

# ESTIMATION OF FATIGUE LIFE OF STRUCTURAL MEMBERS UNDER IRREGULARLY VARYING LOADS

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## ABSTRACT

This paper presents methods of increasing the accuracy of prediction of fatigue damage in aircraft structures by special random load tests, taking into account the low-amplitude loads and the structure of service load sequence. Procedure based on local uniaxial stress state is discussed for cycle representation at multiaxial random loading.

## KEYWORDS

Random test, multiaxial fatigue, fatigue damage accumulation, multiaxial fatigue criteria.

## INTRODUCTION

The determination of load sequence equivalence, the prediction of individual fatigue damages of structures, and other related problems require the development of accurate, reliable and, at the same time, relatively simple methods of fatigue damage estimation at irregularly varying loads. Naturally, the requirements for accuracy, reliability and simplicity are mutually exclusive in many aspects. This paper proposes some ways to solve of this problem.

## UNIAXIAL LOADING

A uniaxial loading is considered as the basic one because there exist a lot of experimental and theoretical results for this case. This method is based on the determination of constants to be used for calculation of fatigue life in order that the best possible fitting for fatigue life values at load histories typical of service can be evaluated. Some details of this method are as follows.

a) Fatigue life values are determined at typical service load sequence and its variations. "Typical service load sequence" is a combination of functional and random loads the

"rainflow matrix" for this combination is same as a matrix defined or predicted for the average service load conditions with 2 exceptions. They are, 1) all stresses have to be in the interval [ S-, S+ ] and the rare load peaks and troughs, which exceed one or both of these boundaries, are excluded from load sequence; and 2) the smallest amplitude is defined by the possibilities of test equipment, its value is usually 5 - 10 % of the highest amplitude. Values of S+, S- are defined by the requirement to obtain a conservative result. A rare tensile stress S+ is considered to be capable of prolonging fatigue life, and its value is defined by the adopted probability p+ of absence of this stress throughout the service life. A rare compressive stress S- is taken to be capable of reducing the fatigue life, and its value is defined by the adopted probability p- of appearance of this stress throughout the service life. For usual size of aircraft fleet and Poisson distribution taken for S+ and S- at p+ = p- = 0.01 the tensile stress S+ and the compressive stress S- must be equal to stresses that are encountered five times per service life and once per 100 service lives, respectively. At p+ = p- = 0.1 these values are equal to stresses those are encountered twice per service life and once per 10 service lives respectively ( Svirsky and Raikher, 1992 ).

b) Variations of typical service load sequence may be different. The best variation for definition of constants is a scaling of typical service load sequence, i. e. all stresses of the latter are multiplied by a scale factor. Theoretically the total number of variations must be equal to or greater than the number of the constants to be defined. Parameters at variations of typical service load sequence are selected so as to provide fatigue life within 0.25 - 4.0 service lives.

c) The basis for determination of required constants is a rainflow matrix which has a structure as presented in Table 1.

Table 1. Structure of rainflow matrix

Item	Description
NE	The total number of matrix entries
$\sigma_a$	Stress amplitude of the 1st entry
$\sigma_{max}$	Maximum stress of the 1st entry
R	The number of reversals of the 1st entry
$(\sigma_a, \sigma_{max}, R)$	of next entries ( 2, 3, ..., NE )

Two types of matrices are used. They are, a calculated one and a "feedback" one. The first type of matrix is obtained by rainflow representation of the typical service load sequence generated by a computer. For obtaining the second type, a feedback signal at specimen loading is used. The usage of the second type of matrix gives the possibility to provide data reliability and to improve the accuracy because the accuracy of feedback

measurement is 10 - 20 times higher than the accuracy of control in servohydraulic machines used for such tests. For tests of "smooth" specimens, the strain amplitude  $\epsilon_a$  is used instead of  $\sigma_a$ . In this case the usage of the second type rainflow matrix gives the possibility to account for a change in mechanical properties of the material in cyclic loading.

## DESIGN FATIGUE LIFE CURVES

Determination of material/structure constants from typical service load sequence gives a possibility to use simple linear models of fatigue damage accumulation for the estimation of fatigue life and the equivalents of various load sequences. Indeed, we use a method of linearisation of non-linear damage accumulation laws which provides an improved accuracy by decreasing a range of the load sequence variation. The typical service load sequence is considered to be an average in that range. The usage of simple linear damage accumulation models means that the constants of fatigue life estimation procedure are mainly the constants of fatigue life curve which will be referred to as "design fatigue life curve" henceforth. Some results on the determination of design fatigue life curves in random loading of sheet specimens with central holes ( Svirsky et al., 1988 ) and one-shear bolted joint specimens are shown in Figures 1 and 2. Sheet specimens with

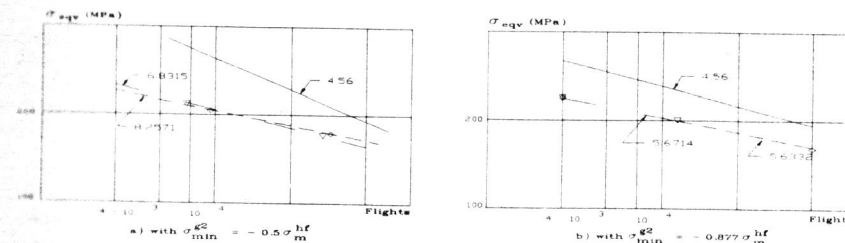


Fig. 1 Design fatigue life curves of free hole specimen

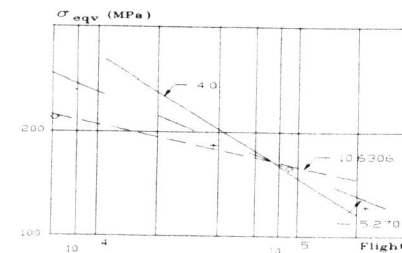


Fig. 2 Design fatigue life curves of joint with  $\sigma_{min} = -0.5\sigma_m$

central holes ( free hole specimens ) were made from aluminum alloy D16chT ( 4 mm thickness, 30 mm wide and hole diameter 5 mm ). One-shear bolted joint specimens ( or joint specimens ) were made from D16AT aluminum alloy D16AT ( 6 mm thick, 36 mm wide and 6 mm diameter holes for fastening an overlap by 2 neat-fitted titanium bolts which transfer 8 % of the total load applied to the specimen ). Flight equivalent stresses  $\sigma_{eqv}$  versus flights-to-failure ( Flights in Figures 1 - 4 ) have been plotted in double logarithmic coordinates. The flight equivalent stress is defined as the maximum stress of a pulsating cycle at regular tensile loading resulting in the same damage as that of the average flight ( according to an adopted S-N curve and the linear fatigue damage accumulation model ( Miner's Rule ) ). Some other details of damage estimation are by Svirsky et al. ( 1988 ). Dashed lines ( "TWIST curves" ) stand for design curves got from a standardized load sequence TWIST ( De Jonge et al., 1973 ). Dash-and-dot lines are for TWIST where loads greater than 2.15 times the horizontal flight mean stress  $\sigma_m$  are replaced by  $2.15 \cdot \sigma_m$  ( "truncated TWIST curves" ). It may be mentioned here that for joints, the loads with the smallest amplitude were almost excluded. Solid lines stand for reference S-N curves obtained at regular loading ( "regular curves" ). Numbers in Figures 1 - 2 stand for the curve slopes in log-log coordinates. It is clear that S-N life curves obtained at regular loading sometimes does not give accurate representation about material/structure properties at service loading. Design fatigue life curves for various spectra are different, so equivalence of full-scale test load sequence to service loads must be provided for the wide range of S-N curve slopes; for example, use made be of methods proposed by Raikher ( 1970 ).

The results of test data processing by using the design curves are shown in Figures 3 and 4 . Results from various modifications to TWIST program ( truncations of high loads down to  $2.15 \cdot \sigma_m$ , omission of cycles with small amplitudes, variation of minimum stress at ground loads  $\sigma_{min}$  from 0 down to  $-0.877 \cdot \sigma_m$  ) were processed.

It is clear that the use of design life curves gives the best fit for test data which are then used to derive design life curves. For free hole specimens, maximum deviation of accumulated fatigue damage sum decreased from 2.04 ( for a regular curve ) to 1.54, and for joints, from 3.73 to 1.40. The usage of design life curves for other typical service load sequence in some cases gives a deviation much higher than that at the usage of regular S-N curves. The highest deviations were for joints estimated by using the design life curve for standard service load sequence ( de Jonge et al., 1973 ) ( see Fig. 4a ). At truncation of maximum amplitude from 2.6 to 2.15 times the horizontal flight mean stress the ratio is equal to 20 ( non-conservative ), at the same truncation and with minimum stress of ground loads lowered to  $-0.877 \cdot \sigma_m$ , the ratio is as great as 92 ( non-conservative ). Thus the usage of standard service load sequence with rare high stresses which are not characteristic of service loads of some transport aircraft ( according to above mentioned requirements to S+ ) may lead to non-conservative fatigue life estimates. It is true not only for design curves, but for relative Miner's Rule and for fatigue damage equivalence definition of various load sequences.

Local elasto-plastic stress-strain approach gives the possibility to widen the range of validity of a design fatigue life curve ( as shown by Svirsky et al. ( 1988 ) for free hole specimens ) taking into account the effect of rare high stresses which are responsible for most non-linear effects in damage summation.

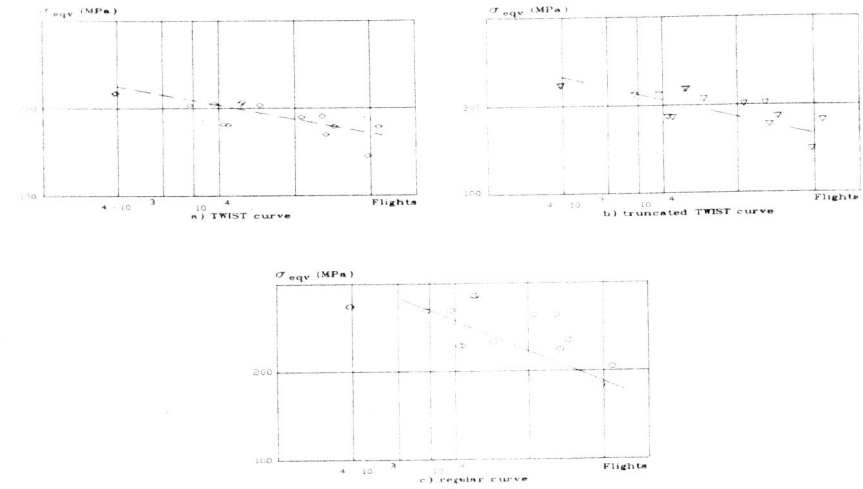


Fig. 3 Free hole specimen test data processed by using various fatigue life curves shown on Fig.1,a

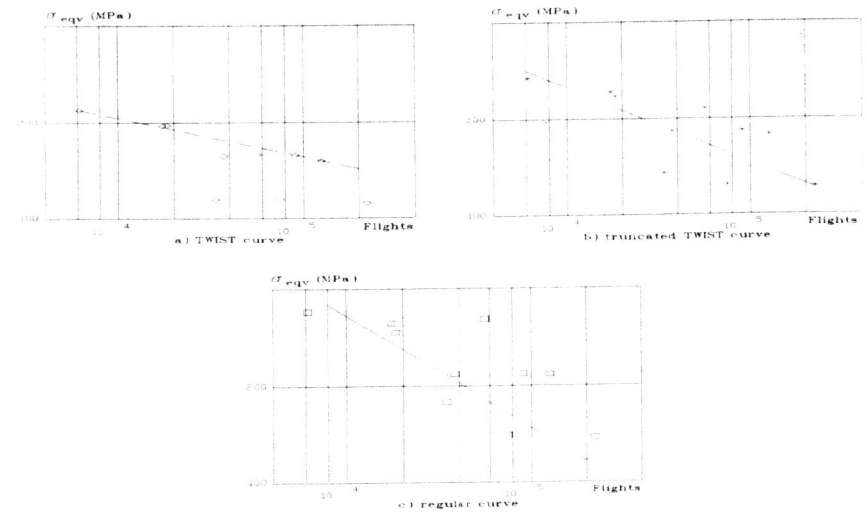


Fig. 4 Joint specimen test data processed by using various fatigue life curves

## MULTIAXIAL LOADING

At random multiaxial loading there is one very difficult question: what is the fatigue cycle or reversal. Our experiments on tubular specimens have shown fatigue failures at constant modules of principal stresses and rotation of their directions at biaxial plain-stress state. This means that the defining of the equivalent uniaxial stress on the basis of modules of principal stresses ( the stress intensity, the maximum shear stress etc. ) may give qualitatively wrong results, i. e. static loading instead of cyclic loading. There are two cases of random multiaxial loading where this question may be solved by reduction to a uniaxial case without additional hypotheses, i. e. a proportional stress state and an approximately uniaxial local stress state. At a proportional stress state one can use any stress component for rainflow analysis. The important cases of the proportional stress state are many versions of plane-strain state and single-component external loading, for example, pressurization of a fuselage with small bending and torsion moments. An approximately uniaxial local stress state is characteristic of many aircraft elements critical from fatigue standpoint ( for example, holes in sheets ). In this case, for rainflow analysis, the uniaxial local stress  $\sigma_{loc} = f ( X_1, X_2, \dots, X_n )$  is used, where  $X_i$  is the  $i$ -th component of nominal stresses or loads,  $f$  is a function depending on geometry. If a failure point is not known, there is need to make such analysis for all suspicious points. If there is a set of points on the stress concentrator contour with approximately equal damages, we must account for multiplicity of sites of probable failure according to Raikher et al. (1982), when damage is estimated by the S-N curve got at uniaxial loading. The number of contour points to be analyzed may be reduced by the use of parabolic approximation of damage along the contour. For example, at equal intervals between contour points  $x_1, x_2, x_3$  and damages in these points  $D_1, D_2, D_3$ , an abscissa of a point with maximum damage  $D_{max}$  is equal to

$$x_{max} = x_2 + \frac{(D_3 - D_1) \cdot d}{2 \cdot (2D_2 - D_1 - D_3)}$$

and a damage

$$D_{max} = D_2 + \frac{(D_3 - D_1)^2}{8 \cdot (2D_2 - D_1 - D_3)}$$

where  $d = x_2 - x_1 = x_3 - x_2$  is the distance between points.

Similar but more complicated formulae may be derived for arbitrary spaced points.

## CONCLUSIONS

Design fatigue life curves obtained at typical service load sequences give the possibility to use some simple linear models of fatigue damage accumulation and, at the same time, provide fair accuracy.

A "feedback rainflow matrix" is a very useful method of random test data presentation, which helps improving reliability and accuracy of random test data, accounting for a changing of material deformation properties and obtaining design fatigue life curves.

The problem of inclusion of rare peaks and troughs into typical service load sequences is recommended to be solved as the problem of absence of a favourable overload and presence of an unfavourable overload in service life with an adopted probability.

Equivalence of full-scale test load sequence to service loads must be provided in the wide range of slopes of S-N curves because the slopes of these curves are dependent on a particular service load spectrum.

At multiaxial random loading, a problem of fatigue cycle definition can be solved by using local stress approach or by considering a proportional loading case for the most critical elements of aircraft structures. The number of points to be examined may be reduced by application of interpolation.

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