/L<0.2$ and $\sigma_p/\sigma_c<2.0$ (with

error up to 5%) $K_I \approx K_I^{(0)}$ with corresponding tensile stresses σ_p=const. $K_I<K_I^{(0)}$ at all values of l/L and σ_p/σ_c. It is known that under tensile stresses influence the propagating crack penetrates into a zone of compressive stresses and arrests (Zlachevsky A.B., Shuvalov Y.A., 1985). We assume $K_I(l_*)=0$, where l_* is arrested crack size to be a condition of crack arrest. Values l_*/L at various σ_p/σ_c in case of distributions (1) and (2) for CC and in case of distribution (2) for EC are given in Table.

	σ_p/σ_c	0.1	0.5	0.8	1.0	2.0
(5)	CC	1.45	1.58	1.67	1.73	2.0
(6)		1.44	1.51	1.57	1.60	1.80
(7)	EC	1.49	1.57	1.64	1.68	1.93

Having determined critical crack size l_* from equation $K_I^{(c)}(l_*)=0$ we obtain for CC $l_*/L=1.53$, and for EC $l_*/L=1.5$. At $kl/T>8.0$, $K_I^{(c)}/K_I^{(0)}<0.1$, it means that at greater length l or at higher frequency of stress sign variation $K_I^{(c)}/K_I^{(0)} \to 0$ and $K_I^{(s)}/K_I^{(0)} \to 0$.

As an example, we consider distribution of normal stresses shown in Fig 2a. Variation of $K_I/K_I^{(0)}$ ($K_I^{(0)}=\sigma\sqrt{\pi l}$) are determined and investigated, depending on l at different $\eta=\sigma_p/\sigma_c$.

As it is seen from the curves $K_I\approx\sigma_p\sqrt{\pi l}$ at $l<\pi/2$; if $l\to\infty$ that $K_I=(\sigma_p-\sigma_c)\sqrt{\pi l}/2$. Values $K_I/K_I^{(0)}$ change about the value $(1-1/\eta)/2$, $(\eta=\sigma_p/\sigma_c)$. In this case amplitude value decreases with the growth of l, however it substantially depends on η. We can choose such values of η_0 at $l\geq l_*$ ($K_I(l_*)=0$) at which condition $K_I(l)=0$ is satisfied, i.e. the crack penetrates into the compressive stresses area and arrests. In our case $\eta_0=(\pi-2)/(\pi+2)$. To study the influence of the shape of the residual stresses pattern on SIF value we considered distribution of stresses, for which $\sigma_p=\sigma_c=p$ and was equal by length (π) to irregular areas of tensile and compressive stresses. At $l>5\pi$ influence of pattern shape on relation $K_I/K_I^{(0)}$ is insignificant and at $l\to\infty$ $K_I/K_I^{(0)}\to 0$.

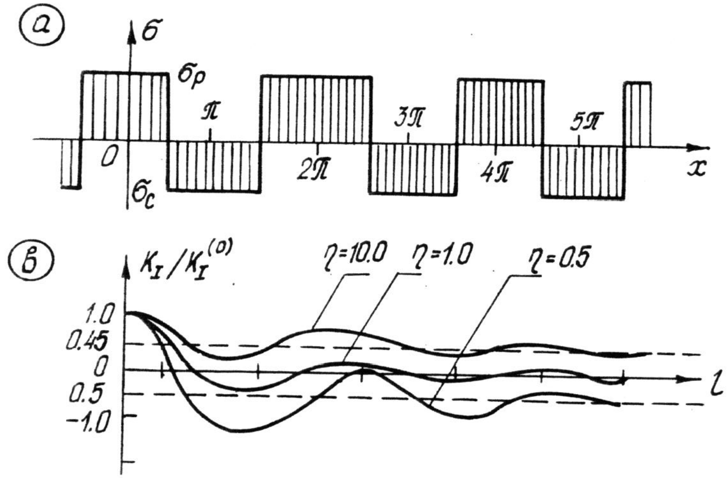

Fig.2. b - Variation of relation $K_I/K_I^{(0)}$ depending on CC length of at various values $\eta=\sigma_p/\sigma_c$; a - internal stresses pattern.

Note, that for more accurate evaluation of brittle strength of welded joints it is necessary: 1) to take into account the partial contact of crack edges; 2) to determine critical length l_* from equation $K_I(l_*)=K_{Ic}$; 3) to study influence of the prefracture redistribution of residual stresses with size l increase. The influence of shear residual stresses on the brittle strength is taken into account in much the same way.

REFERENCES

Zlachevsky A.B., Shuvalov I.A. (1985). Factors of fatigue crack growth retardation. Fiziko-Khimicheskaya Mechanika Materialov, 21, No 2, P.41-45 (in Russian).

Panasyuk V.V. (1968). Limit equilibrium of brittle bodies with cracks. Kiev, Naukova Dumka Publishing House, 238p. (in Russian).

Panasyuk V.V., Savruk M.P., Datsyshyn A.P. (1976). Distribution of stresses near cracks in plates and shells. Naukova Dumka Publishing House. 442p. (in Russian).

Trush I.I., Berezhnytska M.F., and Berezhnytsky L.T.(1987). Valuation of SIF at the given residual stresses distribution. Fiziko-Khimicheskaya Mekhanika Materialov, 23, No 5, P.85-96.

Trush I.I., Berezhnytska M.P., Babei Yu.I., Berezhnytsky L.T. (1987). To the influence of residual stresses under quasibrittle fracture. Booklet No 136, Lviv, 56p. (in Russian).